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Quantity vs. Quality and Exclusion by Two-Sided Platforms

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Abstract

This paper provides a simple model of two-sided platforms, in which one side (W) values not just the *quantity* (i.e. number) of users on the other side (M), but also their *average quality* in some dimension. In this context, platforms might find it profitable to exclude low-quality users on side M, even though some would be willing to pay the platform access prices.

Platforms are more likely to engage in exclusion of low-quality M users when W users place more value on the average quality and less value on the total quantity on side M. Exclusion incentives also depend on the proportion of high-quality users in the overall M population and on their cost advantage in joining the platform, relative to low-quality M users. The net effect of these two factors is ambiguous: it generally depends on whether they have a stronger impact on the gains from exclusion (higher average quality) or on its costs (lower quantity).

Keywords: two-sided platforms, exclusion, quality and quantity, indirect network effects.
JEL Classifications: L1, L2, L8

1 Introduction

An important part of many real-world two-sided platform strategies are non-price "governance rules," which regulate access *to* and transactions *on* the platforms (cf. Boudreau and Hagiu (2009)). One of the most common two-sided platform governance rules is the restriction of access on at least one side, resulting in the exclusion of some customers who would otherwise be willing to pay the plaform's access and/or transaction fees. For example: videogame console manufacturers such as Microsoft, Sony and Nintendo restrict access to a select set of game developers and exclude many others (by including security chips in their consoles), even though the latter would also be willing to pay the per game royalties levied by the manufacturers[1]; some romantic matchmaking sites like eHarmony carefully screen and reject a sizeable fraction of applicants who would be willing to

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[1]For a detailed analysis of videogame platform strategies, see chapter 5 in Evans Hagiu and Schmalensee (2006) and Hagiu and Halaburda (2009).

pay their membership fees (cf. Piskorski, Halaburda and Smith (2008)); Apple routinely excludes certain application developers from its highly popular iPhone store; etc.

The economics and strategy literature on two-sided markets to date has devoted most of its attention to two-sided pricing strategies (e.g. Armstrong (2006), Caillaud and Jullien (2003), Parker and Van Alstyne (2005), Rochet and Tirole (2003) and (2006)) and although some recent papers have started to tackle certain design issues (cf. Hagiu and Jullien (2009), Parker and Van Alstyne (2008)), there has been virtually no formal work on two-sided platform governance rules and the factors that drive two-sided platforms to restrict access beyond what they can achieve through pricing alone.

This paper aims to start filling this gap: it builds a simple model formalizing profit-maximizing two-sided platforms' choice of exclusion policies and explores how several factors affect this choice. First and most fundamentally, two-sided platforms are more likely to restrict access on one side - denoted M - when users on the other side - denoted W - place more value on some *quality* attribute of M users relative to their *quantity* (or number). By contrast, most of the existing models of two-sided platforms assume that indirect network effects are exclusively captured by a preference for quantity. It is worth emphasizing that absent the preference of at least one side for quality on the other side, it would never be in the platforms' interest to exclude customers who are willing to pay their prices (access or transaction fees).

Second, platforms' exclusion incentives also depend on the proportion of high-quality users in the overall M population and on their cost advantage in joining the platform, relative to low-quality M users. The net effect of these two factors is ambiguous: it generally depends on whether they have a stronger impact on the gains from exclusion (higher average quality) or on its costs (lower quantity). The broader implication is that the correlation between quality and willingness to pay for (or cost of) joining the platform on the side whose quality matters is a key factor affecting the platform's incentives to exclude, but its effect is non-monotonic.

The quality vs. quantity tradeoff emphasized in my model is related to the "lemons market failure" first studied by Akerlof (1970). The key difference is that here a profit-maximizing two-sided platform plays a role similar to that of a "public authority" (cf. Boudreau and Hagiu (2009)) imposing a form of non-price regulation (restriction of access) in order to reduce the negative impact of low quality users on one side of the market. Since indirect network effects are determined by a combination of quality and quantity, the two-sided platform has an incentive to use an additional instrument other than its prices on the two sides in order to achieve the "right mix".

I use a simple model of two-sided platforms, with linear demands and access prices on both sides (cf. Armstrong (2006)). The novelty is the introduction of a quality parameter on side M that side W values positively. In this context, exclusion by the two-sided platform takes the natural form of refusing access to all M users of quality below a certain threshold - even though some of them would be willing to pay the price of access. This modeling of exclusion is similar to the minimum quality standards studied by Leland (1979), who uses a version of Akerlof (1970)'s model with a continuum of quality types. In that paper however, the quality standards are imposed by a public regulator, whereas here they are set by a profit-maximizing two-sided platform *in addition* to its prices.

To the best of my knowledge, the only papers in the two-sided market literature which study profit-maximizing two-sided platforms in contexts in which the quality of the two sides matters are Damiano and Li (2007) and (2008). The key difference is that in their model the quality of users is unobservable to the platforms and the latter can only use prices in order to sort quality (there is no non-price exclusion), whereas in my model average quality does not depend on the platform's prices, but only on its exclusion policy.

The remainder of the paper is organized as follows: the next section focuses on the case of a monopoly two-sided platform's choices of prices and exclusion level; section 3 studies the exclusion levels chosen by two competing two-sided platforms in the symmetric equilibrium; section 4 concludes.

2 Exclusion by a platform monopolist

2.1 Model set up

To fix ideas, the two sides of the market are denoted by M and W. As is standard in two-sided market models, I assume that the utility derived by M users from joining the platform is increasing in the *number* of W users who join, and viceversa. This captures the indirect network effects based on *quantity* - i.e. number of users on the other side of the market. The novel ingredient of my model is the introduction of a preference for the *quality* of side M by side W: the utility derived by W users from joining the platform is increasing not only in the number of M users but also in the *average quality* of the M users who join the platform.

Specifically, the respective utilities of M users and W users from joining the platform when there are N_W users on side W and N_M users on side M are given by:

$$U_M(\theta_M, q) = \alpha_M N_W - P_M - \theta_M c(q)$$

$$U_W(\theta_W) = V_W(\overline{q_M}) + \alpha_W(\overline{q_M}) N_M - P_W - \theta_W$$

where:

- θ_M and θ_W are horizontal differentiation parameters, both uniformly distributed on $[0, 1]$
- q is the "quality" of an individual M user from the perspective of W users, distributed *independently of* θ_M, with cdf $F(.)$ and density $f(.)$ over $[0, +\infty[$, such that the average quality is finite: $\int_0^\infty qf(q)\,dq < \infty$
- $\theta_M c(q)$ and θ_W are to be interpreted as the opportunity costs of joining the platform for the two types of users
- $\overline{q_M}$ is the *average quality* of M users who join the platform (see below for the derivation of its expression)
- $\alpha_M > 0$ and $\alpha_W(\overline{q_M}) > 0$ are the indirect network effect parameters on the two sides, with $\alpha'_W(\overline{q_M}) \equiv \frac{d\alpha'_W}{d\overline{q_M}} \geq 0$
- $V_W(\overline{q_M})$ is the standalone utility derived by W users from joining the platform: $V'_W(\overline{q_M}) \equiv \frac{dV'_W}{d\overline{q_M}} \geq 0$
- P_M and P_W are the access prices charged by the platform to the two sides

Thus, M users are differentiated in two dimensions, θ_M and q, independently distributed of each other, and their opportunity cost of joining the platform - $\theta_M c(q)$ - depends on both characteristics.

