Strategic Interactions in Two-Sided Market Oligopolies

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Abstract

Strategic interactions between two-sided platforms depend not only on whether their decision variables are strategic complements or substitutes as for one-sided firms, but also -and crucially so- on whether or not the platforms subsidize one side of the market in equilibrium. For example, with prices being strategic complements across platforms, we show that a cost-reducing investment by one firm may have a *positive* effect on its rival's profits and a *negative* effect on its own profits when one side is subsidized in equilibrium. By contrast, if platforms make positive margins on both sides, the same investment has the regular, expected effects. Our analysis implies that the strategy space and the logic of competitive advantage are fundamentally different in two-sided markets relative to one-sided markets.

Keywords: Two-Sided Markets, Two-Sided Platforms, Strategic Complements, Strategic Substitutes, Competitive Advantage.

JEL Classifications: L1, L2, L4, L8

1   Introduction

Fudenberg Tirole (1984) (hereafter FT) and Bulow Geanakoplos and Klemperer (1985) (hereafter BGK) have proposed a typology of strategic interactions in one-sided markets oligopolies. They analyze a competition game preceded by an investment stage. Three factors are shown to determine whether an incumbent will over-invest or under-invest: whether the objective of the incumbent is to accomodate or to deter entry, whether actions in the competition game are strategic completements or substitutes and whether investment increases or decreases rivals' profits (cf. Tirole 1988).

In this paper, we show that the possibility of subsidization of one side in a two-sided market can lead to fundamentally new (and somewhat surprising) strategic configurations in oligopoly. For instance, we show that a cost-reducing investment by a two-sided platform may be a successful entry accomodation strategy and at the same time raise the profits of its rival (in a one-sided market, cost reductions by one firm unambiguously hurt its competitors). The intuition behind this result is as follows. A cost reduction by one of the platforms gives it a competitive advantage relative to its rival, so that in the new equilibrium its prices on both sides will be lower than its rival's and it will steal customers from its rival on both sides of the market relative to the initial

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equilibrium. However, the ratio between the number of customers stolen from the subsidized side and the number of customers stolen from the other (profitable) side may be sufficiently high so that the rival is happy to get rid of both, and sufficiently low so that the more cost-effective platform can serve them profitably at the new equilibrium prices. In other words, the cost reduction allows a "price-rebalancing" act by both platforms[1], which may end up being beneficial to both of them - in stark contrast with the one-sided case.

An important application of this result concerns tying in the spirit of Whinston (1990): a two-sided platform A also has a monopoly power over another product M which is homogenously valued by all customers on one side of A. Tying M and the purchase of the platfom on this side of A then acts as a commitment to price agressively by raising the opportunity cost of a foregone sale. In the pricing game that follows, it has the same effect as a reduction of the marginal cost of distribution of A on the side of the market which buys M, relative to rival two-sided platforms. In a one-sided market with price competition and homogenous valuations of the tying good, tying is always a "top dog" strategy: it decreases rivals' profits while increasing one's own. By contrast, the result mentioned above implies that in a two-sided market, tying can be part of a "fat cat" strategy: a profitable way to accomodate entry while at the same time being "soft" (i.e. benefitting rivals as well).

More broadly, our analysis reveals that the set of strategic configurations in a two-sided market is strictly larger than in a one-sided market - not in terms of the nature of the strategies but in terms of the conditions under which they emerge. This is due to the fact that in two-sided markets, the sign of the strategic effect – that determines whether the incumbent will over- or under-invest – can no longer be entirely determined by the effect on rivals' profits as in one-sided markets.

This conclusion lends significant substance and support to the contention that two-sided markets have new implications, both from a strategy and from a public policy perspective. Indeed, the recent and burgeoning literature on two-sided markets is built on this premise, but the main argument put forward up to now is that two-sided platforms are different because they might end up subsidizing one side of the market in order to recoup on the other. This argument has received significant attention from antitrust scholars (e.g. Evans (2003), Wright (2004)), which have pointed out several implications, such as for instance that below-cost pricing in a market may not be indicative of predation, but of a two-sided pricing strategy, which can be profitable regardless of the presence of competition. Our paper takes a step beyond exhibiting this type of skewed two-sided pricing structure by deriving deeper implications of this feature for strategic interactions among platforms and a systematic categorization of strategies in two-sided markets oligopolies (which we show can be very different from one-sided markets).

Relation to the literature. This paper contributes to the recent two-sided markets literature, intiated by Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003). Most of this literature has analyzed how platforms might solve the chicken-and-egg problem associated

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[1]Cf. Rochet and Tirole (2006).

with two sided-markets and focused almost exclusively on the conditions determining which (if any) side is subsidized and how much. We study the implications of the two-sided pricing game between two platforms on strategic investment choices that must be made prior to the pricing game. To illustrate the conclusions of our general analysis, we borrow from Armstrong (2006)'s two-sided Hotelling competition setting, but we extend his model by allowing for "hinterlands" - this extension is absolutely necessary in order to exhibit results which are different from the ones occurring in one-sided markets.

This paper is also related to the literature on tying. An important benchmark in this literature is Whinston (1990). Whinston shows that when the tying good is homogenous, then tying acts as a commitment to be more aggressive in the competition on the tied good market. As a result, tying can be a profitable strategy to deter entry. However, in this case, tying is never a profitable strategy to accomodate entry. Our results show that this insight can be overturned in a two-sided setting.