In particular, the function $c(.)$ can be increasing or decreasing. When $c(.)$ is decreasing (increasing), M users of high quality have a higher (lower) opportunity cost of using the platform's service than M users of low quality.

W users' preference for quality of M users is captured by the fact that both $\alpha_W(\overline{q_M})$ and $V_W(\overline{q_M})$ are non-decreasing functions of $\overline{q_M}$.

An user on either side joins the platform if her expected utility is non-negative. If the platform only uses prices (P_W, P_M) to regulate entry on both sides of the market, then the average quality of the M users who join the platform is:

$$\overline{q_M} = \frac{\int_0^\infty \frac{\alpha_M N_W - P_M}{c(q)} f(q)\, q dq}{\int_0^\infty \frac{\alpha_M N_W - P_M}{c(q)} f(q)\, dq} = \frac{\int_0^\infty \frac{f(q)q}{c(q)} dq}{\int_0^\infty \frac{f(q)}{c(q)} dq}$$

Because W users care about the average quality of M users however, the platform may find it profitable to exclude a positive measure of M users, even though they would be willing to pay the price of admission P_M. In particular, if the platform decides to exclude some M users, it will always start by excluding the lowest quality ones. Thus, in addition to prices (P_W, P_M), I also allow the platform to set L_M, the quality threshold of admission on side M, such that only M users of quality $q \geq L_M$ are allowed to join.

Consequently, the effective participations on both sides of the market (N_W, N_M) are given by:

$$N_W = V_W(\overline{q_M}(L_M)) + \alpha_W(\overline{q_M}(L_M)) N_M - P_W \tag{1}$$

$$N_M = \lambda_M(L_M)(\alpha_M N_W - P_M) \tag{2}$$

where:

$$\lambda_M(L_M) = \int_{L_M}^\infty \frac{f(q)}{c(q)} dq$$

is the fraction of M users who are actually allowed to join the platform among those *willing* to join given P_M and N_W and:

$$\overline{q_M}(L_M) = E[q \mid L_M] = \frac{\int_{L_M}^\infty \frac{qf(q)}{c(q)} dq}{\lambda_M(L_M)}$$

is the average quality of M users conditional on the platform's exclusion policy (L_M).

Naturally, $\lambda_M(L_M)$ is decreasing and $\overline{q_M}(L_M)$ is increasing in L_M. Note that the "cost function" $c(q)$ affects the fraction of M users excluded λ_M and the average quality of participating M users $\overline{q}_M$ for any given level of exclusion L_M in a straightforward way, by placing different "weights" on the density function of M users' quality.

I assume for simplicity that the platform has zero marginal costs of serving users on both sides, therefore its profits are:

$$\Pi^P = P_M N_M + P_W N_W$$

which it maximizes over (P_M, P_W, L_M). The focus of the paper is on determining the optimal level of exclusion chosen by the platform, L_M^*.

Note that this formulation assumes away implementation costs of the exclusion mechanism - e.g. costs of screening quality or restricting access through technological locks - and focuses instead on the inherent economic tradeoffs which are independent of such costs. Adding implementation costs would have the unsurprising effect of shifting the balance towards less exclusion.

Before proceeding, several observations on the modeling set-up are in order. First, in my model the quality of M users is assumed to only affect their opportunity cost of joining the platform. In reality, it may also affect the utility they derive from their interactions with W users on the platform, i.e. α_M might also depend on q. The reason for this assumption is simplification: most importantly, it implies that the average quality $\overline{q_M}$ only depends on L_M and not on the prices P_M and P_W. This eliminates the role played by prices in sorting quality, which is the focus of Damiano and Li (2007) and (2008). Instead, I wish to isolate the effects of various exogenous factors on the platform's choice of L_M. Thus, $c(.)$ is best interpreted as capturing the *net* effect of quality on M users' payoffs from joining the platform. For instance, in the case of software platforms like iPhone or Facebook (side W being users and side M being third-party application developers), $c(q)$ is *decreasing* in q if and only if the revenues of an application developer from selling his application on the platform *net* of his development and porting costs are *increasing* in q.

Second, users on both sides of the market can observe the platform's prices and exclusion policy on side M, so that everyone can correctly infer the average equilibrium quality of M users who adopt the platform.

Third, the platform is assumed to observe the quality of each M user prior to admitting them and this quality is set in stone, i.e. I am not studying here incentives to invest in quality enhancements by the users (before or after the platform sets prices and access policies). Also, note that in this paper the notion of "quality" of M users refers to any measurable characteristic/attribute that increases the utility derived by W users. This can therefore be different (more general) than "objective" quality. For example, the professional social network LinkedIn approves third-party applications which are most relevant to professional social networking and may turn down what some might regard as "high-quality" applications, if they do not fit this profile.

Fourth, the main reason for which I chose to focus on the case in which only W users care about the quality of M users, whereas M users care solely about the quantity of M users, is simplicity (and has nothing to do with the labeling of the two sides as M (men) and W (women)). In the appendix, for completeness, I derive the general expression of platform profits when each side cares about both quality and quantity on the other side and the platform can exclude users on both sides. Working with this more general expression would be more cumbersome but the main conclusions would remain unchanged.

Using (1) and (2) to switch price and demand variables, Ican optimize the platform's profits over (N_M, N_W, L_M):

$$\max_{N_M,N_W,L_M} \left\{ \begin{array}{c} N_W \left(V_W \left(\overline{q_M}\left(L_M\right)\right) + \alpha_W \left(\overline{q_M}\left(L_M\right)\right) N_M - N_W\right) \\ +N_M \left(\alpha_M N_W - \frac{N_M}{\lambda_M(L_M)}\right) \end{array} \right\}$$

Taking the first-order conditions in N_W and N_M respectively, Iobtain (omitting functional arguments for simplicity):

$$-\left(\alpha_M + \alpha_W\right) N_M + 2N_W = V_W$$

$$\frac{2}{\lambda_M} N_M - \left(\alpha_M + \alpha_W\right) N_W = 0$$

The second order condition for the maximization problem to be well-defined and have an interior solution requires:

$$\overline{\alpha}\left(\overline{q_M}\left(L_M\right)\right)^2 < 1 \quad \text{for all } L_M$$

which I assume holds throughout the paper.