There are three papers in the two-sided market literature that focus on tying. Rochet and Tirole (2003b) provide an economic analysis of the tying practice initiated by payments card associations Visa and MasterCard in which merchants who accept their credit cards were forced also to accept their debit cards. They show that in the absence of tying, the interchange fee between the merchant's and the cardholder's banks on debit is too low and tends to be too high on credit compared to the social optimum. Tying is shown to be a mechanism to rebalance the interchange fee structure and raise social welfare. Choi (2007) analyzes the welfare effects of tying in a model of competition between two-sided platforms (connecting consumers and content providers), when one or both sides can multihome. In his model, tying simply allows one of the platforms to reach all consumers by bundling the platform product in question with another product that all consumers need (the motivating example is the tying of Windows Media Player to the Windows Operating System, which every PC user has). The impact of tying on social welfare depends on whether consumers can multihome or not, but in all cases, tying unambiguously hurts the rival platform.

Amelio and Jullien (2007) consider a setting in which two-sided platforms would like to set prices below zero on one side of the market in order to solve the demand coordination problem, but are constrained to set non-negative prices. Tying can then serve as a mechanism to introduce implicit subsidies on one side of the market in order to solve the aforementioned coordination failure. As a result, tying can raise participation on both sides and can benefit consumers in the case of monopoly platform. In a duopoly context tying also has a strategic effect on competition. Contrary to the monopoly case, tying may not be ex-post and/or ex-ante optimal for a contested platform. Moreover, the competing platform benefits from it if the equilibrium implicit subsidy is large enough. We also obtain this result, although as a particular case of a broader setting.

The remainder of the paper is organized as follows. In Section 2, we briefly review the typology introduced by FT for one sided-markets. In Section 3, we lay out a very general two-sided market oligopoly setting, derive the corresponding characterization of the strategic space and explain the key differences relative to the one-sided environment. In Section 4, we illustrate the general analysis with specific, micro-founded models, which also allow us to derive more intuition for our main results.

We conclude in Section 5.

2   One-sided markets

This section provides a brief summary of the framework and animal strategy nomenclature introduced by FT; it draws heavily on Tirole (1988), to which the reader is referred for more details.

There are two firms, an incumbent, A, and an entrant, B. The timing of the duopoly game comprises three stages. In stage 1, firm A chooses a variable K_A – for example a capacity, an investment in product quality or in a cost reducing technology. In stage 2, firm B observes K_A and decides whether or not to enter. In stage 3, the stage game depends on wether or not B has entered at stage 2. If B has not entered at stage 2, A chooses x_A – which could be a price or a quantity– to maximize profits as a monopolist. If B has entered then A and B simultaneously choose x_A and x_B.

In stage 3, if only firm A is active then its profits are denoted by $\Pi^M(K_A, x_A)$. If both firms are active, firm i's profits are denoted by $\Pi^i(K_A, x_A, x_B)$. The best response functions for the two firms in this stage are denoted by $x_A^*(x_B, K_A)$ and $x_B^*(x_A, K_A)$ respectively:

$$x_i^*(x_j, K_A) = \arg\max_{x_i} \Pi^i(K_A, x_A, x_B) \text{ for } i, j \in \{A, B\}, i \neq j$$

Also, denote by $(x_A^{**}(K_A), x_B^{**}(K_A))$ the resulting Nash equilibrium in stage 3 given A's choice of K_A in stage 1.

There are two cases to consider. If entry deterrence is the most profitable strategy for A then its choice of K_A is driven by Π^B. Firm A will choose K_A so as to just deter entry by firm B:

$$K_A^M = \arg\max_{K_A} \Pi^M\left(K_A, x_A^M(K_A)\right)$$

subject to:

$$\Pi^B\left(K_A, x_A^{**}(K_A), x_B^{**}(K_A)\right) \leq 0 \tag{1}$$

where $x_A^M(K_A) = \arg\max_{x_A} \Pi^M(K_A, x_A)$.

By contrast, if entry accomodation is the most profitable strategy for firm A, then its choice of K_A is driven by Π^A. In this case, K_A is chosen as follows:

$$K_A^D = \arg\max_{K_A} \Pi_A\left(K_A, x_A^{**}(K_A), x_B^{**}(K_A)\right)$$

subject to

$$\Pi_B\left(K_A, x_A^{**}(K_A), x_B^{**}(K_A)\right) \geq 0 \tag{2}$$

In the rest of the paper we assume that B's participation constraint (2) is not binding. Conse-

quently, the stage 3 Nash equilibrium $(x_A^{**}(K_A), x_B^{**}(K_A))$ is characterized by:

$$\frac{\partial \Pi^A}{\partial x_A} = \frac{\partial \Pi^B}{\partial x_B} = 0$$

Following FT, in the case of entry accomodation, the overall effect of K_A on Π^A can be decomposed into a direct effect and a strategic effect:

$$\frac{d\Pi^A}{dK_A} = \underbrace{\frac{\partial \Pi^A}{\partial K_A}}_{\text{direct effect}} + \underbrace{\frac{\partial \Pi^A}{\partial x_B}\frac{dx_B^{**}}{dK_A}}_{\text{strategic effect}}$$

Meanwhile, the influence of K_A on firm B's profits, both under entry deterrence and accomodation, is given by:

$$\frac{d\Pi^B}{dK_A} = \underbrace{\frac{\partial \Pi^B}{\partial K_A}}_{\text{direct effect}} + \underbrace{\frac{\partial \Pi^B}{\partial x_A}\frac{dx_A^{**}}{dK_A}}_{\text{strategic effect}} \tag{3}$$

Assuming by symmetry that $sign\left(\frac{\partial \Pi^A}{\partial x_B}\right) = sign\left(\frac{\partial \Pi^B}{\partial x_A}\right)$, we have:

$$sign\left(\frac{\partial \Pi^A}{\partial x_B}\frac{dx_B^{**}}{dK_A}\right) = sign\left(\frac{\partial \Pi^B}{\partial x_A}\frac{\partial x_B^*}{\partial x_A}\right) sign\left(\frac{dx_A^{**}}{dK_A}\right) \tag{4}$$

where $\frac{\partial x_B^*}{\partial x_A} > 0 \; (< 0)$ if the variables (x_A, x_B) are strategic complements (substitutes).

The focus of the analysis is on the strategic effects. Direct effects would exist even if firm B did not observe K_A (open loop solution). Therefore, the sign of the strategic effect - under both entry deterrence and accomodation - determines whether firm A should over- or under-invest relative to the level of investment which would prevail in the open loop equilibrium. FT adopt the convention that investment makes firm A "tough" ("soft") if the strategic effect of K_A on firm B's profits is negative (positive). We will maintain this convention throughout our paper. They then define the following strategies: (i) "top dog", be big (i.e. overinvest) in order to look tough (aggressive); (ii)"lean and hungry look", stay small (i.e. underinvest) in order to look tough (aggressive), (iii) "puppy dog": stay small in order to look soft (inoffensive) and (iv) "fat cat": be big in order to look soft (inoffensive). These four strategies turn out to be sufficient for fuly describing firm A's desired behavior in all cases, as shown in the following table:

		Investment K_A is:	
		Tough	Soft
Actions are:	Strategic complements	**A**: *Puppy Dog* **D**: *Top Dog*	**A**: *Fat Cat* **D**: *Lean and Hungry*
	Strategic substitutes	**A**: *Top Dog* **D**: *Top Dog*	**A**: *Lean and Hungry* **D**: *Lean and Hungry*

Table 1

where **A** stands for entry accomodation and **D** for entry deterrence.

We now specialize the model to the following application, to which we will refer in the next sections of the paper. Actions x_i are prices - $x_i \equiv p_i$ for $i = A, B$ - and K_A is an investment which reduces the marginal cost c_A of firm A. We also make three assumptions. First, profit functions are concave in own prices so that best response correspondances $p_A^*(p_B, c_A)$ and $p_B^*(p_A, c_A)$ are well-defined and single-valued. Second, for each c_A there is a unique Nash equilibrium in prices $(p_A^{**}(c_A), p_B^{**}(c_A))$ and it is stable. Third, prices are strategic complements (this is the case with linear demand models): $\frac{\partial p_i^*}{\partial p_{-i}} > 0$ for $i \in \{1, 2\}$.

In the appendix we prove that these assumptions guarantee that:

$$\frac{dp_A^{**}}{dc_A} > 0$$

Denoting by $n_i(p_A, p_B)$ the demand for firm i – with the property that $\frac{\partial n_i}{\partial p_{-i}} > 0 > \frac{\partial n_i}{\partial p_i}$, for $i \in \{1, 2\}$ – we have:

$$\frac{d\Pi^B}{dK_A} = \frac{d}{dK_A}\left((p_B - c_B)n_B\right) = \underbrace{(p_B^* - c_B)}_{>0}\underbrace{\frac{\partial n_B}{\partial p_A}}_{>0}\underbrace{\frac{dp_A^*}{dc_A}}_{>0}\underbrace{\frac{dc_A}{dK_A}}_{<0} < 0$$

where the first term is positive because firm B's profits have to be positive in equilibrium in order for firm B to be active.

Thus, a cost-reducing investment by firm A in a one-sided market can only be tough. This is quite intuitive: a marginal cost reduction by one firm can only hurt its rival in a one-sided context when firms compete in prices[2]. As we will see in the next section, things change radically in a two-sided context.

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[2]It can easily be shown that the same holds when firms compete in quantities.

3 Two-sided markets

We now move to a two-sided context, in which the two firms A and B are competing not for just one type of customers, but for two interrelated groups of customers 1 and 2. Each platform $i \in \{A, B\}$ chooses 2 actions p_1^i and p_2^i, which correspond to sides 1 and 2 respectively. Although in principle these could be any strategic actions, we will henceforth only consider the case in which they correspond to prices that the platforms have to set on both sides of the market.[3]

Two-sidedness is captured by assuming that the demand a platform faces on each side is *decreasing* in the price it charges to that side, *increasing* in the price charged by the rival platform on the same side, *increasing* in the *realized* demand on the other side of the same platform and *decreasing* in the *realized* demand on the other side of the rival platform. In short, the demand for platform i on side j is given by:

$$N_j^i = \hat{n}_j^i \left(p_j^i, p_j^{-i}, N_{-j}^i, N_{-j}^{-i} \right) \tag{5}$$

with $\frac{\partial \hat{n}_j^i}{\partial p_j^i} < 0 < \frac{\partial \hat{n}_j^i}{\partial p_j^{-i}}$; $\frac{\partial \hat{n}_j^i}{\partial N_{-j}^i} > 0 > \frac{\partial \hat{n}_j^i}{\partial N_{-j}^{-i}}$.

Let c_j^i denote the marginal costs of platform i on side j, which we all assume to be constant.

The timing of the game is the same as in the previous section. In stage 1, firm A chooses an investment K_A. In stage 2, firm B observes K_A and decides whether or not to enter. In stage 3: if B has not entered at stage 2, then A chooses p_1^A and p_2^A as a monopolist; if B has entered then A and B simultaneously choose p_1^i and p_2^i, $i \in \{A, B\}$.