Solving the above two equations for (N_W, N_M), Iobtain:

$$N_M = \lambda_M \frac{\overline{\alpha} V_W}{2\left(1 - \lambda_M \lambda_W \overline{\alpha}^2\right)}$$

$$N_W = \frac{V_W}{2\left(1 - \lambda_M \lambda_W \overline{\alpha}^2\right)}$$

where:

$$\overline{\alpha} \equiv \overline{\alpha}\left(\overline{q_M}\left(L_M\right)\right) = \frac{1}{2}\left[\alpha_M + \alpha_W\left(\overline{q_M}\left(L_M\right)\right)\right]$$

is the average indirect network effect parameter.

I can then derive the expressions of optimal prices given L_M:

$$P_W = \frac{V_W\left(1 - \lambda_M \overline{\alpha} \alpha_M\right) + V_M \lambda_M\left(\alpha_W - \overline{\alpha}\right)}{2\left(1 - \lambda_M \overline{\alpha}^2\right)}$$

$$P_M = \frac{V_M\left(1 - \lambda_M \overline{\alpha} \alpha_W\right) + V_W\left(\alpha_M - \overline{\alpha}\right)}{2\left(1 - \lambda_M \overline{\alpha}^2\right)}$$

Note that P_i is higher and P_j is lower whenever α_i is higher and is α_j lower - holding $\overline{\alpha}$ constant. This is a familiar result in the two-sided market literature: profit maximizing platforms charge more to the side which values the participation of the other side more.

Finally, I obtain:

Proposition 1 *The expression of platform profits as a function of L_M, the level of exclusion on side M, is:*

$$\Pi^P\left(L_M\right) = \frac{V_W^2\left(\overline{q_M}\left(L_M\right)\right)}{4\left[1 - \lambda_M\left(L_M\right) \times \overline{\alpha}^2\left(\overline{q_M}\left(L_M\right)\right)\right]} \tag{3}$$

■

Using the envelope theorem, the optimal level of exclusion L_M is given by maximizing expression (3) with respect to L_M. Note that this expression is increasing in λ_M, V_W, α_M and α_W (through $\overline{\alpha}$).

There is therefore a clear tradeoff between quality and quantity involved in choosing the optimal exclusion level L_M. Indeed, an increase in L_M (the quality threshold for admission) has two opposite effects on profits: a negative effect through a reduction in λ_M - i.e. by decreasing quantity on side M - and a positive effect through an increase in the average quality $\overline{q_M}(L_M)$, which in turn increases participation on side W (V_W and α_W are non-decreasing in $\overline{q_M}$).

In the appendix Ishow that this tradeoff generalizes to the case in which each side cares about average quality on the other side and the platform can exclude users on both sides.

2.2 Quality vs. quantity

In order to reach a better understanding of how various factors affect the platform's choice of exclusion L_M and thereby its choice of quality vs. quantity on side M, the model needs to be specified a bit further.

In particular, I assume that the distribution of quality types on side M is binary: a fraction λ of M users are of quality $q = 1$ (high) and a fraction $(1 - \lambda)$ are of quality $q = 0$ (low). Let then:

$$c(q) = \begin{cases} c & \text{if} \quad q = 0 \\ \\ \frac{c}{\beta} & \text{if} \quad q = 1 \end{cases}$$

where $c, \beta > 0$ and β is to be interpreted as a measure of high quality M users' cost advantage of joining the platform relative to low quality M users. In particular, if $\beta > 1$ then high quality M users have a lower opportunity cost of participating, all other things being equal (in particular, holding constant the horizontal differentiation parameter θ_M). On the other hand, if $\beta < 1$ then high quality users have a higher opportunity cost. This is the case if side M represents developers (e.g. of videogames or software applications) and making a better quality product available on the platform is more costly than supplying a lower quality one.

Since there are only two quality levels of users M possible - 0 and 1 -, the platform's choice of exclusion is limited to two options: $L_M = 0$ which means all M users who are willing to pay are allowed to access the platform and $L_M = 1$, which means the platform excludes all M users of low quality ($q = 0$).

It is then straightforward to derive the expressions of λ_M and $\overline{q_M}$ under the two governance regimes ($L_M = 0$ - no exclusion; $L_M = 1$ - access restricted to high-quality users):

$$\lambda_M \left(L_M, \beta\right) = \begin{cases} \frac{\lambda\beta+(1-\lambda)}{c} \equiv \lambda_{NE} & \text{if} \quad L_M = 0 \\ \\ \frac{\lambda\beta}{c} \equiv \lambda_E & \text{if} \quad L_M = 1 \end{cases}$$

$$\overline{q_M} \left(L_M, \beta\right) = \begin{cases} \frac{\lambda\beta}{\lambda\beta+(1-\lambda)} \equiv \overline{q_{NE}} & \text{if} \quad L_M = 0 \\ \\ 1 & \text{if} \quad L_M = 1 \end{cases}$$

Note that:

- λ_{NE} and λ_E are both increasing in β
- λ_{NE} is increasing in λ if and only if $\beta > 1$, i.e. if and only if high-quality M users have a competitive advantage over low-quality M users
- $\overline{q_{NE}}$ is increasing in both β and λ as expected: when β increases, the proportion of high quality users who join the platform under the no exclusion regime ($L_M = 0$) increases, which results in higher average quality. Similarly for λ.

Using expression (3) and comparing the two governance regimes, Iconclude that the platform chooses to exclude low quality M users if and only if:

$$\frac{V_W^2\left(\overline{q_{NE}}\right)}{V_W^2\left(1\right)} \leq \frac{1 - \lambda_{NE} \times \overline{\alpha}^2\left(\overline{q_{NE}}\right)}{1 - \lambda_E \times \overline{\alpha}^2\left(1\right)} \tag{4}$$

Note that this condition holds trivially (with equality) when $\lambda = 1$, i.e. when there are no low quality M users.

In what follows, I analyze two polar cases: (i) α_W is constant (which implies $\overline{\alpha}$ is constant) and therefore $\overline{q_M}$ only afects V_W; (ii) V_W is constant and therefore $\overline{q_M}$ only afects α_W and hence $\overline{\alpha}$. Both of these cases are empirically relevant (they represent different utility micro-foundations). For my purposes, it is remarkable that they lead to very different results in certain aspects that I discuss below.