Throughout the rest of the paper, we focus on the case in which K_A is a cost-reducing investment on side 1 for platform A:

$$\frac{dc_1^A}{dK_A} < 0 \text{ and } \frac{dc_2^A}{dK_A} = \frac{dc_1^B}{dK_A} = \frac{dc_2^B}{dK_A} = 0$$

To simplify things, we assume that K_A has no direct effect on platform B's profits, so the total effect is equal to

We also make the following four assumptions in order simplify the analysis (the substance of our conclusions is not affected).

Assumption 1 *(non-singularity) For any set of prices $\{p_j^i\}$, the four equations (5), determine a unique, stable, configuration of demands:*

$$N_j^i = n_j^i \left(\{p_l^k\} \right)$$

where $\frac{\partial n_j^i}{\partial p_j^i} < 0 < \frac{\partial n_j^i}{\partial p_j^{-i}}$ and $\frac{\partial n_j^i}{\partial p_{-j}^i} < 0 < \frac{\partial n_j^i}{\partial p_{-j}^{-i}}$.

The profits of platform i can then be written as:

$$\Pi^i = \left(p_1^i - c_1^i \right) n_1^i + \left(p_2^i - c_2^i \right) n_2^i$$

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[3]In fact, we are unaware of any model of two-sided markets in which platforms compete in variables other than prices.

Assumption 2 *(concavity)* *Each platform's profits are concave in both of its prices holding the rival platform's prices constant and there always exists a unique Nash equilibrium in the stage 3 pricing game denoted by $\{p_j^{**i}\}$ with $i \in \{1, 2\}$, $j \in \{A, B\}$.*

We continue to denote the platforms' best response functions by $p_j^{i*}\left(p_1^{-i}, p_2^{-i}, c_1^i, c_2^i\right)$, $i \in \{A, B\}$, $j \in \{1, 2\}$.

Assumption 3 *(strategic complementarity)* *Prices are strategic complements: $\partial p_j^{i*}/\partial p_l^{-i} > 0$ and equilibrium prices for each platform are increasing in its own costs:*

$$\frac{\partial p_j^{i**}}{\partial c_{j'}^i} > 0 \text{ for all } i \in \{A, B\}, \ j, j' \in \{1, 2\} \tag{6}$$

Note that this assumption implies that[4]:

$$\frac{dp_i^{A**}}{dK_A} = \frac{\partial p_i^{A**}}{\partial c_1^A} \frac{dc_1^A}{dK_A} < 0$$

$$\frac{dp_i^{B**}}{dK_A} = \frac{\partial p_i^{B*}}{\partial p_1^A} \frac{dp_1^{A**}}{dK_A} + \frac{\partial p_i^{B*}}{\partial p_2^A} \frac{dp_2^{A**}}{dK_A} < 0$$

for $i = 1, 2$.

In a one-sided market, the condition corresponding to (6) is automatically verified and can be seen at the most general level as an application of the monotone comparative statics results in Milgrom and Shannon (1994). These principles, however, cannot be invoked in the two-sided context analyzed in this section.

Assumption 4 *(symmetry)* *For $K_A = 0$ and the corresponding Nash equilibrium, we have:*

$$\frac{\partial \Pi^i}{\partial p_j^{-i}}\left(\{p_l^{**k}\}\right) = \frac{\partial \Pi^{-i}}{\partial p_j^i}\left(\{p_l^{**k}\}\right)$$

Indeed, instead of solving the last two stages of the game for all values of K_A, we will only perform a local comparative statics analysis in K_A around $K_A = 0$, which is sufficient for our purposes.

3.1   Strategic interactions with variable prices on both sides

We are interested in determining the strategic effects of K_A on the rival platform (B)'s profits as well as on platform A's own profits. The notion of under/over-investment is relative to a situation

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[4]The Nash equilibrium prices $\left(p_j^{i**}\right)$ are a function of the four costs $\left(c_j^i\right)$, which in turn depend solely on K_A. With a slight abuse of notation, we denote by $p_j^{i**}(K_A)$ the resulting function of K_A. The total derivative $\frac{dp_j^{i**}}{dK_A}$ refers to this latter function whereas the partial derivatives $\frac{\partial p_j^{i**}}{\partial c_{j'}^i}$ refer to the function of costs.

in which only the direct effects exists. Hence our interest lies in determining the signs of $\frac{d\Pi^A}{dK_A} - \frac{\partial\Pi^A}{\partial K_A}$ and $\frac{d\Pi^B}{dK_A} - \frac{\partial\Pi^B}{\partial K_A}$[5].

Using the envelope theorem, the sign of the strategic effect on platform B's profits is:

$$sign\left\{\frac{d\Pi^B}{dK_A} - \frac{\partial\Pi^B}{\partial K_A}\right\} = sign\left\{\frac{\partial\Pi^B}{\partial p_1^A}\frac{dp_1^{A**}}{dK_A} + \frac{\partial\Pi^B}{\partial p_2^A}\frac{dp_2^{A**}}{dK_A}\right\}$$

$$= sign\left\{\sum_{j\in\{1,2\}}\left(\left(p_1^B - c_1^B\right)\frac{\partial n_1^B}{\partial p_j^A} + \left(p_2^B - c_2^B\right)\frac{\partial n_2^B}{\partial p_j^A}\right)\frac{dp_j^{A**}}{dK_A}\right\} \tag{7}$$