2.2.1 Quality only affects V_W

In this case, condition (4) can be written as:

$$\left[\frac{V_W\left(\overline{q_{NE}}\right)}{V_W\left(1\right)}\right]^2 \leq \frac{1-\lambda_{NE}\overline{\alpha}^2}{1-\lambda_E\overline{\alpha}^2}$$

The right hand side of the above inequality is decreasing in β and decreasing in $\overline{\alpha}$, whereas the left-hand side is increasing in β and constant in $\overline{\alpha}$. The following proposition follows immediately:

Proposition 2 *When the average quality of M users only affects the standalone utility V_W of W users, the platform is more likely to exclude low quality M users when β decreases and/or $\overline{\alpha}$ decreases.*

If $V_W\left(\overline{q_M}\right) = \sqrt{\overline{q_M}}$ then the platform excludes low quality M users if and only if:

$$2\lambda\beta + (1-\lambda) \leq \frac{c}{\overline{\alpha}^2} \tag{5}$$

■

First, note the highly intuitive but important result that exclusion is more likely to be optimal when $\overline{\alpha}$ - a measure of W users' preference for quantity in this case - *decreases.* On the other hand, the preference for quality is captured by the term $\frac{V_W(\overline{q_{NE}})}{V_W(1)}$. To see this effect clearly, suppose that $V_W\left(\overline{q}\right) = V_0 + V\overline{q}$: then V is a measure of W users' preference for quality. Condition (4) then becomes:

$$\left[\frac{V_0}{V_0+V} + \frac{V}{V_0+V}\overline{q_{NE}}\right]^2 \leq \frac{1-\lambda_{NE}\overline{\alpha}^2}{1-\lambda_E\overline{\alpha}^2}$$

which means exclusion is more likely when V *increases* and V_0 *decreases.*

Together, these two effects capture the fundamental quality vs. quantity tradeoff involved in choosing the optimal level of exclusion.

Turning attention to condition (5), note that the effect of λ depends on whether $\beta <> \frac{1}{2}$. If $\beta > \frac{1}{2}$ then exclusion is optimal for low λ, i.e. when the proportion of high-quality M users in the overall population is sufficiently low. Conversely, if $\beta < \frac{1}{2}$ then exclusion is optimal when the proportion of high-quality M users is sufficiently high. The interpretation of this effect and that of β are discussed in section 2.2.3 below.

2.2.2 Quality only affects α_W

If V_W does not depend on $\overline{q_M}$ then condition (4) becomes:

$$\left[\frac{\overline{\alpha}\left(\overline{q_{NE}}\right)}{\overline{\alpha}\left(1\right)}\right]^2 \leq \frac{\lambda_E}{\lambda_{NE}}$$

Both sides of this inequality are increasing in β and λ, therefore, a priori, both of these parameters have an ambiguous effect on the likelihood of exclusion.

Proposition 3 *Suppose $\overline{\alpha}\left(\overline{q_M}\right) = \alpha_0 + \alpha\overline{q_M}$.*[2] *Then exclusion of low quality M users is optimal for the platform if and only if:*

$$\frac{\lambda\beta}{\lambda\beta + (1-\lambda)} \geq \left(\frac{\alpha_0}{\alpha}\right)^2 \tag{6}$$

■

Thus, exclusion is more likely when α - the preference for quality here - is higher and α_0 - the preference for quantity *independent of* quality - is lower. This confirms the intuition from the previous case.

By contrast, the effects of β and λ are now quite different: exclusion is more likely when both β and λ are higher.

2.2.3 Interpretation

First, it should be clear that the effects of preferences for quality vs. quantity are robust and generalize to any utility formulation. Exclusion will always be more likely when W users place more value on quality and less value on quantity.

Second, comparing Propositions 2 and 3, things are very different when it comes to the effects of λ and β.

[2] This is the case if $\alpha_W\left(\overline{q_M}\right) = \alpha_W^0 + 2\alpha\overline{q_M}$ and we denote $\alpha_0 \equiv \frac{\alpha_W^0 + \alpha_M}{2}$.

To isolate the effect of λ, suppose $\beta = 1$, i.e. the quality of M users has no bearing on their opportunity cost of joining the platform. In this case, the conditions (5) and (6) for exclusion to be profit-maximizing become:

$$\lambda \leq \frac{c}{\overline{\alpha}^2}$$

when $V_W\left(\overline{q_M}\right) = \sqrt{\overline{q_M}}$ and α_W does not depend on $\overline{q_M}$ and:

$$\lambda \geq \left(\frac{\alpha_0}{\alpha}\right)^2$$

when $\overline{\alpha}\left(\overline{q_M}\right) = \alpha_0 + \alpha\overline{q_M}$ and V_W does not depend on $\overline{q_M}$.

Thus, in the first case the exclusion regime is optimal when the proportion of high quality M users (λ) is sufficiently low, whereas in the second case the exclusion regime is optimal when λ is sufficiently high.

The reason λ can have opposite effects on the platform's exclusion policy depending on the formulation of user preferences is that there are two channels through which λ affects the change in platform profits when going from a no-exclusion ($L_M = 0$) to a regime with exclusion ($L_M = 1$). On the one hand - and perhaps most intuitively - a higher λ makes exclusion less attractive by reducing its benefits (smaller increase in average quality) relative to the no-exclusion regime. On the other hand however, a higher λ also decreases the costs of exclusion by reducing the loss of quantity $(1 - \lambda)$ on side M, which tends to make exclusion relatively more attractive. The net result of these two mechanisms can go either way in general. In my model, when quality does not affect the indirect network effect $\overline{\alpha}$ but only the standalone utility term V_W, the first mechanism dominates. Conversely, when V_W is constant and $\overline{\alpha}$ is linear in $\overline{q_M}$, the second mechanism prevails.

With this in mind, the interpretation of the effects of β - the cost advantage of high-quality M agents - becomes straightforward, since it relies on a very similar tradeoff. A higher β increases the fraction of high-quality M users joining the platform for any given pair of prices (P_M, P_W) and therefore increases the average quality on side M. This means that the potential gains from exclusion are smaller (there is less to gain in terms of average quality) but so are the costs (exclusion would entail a less onerous quantity sacrifice since there are fewer M users of low quality willing to participate to begin with. Again, the first effect dominates in the version of the model with constant $\overline{\alpha}$, whereas the second mechanism prevails in the version with constant V_W.

In the appendix, I show that all the results and effects discussed above also apply to the (somewhat simpler) case of a one-sided platform with direct network effects.

3 Exclusion with competing platforms

I now turn to the case of platform competition. This section shows that the results derived in the monopoly platform case also hold in the symmetric equilibrium of a platform duopoly setting.

In order to keep things as simple as possible, I assume competition only occurs on side W, while M users are allowed to multihome between the two platforms. In particular, the two platforms are differentiated a la Hotelling on side W, such that the utility derived by user $x \in [0,1]$ adopting platform $i \in \{1,2\}$ is:

$$U_W(x,i) = V_W^i\left(\overline{q}_M^i\right) + \alpha_W^i\left(\overline{q}_M^i\right) N_M^i - P_W^i - t\left[(2-i)x + (i-1)(1-x)\right]$$

where $\overline{q}_M^i$ is the average quality of M users on platform i.

The utility of an user of type M, indexed by θ_M and quality q_M from joining platform i is:

$$U_M(\theta_M, q_M, i) = \alpha_M N_W^i - P_M^i - \theta_M c(q_M, \beta)$$

where θ_M is uniformly distributed over $[0,1]$ whereas q_M is distributed with c.d.f. F and density f. The parameter β is assumed to decrease the cost of every quality type, but more so for higher quality types:

$$\frac{\partial c(q,\beta)}{\partial \beta} < 0 \text{ and } \frac{\partial^2 c(q,\beta)}{\partial q \partial \beta} < 0 \text{ for all } q \text{ and } \beta$$

Thus, the effective participations on the two sides of plaform i are determined by:

$$N_W^i = \frac{1}{2} + \frac{V_W\left(\overline{q}_M^i\right) - V_W\left(\overline{q}_M^j\right) + \alpha_W\left(\overline{q}_M^i\right) N_M^i - \alpha_W\left(\overline{q}_M^j\right) N_M^j + P_W^j - P_W^i}{2t} \tag{7}$$

$$N_M^i = \lambda_M^i\left(\alpha_M N_W^i - P_M^i\right) \tag{8}$$

where $V_W(.)$ and $\alpha_W(.)$ are non-decreasing functions and:

$$\lambda_M^i \equiv \lambda_M\left(L_M^i\right) = \int_{L_M^i}^{\infty} \frac{f(q_M)\, dq_M}{c(q_M, \beta)}$$

$$\overline{q}_M^i \equiv \overline{q}_M\left(L_M^i\right) = \frac{\int_{L_M^i}^{\infty} \frac{q_M f(q_M) dq_M}{c(q_M,\beta)}}{\lambda_M\left(L_M^i\right)}$$

Note that $\frac{\partial^2 c(q,\beta)}{\partial q \partial \beta} < 0$ for all q and β implies:

$$\frac{\partial \overline{q}_M(L_M)}{\partial \beta} > 0 \text{ for all } L_M$$

which is natural: when the cost advantage of higher quality M users increases, the composition of participating M users for any given L_M shifts towards higher quality and hence the average quality of participating M users increases.

The two platforms simultaneously choose their respective prices on both sides and their respective exclusion levels L_M^i on side M. Assuming 0 marginal costs, platform i's profits are:

$$\begin{aligned} \Pi_P^i &= P_W^i N_W^i + P_M^i N_M^i \\ &= \left(P_W^i + P_M^i \lambda_M^i \alpha_M\right) N_W^i - \lambda_M^i \left(P_M^i\right)^2 \end{aligned} \quad (9)$$

In all that follows I will restrict attention to the symmetric equilibrium, in which:

$$P_W^1 = P_W^1 = P_W^* \ ; \ P_M^1 = P_M^1 = P_M^* \ ; \ L_M^1 = L_M^1 = L_M^*$$

Finally, I assume the necessary second order conditions for well-behaved maximization problems are satisfied. In particular, this requires:

$$t > \alpha_M \alpha_W\left(\overline{q}_M\right) \text{ for all } \overline{q}_M$$

As in the monopoly platform analysis, I distinguish between two polar cases: (i) α_W is constant (which implies $\overline{\alpha}$ is constant) and therefore $\overline{q}_M$ only afects V_W; (ii) V_W is constant and therefore $\overline{q}_M$ only afects α_W and hence $\overline{\alpha}$.

3.1 Quality only affects V_W

In this case, $\alpha_W(\overline{q}_M) \equiv \alpha_W$ for all $\overline{q}_M$. Let:

$$\overline{\alpha} = \frac{\alpha_M + \alpha_W}{2}$$

The following proposition (proven in the appendix) defines the symmetric equilibrium:

Proposition 4 *When the average quality of M users only affects V_W, the symmetric equilibrium prices are $P_W^* = t - \frac{\alpha_M \lambda_M(L_M^*)}{4}(\alpha_M + 3\alpha_W)$ and $P_M^* = \frac{\alpha_M - \alpha_W}{4}$.*

The symmetric equilibrium level of exclusion L_M^* chosen by both platforms is the solution to:

$$\frac{d\lambda_M}{dL_M}(L_M^*) \times \frac{\overline{\alpha}^2}{2} + \frac{dV_W}{d\overline{q}_M}(\overline{q}_M(L_M^*)) \times \frac{d\overline{q}_M}{dL_M}(L_M^*) = 0 \tag{10}$$

■

The second order condition requires that the left hand side of (10) is decreasing in L_M^*, which leads to the following corollary:

Corollary *The symmetric equilibrium level of exclusion L_M^* defined by (10) is decreasing in $\overline{\alpha}$.*

If $V_W(.)$ is concave and $\frac{\partial^2 \overline{q}_M}{\partial L_M \partial \beta} \leq 0$ then L_M^* is decreasing in β.

■

Proof of Corollary: Since the left hand side of (10) is decreasing in L_M^* by concavity and decreasing in $\overline{\alpha}$ (recall $\frac{d\lambda_M}{dL_M}(L_M^*) = -\frac{f(L_M^*)}{c(L_M^*)} < 0$), the implicit function theorem implies that L_M^* is decreasing in $\overline{\alpha}$.

The derivative of the left hand side of (10) in β writes:

$$\frac{\partial^2 \lambda_M}{\partial L_M \partial \beta} \times \frac{\overline{\alpha}^2}{2} + \frac{d^2 V_W}{(d\overline{q}_M)^2} \frac{\partial \overline{q}_M}{\partial L_M} \frac{\partial \overline{q}_M}{\partial \beta} + \frac{dV_W}{d\overline{q}_M} \frac{\partial^2 \overline{q}_M}{\partial L_M \partial \beta}$$

and is negative because $\frac{\partial^2 \lambda_M}{\partial L_M \partial \beta} = -\frac{\partial\left(\frac{f(L_M^*)}{c(L_M^*, \beta)}\right)}{\partial \beta} < 0$; $\frac{d^2 V_W}{(d\overline{q}_M)^2} < 0$; $\frac{\partial^2 \overline{q}_M}{\partial L_M \partial \beta} \leq 0$; $\frac{\partial \overline{q}_M}{\partial L_M} > 0$; $\frac{\partial \overline{q}_M}{\partial \beta} > 0$; $\frac{dV_W}{d\overline{q}_M} > 0$.

■

The results in Proposition 4 and its corollary parallel those obtained for a monopoly platform in Proposition 2. In particular, the preference for quantity $\overline{\alpha}$ has the expected effect of decreasing the level of exclusion, while β - which is positively correlated with a cost advantage for higher quality M users - has a stronger impact on the gains from exclusion (higher average quality) than on its costs (lower quantity), so that an increase in β reduces the need for exclusion.