The sign of the strategic effect of K_A on firm A's own profits is:

$$sign\left\{\frac{d\Pi^A}{dK_A} - \frac{\partial\Pi^A}{\partial K_A}\right\} = sign\left\{\frac{\partial\Pi^A}{\partial p_1^B}\frac{dp_1^{B*}}{dK_A} + \frac{\partial\Pi^A}{\partial p_2^B}\frac{dp_2^{B**}}{dK_A}\right\} = sign\left\{\frac{\partial\Pi^B}{\partial p_1^A}\frac{dp_1^{B**}}{dK_A} + \frac{\partial\Pi^B}{\partial p_2^A}\frac{dp_2^{B**}}{dK_A}\right\}$$

$$= sign\left\{\sum_{j\in\{1,2\}}\left(\left(p_1^B - c_1^B\right)\frac{\partial n_1^B}{\partial p_j^A} + \left(p_2^B - c_2^B\right)\frac{\partial n_2^B}{\partial p_j^A}\right)\frac{dp_j^{B**}}{dK_A}\right\} \tag{8}$$

These two expressions constitute the core of our analysis. Note that the respective signs of the strategic effects for firms A and B are disconnected. This is a crucial difference with BGK and FT, which analyze the same kind of strategic interactions, but in one-sided oligopolies. In one-sided contexts the terms $\frac{dp_i^{A**}}{dK_A}$ and $\frac{dp_j^{B**}}{dK_A}$ are negative when the "actions" $\left(p_i^A, p_j^B\right)$ are strategic complements[6]. By contrast, here the fact that the prices $\left(p_i^A, p_j^B\right)$ are strategic complements does not pin down the sign of the terms $\frac{\partial\Pi^B}{\partial p_i^A}$.

As a consequence, the set of strategic configurations in a two-sided market is strictly larger than in a one-sided market - not in terms of the nature of the strategies but in terms of the conditions under which they emerge. Equations (7) and (8) imply that the following four configurations are possible[7]: (i) $\frac{d\Pi^A}{dK_A} - \frac{\partial\Pi^A}{\partial K_A} > 0$ and $\frac{d\Pi^B}{dK_A} - \frac{\partial\Pi^B}{\partial K_A} > 0$; (ii) $\frac{d\Pi^A}{dK_A} - \frac{\partial\Pi^A}{\partial K_A} > 0$ and $\frac{d\Pi^B}{dK_A} - \frac{\partial\Pi^B}{\partial K_A} < 0$; (iii) $\frac{d\Pi^A}{dK_A} - \frac{\partial\Pi^A}{\partial K_A} < 0$ and $\frac{d\Pi^B}{dK_A} - \frac{\partial\Pi^B}{\partial K_A} > 0$; (iv) $\frac{d\Pi^A}{dK_A} - \frac{\partial\Pi^A}{\partial K_A} < 0$ and $\frac{d\Pi^B}{dK_A} - \frac{\partial\Pi^B}{\partial K_A} < 0$.

Under entry accomodation, these cases lead to the following strategies for platform A : (i) fat cat; (ii) top dog; (iii) puppy dog; (iv) lean and hungry look.

By contrast, in a one-sided market with competition in prices, the optimal strategy for entry accomodation by A can only take two forms: fat cat and puppy dog (cf. Table 1). This is because in this case the two strategic effects of K_A necessarily have the same sign.

Consequently, Table 1 above needs to be adjusted in order to allow for the full range of strategic scenarios in a two-sided market. We say that investment K_A is "self-serving" ("self-harming") for firm A if the strategic effect on A is positive (negative). Note that we focus on the case where entry

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[5]Note that by assumption $\frac{\partial\Pi^B}{\partial K_A} = 0$.

[6]In one-sided markets, even in the case when actions are strategic substitutes (e.g. competition in quantities), the signs are flipped - they are negative. Therefore, in one-sided contexts are always entirely determined by whether actions are strategic complements or substitutes.

[7]And while still maintaining the condition that B's profits are positive.

accomodation is the optimal strategy for A.

Prices are strategic complements Entry accommodation regime		Effect of K_A on platform B:	
		Tough	Soft
Effect of K_A on platform A:	Self-serving	*Top Dog*	*Fat Cat*
	Self-harming	*Puppy Dog*	*Lean and Hungry*

Table 2

First, note that it is possible for K_A to have a positive effect on platform B's profits – so that K_A is "soft" – without violating the condition that B's profits are positive in equilibrium:

$$\left(p_1^{B**} - c_1^B\right) n_1^B + \left(p_2^{B**} - c_2^B\right) n_2^B > 0$$

The possibility for a cost reduction by one platform to increase the profits of its rival is novel. This situation never occurs in one-sided markets – as we showed in the previous section. In a two-sided market setting, it is possible that in equilibrium the platforms subsidize one side of the market – say side 2 – and recoup the losses on the other side: $p_2^{B**} - c_2^B < 0$ and $p_1^{B**} - c_1^B > 0$. Note that for K_A to be soft in this case and for B to make positive profits it is necessary that:

$$\frac{n_1^B}{n_2^B} > \frac{-\left(p_2^{B**} - c_2^B\right)}{p_1^{B**} - c_1^B} > \frac{\partial n_1^B / \partial p_j^A}{\partial n_2^B / \partial p_j^A}$$

for at least one $j \in \{1, 2\}$, which implies:

$$\frac{\partial n_2^B / \partial p_j^A}{n_2^B} > \frac{\partial n_1^B / \partial p_j^A}{n_1^B} \tag{9}$$

for at least one $j \in \{1, 2\}$. This condition can be interpreted in the following way: softness is more likely to occur when the "business stealing" effect on the subsidized side $\left(n_2^B\right)^{-1} \left(\partial n_2^B / \partial p_j^A\right)$ is higher relative to the business stealing effect on the profitable side $\left(n_1^B\right)^{-1} \left(\partial n_1^B / \partial p_j^A\right)$.