3.2 Quality only affects α_W

In this case $V_W(\overline{q}_M) \equiv V_W$ for all $\overline{q}_M$. Let:

$$\overline{\alpha}(\overline{q}_M) = \frac{\alpha_M + \alpha_W(\overline{q}_M)}{2}$$

The following proposition (proven in the appendix) defines the symmetric equilibrium:

Proposition 5 *When the average quality of M users only affects α_W, the symmetric equilibrium prices are $P_W^* = t - \frac{\alpha_M \lambda_M(L_M^*)}{4}[\alpha_M + 3\alpha_W(\overline{q}_M(L_M^*))]$ and $P_M^* = \frac{\alpha_M - \alpha_W(\overline{q}_M(L_M^*))}{4}$.*

The symmetric equilibrium level of exclusion L_M^* chosen by both platforms is the solution to:

$$L_M^* = \arg\max_{L_M} \left\{ [\overline{\alpha}(\overline{q}_M(L_M))]^2 \times \lambda_M(L_M) \right\} \tag{11}$$

■

The following corollary is easily derived:

Corollary *Suppose that $\overline{\alpha}(\overline{q}_M) = \alpha_0 + \alpha\overline{q}_M$; $f(q) = \phi e^{-\phi q}$; $c(q) = ce^{-\beta q}$ with $\phi > \max(0, \beta)$. Then $[\overline{\alpha}(\overline{q}_M(L_M))]^2 \times \lambda_M(L_M)$ is concave in L_M and is maximized by:*

$$L_M^* = \frac{1}{\phi - \beta} - \frac{\alpha_0}{\alpha}$$

which is the level of exclusion chosen by the two platforms in the symmetric equilibrium.

■

Proof of Corollary It is easily calculated that $\lambda_M(L_M) = \frac{C\phi}{\phi-\beta}e^{-(\phi-\beta)L_M}$ and $\overline{q}_M(L_M) = L_M + \frac{1}{\phi-\beta}$. Note that $\frac{\partial \overline{q}_M}{\partial L_M} > 0$; $\frac{\partial \overline{q}_M}{\partial \beta} > 0$ and $\frac{\partial^2 \overline{q}_M}{\partial L_M \partial \beta} = 0$.

■

The result in proposition 5 parallels the one in proposition 3 regarding the effect of β (although with very different modelling specifications).

4 Conclusion

I have done two things in this paper. First, I have provided a simple model capturing the incentives that two-sided platforms have to exclude some participants who would be willing to pay the price of admission. The need for exclusion (or enforcing minimum "quality" standards - cf. Leland (1979)) stems here from a fundamental tradeoff between the quality and the quantity of indirect network effects. As soon as at least one side of the market values a quality attribute of the other side (which may or may not be correlated with willingness-to-pay for or cost of joining the platform), the platform may find it optimal to sacrifice quantity to a certain degree in order to increase the average quality of agents on the second side.

Second, I have shown that platforms' incentives to exclude are determined by several important considerations. Users' preferences for quantity unambiguously reduce the incentives to exclude, while preferences for quality have the opposite effect. Meanwhile, the effects of the proportion of high-quality users relative to low-quality users and of the relative cost advantage of high-quality users are ambiguous. Their sign is determined by the interplay of two forces. On the one hand, an increase in either the proportion of high-quality users in the overall population or in the relative cost advantage of high-quality users reduces platforms' benefits from exclusion - the potential gain in average quality is smaller. On the other hand, the same increase also reduces platforms' costs of exclusion - the loss of quantity is also smaller.

Clearly, the current paper represents only an initial effort in exploring the various forces driving two-sided platforms' non-price governance rules. In particular, the need for exclusion in my model arises solely from the introduction of a preference for quality on one side of the market. Another important factor might be too much competition among agents on one side (e.g. among videogame developers for the same console). This suggests developing a model which would explicitly formalize competition between agents of different qualities on one side of the market and would analyze how the nature of competition affects platforms' incentives to exclude. Furthermore, two-sided platforms can use (and often do so in practice) more sophisticated governance rules than the simple imposition of minimum quality standards on one or both sides. For example, they can choose to rely on tiered quality certification systems in order to convey more or less precise information about the quality of agents on one side to agents on the other side. Such certification systems and other types of governance rules regulating information transmission among the two sides of a platform are promising avenues for future research.

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5 Appendix

Derivation of monopoly platform profits with exclusion on both sides

The most general formulation of our model with preferences for quantity and quality on both sides of the market is:

$$U_i(\theta_i, q_i) = V_i(\overline{q_j}) + \alpha_i(\overline{q_j}) N_j - P_i - \theta_i c_i(q_i)$$

for $i \neq j \in \{M, W\}$.

The platform sets threshold quality levels L_M and L_W such that the effective participations on the two sides of the market are given by:

$$N_W = \lambda_W (L_W) \left[V_W \left(\overline{q_M} (L_M) \right) + \alpha_W \left(\overline{q_M} (L_M) \right) N_M - P_W \right]$$

$$N_M = \lambda_M (L_M) \left[V_M \left(\overline{q_W} (L_W) \right) + \alpha_M \left(\overline{q_W} (L_W) \right) N_W - P_M \right]$$

where $\lambda_i (L_i) = \int_{L_i}^{\infty} \frac{f_i(q_i)}{c_i(q_i)} dq_i$ and $\overline{q_i} (L_i) = \frac{\int_{L_i}^{\infty} \frac{f_i(q_i) q_i}{c_i(q_i)} dq_i}{\lambda_i(L_i)}$.

Switching price and demand variables, the platform maximizes profits over (N_M, N_W, L_M, L_W) according to:

$$\max_{N_M, N_W, L_M, L_W} \left\{ \begin{array}{c} N_W \left(V_W \left(\overline{q_M} (L_M) \right) + \alpha_W \left(\overline{q_M} (L_M) \right) N_M - \frac{N_W}{\lambda_W(L_W)} \right) \\ + N_M \left(V_M \left(\overline{q}_W (L_W) \right) + \alpha_M \left(\overline{q}_W (L_W) \right) N_W - \frac{N_M}{\lambda_M(L_M)} \right) \end{array} \right\}$$

Taking the first-order conditions in N_W and N_M respectively (functional arguments are omitted for simplicity):

$$\begin{aligned} -\left(\alpha_M + \alpha_W \right) N_M + \frac{2}{\lambda_W} N_W &= V_W \\ \frac{2}{\lambda_M} N_M - \left(\alpha_M + \alpha_W \right) N_W &= V_M \end{aligned}$$

Solving for (N_W, N_M):

$$N_M = \frac{\lambda_M V_M + \lambda_W \lambda_M \overline{\alpha} V_W}{2 \left(1 - \lambda_M \lambda_W \overline{\alpha}^2 \right)}$$

$$N_W = \frac{\lambda_W V_W + \lambda_W \lambda_M \overline{\alpha} V_M}{2 \left(1 - \lambda_M \lambda_W \overline{\alpha}^2 \right)}$$

where $\overline{\alpha} = \frac{1}{2} \left(\alpha_M + \alpha_W \right)$.

It is then straightforward to determine the optimal prices for a given choice of (L_M, L_W):

$$P_W = \frac{V_W \left(1 - \lambda_M \lambda_W \overline{\alpha} \alpha_M \right) + V_M \lambda_M \left(\alpha_W - \overline{\alpha} \right)}{2 \left(1 - \lambda_M \lambda_W \overline{\alpha}^2 \right)}$$

$$P_M = \frac{V_M \left(1 - \lambda_W \lambda_M \overline{\alpha} \alpha_W \right) + V_W \lambda_W \left(\alpha_M - \overline{\alpha} \right)}{2 \left(1 - \lambda_M \lambda_W \overline{\alpha}^2 \right)}$$

Finally, the expression of platform profits as a function of the levels of exclusion on the two sides is:

$$\Pi^P\left(L_W, L_M\right) = \frac{V_W^2\lambda_W + V_M^2\lambda_M + 2\lambda_W\lambda_M\overline{\alpha}V_W V_M}{4\left(1-\lambda_M\lambda_W\overline{\alpha}^2\right)}$$

where $\lambda_i = \lambda_i\left(L_i\right)$, $V_i = V_i\left(\overline{q_j}\left(L_j\right)\right)$, $\overline{\alpha} = \frac{1}{2}\left[\alpha_M\left(\overline{q_W}\left(L_W\right)\right) + \alpha_W\left(\overline{q_M}\left(L_M\right)\right)\right]$, for $i, j \in \{W, M\}, i \neq j$.

The optimal levels of exclusion on both sides are then given by maximizing the expression of Π^P above with respect to L_W and L_M. Note that this expression is increasing in λ_M, λ_W, V_M, V_W, α_M and α_W (through $\overline{\alpha}$).

■

Exclusion by one-sided platforms with direct network effects

There is only one type of users, whose utility from joining the platform is given by:

$$U\left(\theta\right) = V\left(\overline{q}\left(L\right)\right) + \alpha\left(\overline{q}\left(L\right)\right)N - P - \theta c\left(q\right)$$

where θ is distributed uniformly on $[0,1]$. Again, quality q is distributed independently of θ with cdf. F and density f so that:

$$\overline{q}\left(L\right) = E\left[q \mid L, P\right] = \frac{\int_L^\infty \frac{qf(q)dq}{c(q)}}{\int_L^\infty \frac{f(q)dq}{c(q)}}$$

is the average quality of users conditional on the platform's exclusion policy (L) and its price (P).

The fraction of users allowed to join the platform is:

$$\lambda\left(L\right) = \int_L^\infty \frac{f\left(q\right)dq}{c\left(q\right)}$$

Therefore, effective demand N for the platform's service or product is given by:

$$N = \lambda\left(L\right)\left[V\left(\overline{q}(L) + \alpha\left(\overline{q}\left(L\right)\right)N - P\right]$$

which yields:

$$N = \frac{\lambda\left(L\right)\left[V\left(\overline{q}\left(L\right)\right) - P\right]}{1 - \lambda\left(L\right)\alpha\left(\overline{q}\left(L\right)\right)}$$

The platform solves:

$$\max_{P,L} \frac{\lambda(L)\left[V\left(\overline{q}(L)\right) - P\right] P}{1 - \lambda(L)\,\alpha\left(\overline{q}(L)\right)} = \max_{L} \frac{\lambda(L)\,V^2\left(\overline{q}(L)\right)}{4\left[1 - \lambda(L)\,\alpha\left(\overline{q}(L)\right)\right]}$$

which is very similar to (3).

Here too, assume users can be of two qualities: high quality $q = 1$ with probability λ, and low quality $q = 0$ with probability $(1 - \lambda)$. Let:

$$c(q) = \begin{cases} c & \text{if} \quad q = 0 \\ \frac{c}{\beta} & \text{if} \quad q = 1 \end{cases}$$

The analysis of the various cases is then very similar to the two-sided case. The exclusion regime $(L = 1)$ is profit-maximizing if and only if:

$$\frac{V_W^2\left(\overline{q_{NE}}\right)}{V_W^2(1)} \leq \frac{1 - \lambda_{NE} \times \overline{\alpha}^2\left(\overline{q_{NE}}\right)}{1 - \lambda_E \times \overline{\alpha}^2(1)} \times \frac{\lambda_E}{\lambda_{NE}}$$

where q_{NE}, λ_E and λ_{NE} have the same expressions:

$$\begin{aligned} \lambda_{NE} &= \frac{\lambda\beta + (1-\lambda)}{c} \\ \lambda_E &= \frac{\lambda\beta}{c} \\ \overline{q_{NE}} &= \frac{\lambda\beta}{\lambda\beta + (1-\lambda)} \end{aligned}$$

Suppose $\alpha(\overline{q}) = \alpha$ for all $\overline{q}$:

- If $V(\overline{q}) = \sqrt{\overline{q}}$ then the one-sided platform never excludes low quality users
- If $V(\overline{q}) = \overline{q}$ then the one-sided platform excludes low quality users if and only if:

$$2\lambda\beta + (1 - \lambda) \leq \frac{c}{\alpha}$$

Suppose $V(\overline{q}) = V$ for all $\overline{q}$ and $\alpha(\overline{q}) = \alpha_0 + \alpha\overline{q}$, with $\alpha_0 + \alpha < 1$. Then the one-sided platform excludes low quality users if and only if:

$$\lambda \geq \frac{1-\alpha}{\alpha}$$

These simple illustrations lead to the same results and intuition as the two-sided case regarding the effects of preferences for quality vs. preferences for quality, λ and β.

■

Proof of Proposition 4

Ican solve (7) and (8) for $(N_W^i)_{i=1,2}$ as functions of prices $(P_W^1, P_M^1, P_W^2, P_M^2)$ and exclusion levels (L_M^1, L_M^2):

$$N_W^i = \frac{t - \alpha_M \alpha_W^j \lambda_M^j + V_W^i - V_W^j + P_W^j + \alpha_W^j \lambda_M^j P_M^j - P_W^i - \alpha_W^i \lambda_M^i P_M^i}{2t - \alpha_M \left(\alpha_W^i \lambda_M^i + \alpha_W^j \lambda_M^j\right)}$$

And platform i's profits can be written as (replacing N_M^i by its expression as a function of N_W^i, P_M^i, λ_M^i):

$$\Pi_P^i = \left(P_W^i + P_M^i \lambda_M^i \alpha_M\right) N_W^i - \lambda_M^i \left(P_M^i\right)^2$$

Taking the first order conditions in (P_W^i, P_M^i) and evaluating them at the symmetric equilibrium (in which $N_W^i = \frac{1}{2}$ for $i = 1, 2$), Iobtain the following two equations (to simplify notation, let $\alpha_W^* \equiv \alpha_W \left(\overline{q}_M \left(L_M^*\right)\right)$ and $\lambda_M^* \equiv \lambda_M \left(L_M^*\right)$):

$$P_M^* \alpha_M \lambda_M^* + P_W^* = t - \alpha_M \alpha_W^* \lambda_M^*$$

$$P_M^* \left(4t - 3\alpha_M \alpha_W^* \lambda_M^*\right) + P_W^* \alpha_W^* = \alpha_M \left(t - \alpha_M \alpha_W^* \lambda_M^*\right)$$

Solving this system for (P_W^*, P_M^*) yields:

$$P_W^* = t - \frac{\alpha_M \lambda_M^*}{4} \left(\alpha_M + 3\alpha_W^*\right) \tag{12}$$

$$P_M^* = \frac{\alpha_M - \alpha_W^*}{4} \tag{13}$$

If $\alpha_W \left(\overline{q}_M\right) = \alpha_W$ for all $\overline{q}_M$ then then $\alpha_W^* \equiv \alpha_W$.