Equation (9) holds the key to interpreting the surprising result above. By the envelope theorem, a small change in K_A impacts Π^B only through A's equilibrium prices. Given that K_A reduces c_1^A, we know that A's prices will be lower in the new equilibrium on both sides ($\frac{dp_1^{A**}}{dK_A}, \frac{dp_2^{A**}}{dK_A} < 0$), which means that A steals customers from B on both sides of the market. However, if the proportion of

customers stolen by A from B on the loss-making side is sufficiently high relative to the proportion of customers stolen on the profit-making side, this might increase B's profits. Getting new customers on both sides in these proportions can also be profitable for A because it has now gained some competitive advantage relative to B (recall that $\frac{dc_1^A}{dK_A} < 0$). Thus, the investment K_A can be both soft and self-serving. Inspecting (8), it should be clear however that K_A can also be self-harming, just as in one-sided markets.

A complementary intuition can be obtained in the following way. Imagine that the initial situation is a symmetric equilibrium where prices are given by p_1 and p_2, and market shares are given by n_1 and n_2, with $p_1 - c_1 > 0$ and $p_2 - c_2 < 0$. As we move to the new equilibrium, in which platform A has gained a very small competitive advantage $\Delta c_1^A < 0$, we have[8]:

$$\Delta\Pi^i - \Delta c_1^i n_1 = \Delta p_1^i n_1 + \Delta p_2^i n_2 + (p_1 - c_1)\Delta n_1^i + (p_2 - c_2)\Delta n_2^i$$

for $i = \{A, B\}$. Note that under our assumptions, $\Delta p_1^i < 0$ and $\Delta p_2^i < 0$ so that

$$\Delta p_1^i n_1 + \Delta p_2^i n_2 < 0 \text{ for all } i$$

Hence a necessary condition for $\Delta\Pi^i - \Delta c_1^i n_1$ to be positive for all i (i.e. to be in a fat cat scenario) is that

$$(p_1 - c_1)\Delta n_1^i + (p_2 - c_2)\Delta n_2^i > 0 \text{ for all } i \in \{A, B\} \tag{10}$$

Clearly, if the total market size is fixed on each side, then $\Delta n_j^A = -\Delta n_j^B$ for all $j \in \{1, 2\}$ so that:

$$(p_1 - c_1)\Delta n_1^A + (p_2 - c_2)\Delta n_2^A = -(p_1 - c_1)\Delta n_1^B - (p_2 - c_2)\Delta n_2^B$$

and (10) cannot be verified. Hence, a cost-reducing investment being a fat cat strategy requires market expansion on at least one side. Indeed, suppose that on side 1, total market size is not fixed. Denote by $\Delta m_1 \equiv \Delta m_1^A + \Delta m_1^B > 0$ the *total* expansion in market size on side 1 (it is positive because both platforms' prices decrease). We now have:

$$(p_1 - c_1)\Delta n_1^A + (p_2 - c_2)\Delta n_2^A = -(p_1 - c_1)\Delta n_1^B - (p_2 - c_2)\Delta n_2^B + (p_1 - c_1)\Delta m_1$$

with $(p_1 - c_1)\Delta m_1 > 0$. Therefore, if the market size expansion is strong enough, (10) can be verified. The cost-reducing investment allows for a better balancing act between the two sides which benefits both platforms through a market expansion on the profit-making side.

Tying. Tying can be analyzed as a reduction in marginal costs on one side of the market. Configuration (i) above is then a case in which tying makes the incumbent soft and is profitable at the same time. In other words, tying becomes a fat cat strategy. We have therefore uncovered the

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[8]We ignore second order terms.

possibility that tying can be a profitable entry-accomodation strategy, in stark contrast with the case of one-sided markets.

To put this result in perspective, it is worth reviewing the logic of the argument in Whinston (1990). Whinston considers a model with two firms A and B. Firm A produces two different goods 1 and 2_A. The market for good 1 is monopolized by firm A. Goods 2_A and 2_B are imperfect substitutes and give rise to price competition: the demand for good 2_i is given by $x_2^i(p_2^A, p_2^B)$ with $\frac{\partial x_2^i}{\partial p_2^j} \geq 0$ if $i \neq j$ and $\frac{\partial x_2^i}{\partial p_2^j} < 0$ if $i = j$. Firm B however, needs to pay an entry cost K_B in order to operate. In addition to its pricing decisions, firm A can offer bundles of its products. Two cases have to be distinguished. The *commitment case* occurs when firm A commits to a specific bundle of products before entry and pricing decisions are made. The *no-commitment case* occurs when bundling decisions are made at the same time as pricing decisions.

Whinston's main result corresponds to the case that is closest to our setup, where valuation for the tying good 1 is homogenous across all consumers. Whinston shows that tying is useless in the no-commitment case: firm A can make sure that all customers purchase good 1 and replicate the bundling equilibrium in the independent pricing game (where bundling is prohibited).

In the commitment case tying acts on the best responses as a reduction in marginal cost for firm A: every sale of good 2_A comes with the extra benefit of a sale of good 1, lowering the effective corresponding marginal cost of this sale. Tying therefore acts as a commitment by firm A to be more aggressive. If firm B remains active, this reduces the profits of both firms. Tying is a self-deafeating strategy to accomodate entry. However, tying can result in foreclosure whereby firm B decides not to pay the fixed cost. Firm A then monopolizes the market for good 2. The benfit for firm A is reduced competition for good 2. The potential loss comes from the fact that firm A will be a monopolist who can only offer a bundle. Thus, the presence of a large number of consumers who dislike poduct 2_A may make a commitment to bundling unprofitable, even when it leads to exclusion. The conclusion is that in this context, tying is a top dog strategy that is always unprofitable to accomodate entry, and can sometimes be profitable to deter entry.[9]

3.2   Strategic interactions with prices fixed on one side

It is useful to briefly compare the results above with a situation in which platforms' margins on one side of the market, say side 2, are exogenously fixed, i.e. $p_2^A - c_2^A = p_2^B - c_2^B \equiv \pi_2$. This may be

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[9]Whinston then investigates the consequences of relaxing the assumption that the valuations for good 1 are homogenous. The results are modified as follows. First and most related to this paper, a commitment to tying need not always result in foreclosure: it can be the case that firm B's profits increase as a result of a commitment to tying by firm A. The reason is twofold. Enough consumers may find good 1 unattractive so that the margin on every bundle sale is lower than the margin on an independent sale of good 1_A (it is always higher in the homogenous case) – the monopoly power of firm A is too weak. Also, the elasticity of bundle sales to the price of the bundle is not identical anymore to that arising in market 2 – indeed, when 2_A and 2_B are nearly homogenous, tying essentially transorms the nearly homogenous market 2 into a vertically differentiated market, potentially raising firm B's profits.

Second, tying can be a profitable strategy even in the absence of the ability to commit, and when it is, it may lower firm B's profits.

Note however that with non-homogeneous valuations for good 1, tying in the Whinston model can no longer be interpreted as a marginal cost reduction for good 1, like in our model.

interpreted as a context in which regulation, common practice or other institutions put prices on one side of the market beyond the control of the two platforms. Then each platform i only has one variable to choose - p_1^i - and the demand functions can be written as $N_j^i = n_j^i\left(\{p_1^A, p_1^B\}\right)$.

The strategic effect of K_A on platform B is:

$$\frac{d\Pi^B}{dK_A} = \frac{\partial \Pi^B}{\partial p_1^A}\frac{dp_1^{A*}}{dK_A} = \left[\left(p_1^{B*} - c_1^B\right)\frac{\partial n_1^B}{\partial p_1^A} + \pi_2 \frac{\partial n_2^B}{\partial p_1^A}\right]\frac{dp_1^{A*}}{dK_A}$$

whereas the strategic effect on platform A's own profits is:

$$\frac{\partial \Pi^A}{\partial p_1^B}\frac{dp_1^{B*}}{dK_A} = \frac{\partial \Pi^B}{\partial p_1^A}\frac{dp_1^{B*}}{dp_1^A}\frac{dp_1^{A*}}{dK_A} = \left[\left(p_1^{B*} - c_1^B\right)\frac{\partial n_1^B}{\partial p_1^A} + \pi_2 \frac{\partial n_2^B}{\partial p_1^A}\right]\frac{dp_1^{B*}}{dp_1^A}\frac{dp_1^{A*}}{dK_A} = \frac{d\Pi^B}{dK_A}\frac{dp_1^{B*}}{dp_1^A}$$

The first expression above implies that the possibility of K_A being soft survives. If for example $\pi_2 < 0$ and $\left(n_2^B\right)^{-1}\left(\partial n_2^B/\partial p_1^A\right) > \left(n_1^B\right)^{-1}\left(\partial n_1^B/\partial p_1^A\right)$, then we can have simultaneously $\frac{d\Pi^B}{dK_A} > 0$ and $\Pi^B > 0$ in equilibrium. However, given that $sign\left(\frac{\partial \Pi^A}{\partial p_1^B}\frac{dp_1^{B*}}{dK_A}\right) = sign\left(\frac{d\Pi^B}{dK_A}\right) sign\left(\frac{dp_1^{B*}}{dp_1^A}\right)$, if $\frac{dp_1^{B*}}{dp_1^A} > 0$ – which is always the case with linear demands and prices fixed on one side – then the two strategic effects have the same sign, just like in a one-sided market. Hence, by fixing prices on one side, the set of possible optimal strategies for entry accomodation for platform A is reduced back to {puppy dog, fat cat}.

4 Examples

In this section we use specific models to prove that the results anticipated by the general analysis above are indeed possible. In particular, for each example, our goal is to exhibit the possibility of a small cost reduction by platform A being "soft" (i.e., increasing platform B's profits), while prices are strategic complements across platforms. An additional benefit of working through these examples is that they allow us to gain further intuition regarding the two-sided strategic interactions we are illustrating.

4.1 Fixed prices on one side

We begin with the simpler case, in which platform prices – or markups – are fixed on one side of the market. In this case, we can write without loss of generality the profits of platform $i \in \{A, B\}$ as:

$$\Pi^i = \left(p_1^i - c_1^i\right) n_1^i\left(p_1^i, p_1^j\right) + \pi_2 f\left(n_1^i, n_1^j\right)$$

where $n_1^i\left(p_1^i, p_1^j\right)$ is non-negative, decreasing in p_1^i and increasing in p_1^j; $f\left(.,.\right)$ is non-negative, increasing in its first argument - $f_1 > 0$ - and decreasing in its second argument - $f_2 \leq 0$.

We want to show that the strategic effect of a decrease in c_1^A on Π^B can be positive, i.e.