The first order condition in L_M^i evaluated at the symmetric equilibrium yields:

$$\left[\frac{1}{2}\alpha_M P_M^* - \left(P_M^*\right)^2\right] \times \frac{d\lambda_M}{dL_M}\left(L_M^*\right) + \left(\alpha_M P_M^* \lambda_M^* + P_W^*\right) \times \frac{\partial N_W^i}{\partial L_M^i}\left(P_W^*, P_M^*, L_M^*\right) = 0 \qquad (14)$$

where:

$$\frac{\partial N_W^i}{\partial L_M^i}\left(P_W^*, P_M^*, L_M^*\right) = \frac{2\left[\frac{dV_W}{d\overline{q}_M}\left(\overline{q}_M\left(L_M^*\right)\right)\frac{d\overline{q}_M}{dL_M}\left(L_M^*\right) - \alpha_W P_M^* \frac{d\lambda_M}{dL_M}\left(L_M^*\right)\right] + \alpha_M \alpha_W \frac{d\lambda_M}{dL_M}\left(L_M^*\right)}{4\left(t - \alpha_M \alpha_W \lambda_M^*\right)}$$

Plugging this expression in (14) above and recalling that $\alpha_M P_M^* \lambda_M^* + P_W^* = t - \alpha_M \alpha_W \lambda_M^*$, one obtains:

$$\left[\frac{1}{2}\alpha_M P_M^* - \left(P_M^*\right)^2\right] \times \frac{d\lambda_M}{dL_M}\left(L_M^*\right) + \frac{2\left[\frac{dV_W}{d\overline{q}_M}\left(\overline{q}_M\left(L_M^*\right)\right)\frac{d\overline{q}_M}{dL_M}\left(L_M^*\right) - \alpha_W P_M^* \frac{d\lambda_M}{dL_M}\left(L_M^*\right)\right] + \alpha_M \alpha_W \frac{d\lambda_M}{dL_M}\left(L_M^*\right)}{4} = 0$$

which is equivalent to:

$$\frac{d\lambda_M}{dL_M}\left(L_M^*\right) \times \left[2\alpha_M P_M^* - 4\left(P_M^*\right)^2 - 2\alpha_W P_M^* + \alpha_M \alpha_W\right] + 2\frac{dV_W}{d\overline{q}_M}\left(\overline{q}_M\left(L_M^*\right)\right)\frac{d\overline{q}_M}{dL_M}\left(L_M^*\right) = 0$$

Finally, after using (13) and simplifying:

$$\frac{d\lambda_M}{dL_M}\left(L_M^*\right) \times \frac{\left(\alpha_M + \alpha_W\right)^2}{8} + \frac{dV_W}{d\overline{q}_M}\left(\overline{q}_M\left(L_M^*\right)\right) \times \frac{d\overline{q}_M}{dL_M}\left(L_M^*\right) = 0$$

■

Proof of Proposition 5

The first part of the proof is identical with that of Proposition 4:

$$N_W^i = \frac{t - \alpha_M \alpha_W^j \lambda_M^j + V_W^i - V_W^j + P_W^j + \alpha_W^j \lambda_M^j P_M^j - P_W^i - \alpha_W^i \lambda_M^i P_M^i}{2t - \alpha_M\left(\alpha_W^i \lambda_M^i + \alpha_W^j \lambda_M^j\right)}$$

$$\Pi_P^i = \left(P_W^i + P_M^i \lambda_M^i \alpha_M\right) N_W^i - \lambda_M^i \left(P_M^i\right)^2$$

leading to the same expression for the equilibrium prices (P_M^*, P_W^*):

$$P_W^* = t - \frac{\alpha_M \lambda_M (L_M^*)}{4} [\alpha_M + 3\alpha_W^*]$$

$$P_M^* = \frac{\alpha_M - \alpha_W^*}{4}$$

where $\alpha_W^* = \alpha_W (\overline{q}_M (L_M^*))$.

The first order condition of Π_P^i in L_M^i at the symmetric equilibrium is also the same:

$$\left[\frac{1}{2}\alpha_M P_M^* - (P_M^*)^2\right] \times \frac{d\lambda_M}{dL_M}(L_M^*) + (\alpha_M P_M^* \lambda_M^* + P_W^*) \times \frac{\partial N_W^i}{\partial L_M^i}(P_W^*, P_M^*, L_M^*) = 0 \tag{15}$$

but $\frac{\partial N_W^i}{\partial L_M^i}(P_W^*, P_M^*, L_M^*)$ is different:

$$\frac{\partial N_W^i}{\partial L_M^i}(P_W^*, P_M^*, L_M^*) = \frac{(\alpha_M - 2P_M^*)\left[\alpha_W^* \frac{d\lambda_M}{dL_M}(L_M^*) + \alpha_W^* \frac{d\alpha_W}{d\overline{q}_M}\frac{d\overline{q}_M}{dL_M}(L_M^*)\right]}{4(t - \alpha_M \alpha_W^* \lambda_M^*)}$$

Plugging this expression back into (15), I obtain:

$$\left[\frac{1}{2}\alpha_M P_M^* - (P_M^*)^2\right] \times \frac{d\lambda_M}{dL_M}(L_M^*) + \frac{(\alpha_M - 2P_M^*)\left[\alpha_W^* \frac{d\lambda_M}{dL_M}(L_M^*) + \alpha_W^* \frac{d\alpha_W}{d\overline{q}_M}\frac{d\overline{q}_M}{dL_M}(L_M^*)\right]}{4} = 0$$

Using $P_M^* = \frac{\alpha_M - \alpha_W^*}{4}$ and simplifying, this is equivalent to:

$$[\alpha_M + \alpha_W(\overline{q}_M(L_M^*))]\frac{d\lambda_M}{dL_M}(L_M^*) + 2\lambda_M(L_M^*)\frac{d\alpha_W}{d\overline{q}_M}(\overline{q}_M(L_M^*))\frac{d\overline{q}_M}{dL_M}(L_M^*) = 0$$

which can also be written as:

$$L_M^* = \arg\max_{L_M}\left\{[\overline{\alpha}(\overline{q}_M(L_M))]^2 \times \lambda_M(L_M)\right\}$$

■