$\frac{\partial \Pi^B}{\partial p_1^A} \left(p_1^{B*}, p_1^{A*} \right) < 0$[10]. This is equivalent to[11]:

$$\left(p_1^{B*} - c_1^B \right) + \pi_2 \left[f_1 + f_2 \frac{\partial n_1^A}{\partial p_1^A} \left(\frac{\partial n_1^B}{\partial p_1^A} \right)^{-1} \right] < 0 \tag{11}$$

The first order condition in p_1^B yields:

$$\left(p_1^{B*} - c_1^B \right) + \pi_2 \left[f_1 + f_2 \frac{\partial n_1^A}{\partial p_1^B} \left(\frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} \right] = -n_1^B \left(\frac{\partial n_1^B}{\partial p_1^B} \right)^{-1} > 0 \tag{12}$$

Thus, in order for our example to work, we need to have $f_2 < 0$ and either:

$$\pi_2 > 0 \text{ and } \left| \frac{\partial n_1^A}{\partial p_1^A} \frac{\partial n_1^B}{\partial p_1^B} \right| < \left| \frac{\partial n_1^B}{\partial p_1^A} \frac{\partial n_1^A}{\partial p_1^B} \right| \tag{13}$$

or:

$$\pi_2 < 0 \text{ and } \left| \frac{\partial n_1^A}{\partial p_1^A} \frac{\partial n_1^B}{\partial p_1^B} \right| > \left| \frac{\partial n_1^B}{\partial p_1^A} \frac{\partial n_1^A}{\partial p_1^B} \right| \tag{14}$$

Stability of the demand system in $\left(p_1^A, p_1^B \right)$ requires $\left| \frac{\partial n_1^A}{\partial p_1^A} \frac{\partial n_1^B}{\partial p_1^B} \right| > \left| \frac{\partial n_1^B}{\partial p_1^A} \frac{\partial n_1^A}{\partial p_1^B} \right|$, hence we must have $\pi_2 < 0$[12].

Most models found in the two-sided market literature up to now – Armstrong (2006) in particular – have a symmetic linear strucuture which implies $\left| \frac{\partial n_1^A}{\partial p_1^A} \right| = \left| \frac{\partial n_1^B}{\partial p_1^B} \right| = \left| \frac{\partial n_1^B}{\partial p_1^A} \right| = \left| \frac{\partial n_1^A}{\partial p_1^B} \right|$. Hence they cannot satisify our conditions (13) or (14). We therefore need to depart from the symmetry assumption and look to obtain the following type of expression:

$$n_1^i \left(p_1^i, p_1^j \right) = N - p_1^i + \gamma p_1^j$$

with $0 < \gamma < 1$. In other words, we need the elasticity of a platform's demand in its own price to be strictly higher in absolute value than the elasticity in the rival platform's price. This can be achieved by adding "hinterlands" on side 1 to the two-sided Hotelling model considered in Armstrong (2006).

In the appendix we provide the detailed micro-foundations of this model, which allows us to

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[10] Recall that, with fixed prices on one side, $\frac{dp_1^{A*}}{dc_1^A} > 0$ whenever Π^A is concave in p_1^A.

[11] All terms are evaluated at $\left(p_1^{B*}, p_1^{A*} \right)$.

[12] Note that the requirement $f_2 < 0$ rules out multihoming by all side 2 agents, since in that case demands for each platform on side 2 would be independent of each other (assuming away economies of scale in multihoming) and hence $f \left(n_1^B, n_1^A \right)$ would only depend on n_1^B.

obtain:

$$n_1^i = N_1 - p_1^i + \gamma p_1^j$$

$$n_2^i = \frac{N_2}{2} + \frac{N_2 \alpha_2}{2t_2} \left(n_1^i - n_1^j\right)$$

$$\Pi^i = \left(p_1^i - c_1^i\right) n_1^i + \pi_2 n_2^i \qquad (15)$$

where $N_1 > 0$, $N_2 > 0$, $t_2 > 0$, $\alpha_2 > 0$, $\pi_2 < 0$ (necessary as we have seen above) and $\gamma \in]0, 1[$ are all constants.

The linearity of demand in prices implies that platform i's profits are concave in p_1^i, which in turn implies $\frac{dp_1^A}{dc_1^A} > 0$, and that platform prices are strategic complements, i.e. $\frac{dp_1^i}{dp_1^j} > 0$ for all $i \neq j \in \{A, B\}$ (so that assumptions 1-4 are satisfied).

In the appendix we prove:

Proposition 1 *A necessary and sufficient condition for a cost reduction by platform A to be profit enhancing for platform B – (11) above – is:*

$$N_1 - c_1 (1 - \gamma) - |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) \frac{2(1 - \gamma)}{\gamma} < 0 \qquad (16)$$

∎

Condition (16) can be compared to the condition for n_1 to be positive in equilibrium[13]:

$$N_1 - c_1 (1 - \gamma) - |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) (1 - \gamma) > 0 \qquad (17)$$

and to the condition for platform profits to be positive in equilibrium:

$$\frac{\left[N_1 - c_1 (1 - \gamma) + |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma)\right] \left[N_1 - c_1 (1 - \gamma) - |\pi_2| \frac{N_2 \alpha_2}{2t_2} (1 + \gamma) (1 - \gamma)\right]}{(2 - \gamma)^2} > |\pi_2| \frac{N_2}{2} \qquad (18)$$

It is then easily verified that the last three conditions can hold simultaneously for a range of parameter values. Note that $\gamma < 1$ is absolutely necessary in making it possible that (16) and (17) hold at the same time.

To gain some intuition, note that (16), (17) (18) are clearly satisfied when $\gamma = 0$ and N_1 is sufficiently large relative to N_2. In this case, (15) yields $n_1^i = N_1 - p_1^i$ and so there is no competition between platforms on side 1. Meanwhile, on side 2 platforms are "forced" to lose money and each gains unprofitable side 2 customers in proportion to its market share advantage relative to its rival on side 1. Consequently, plaform A is always happy to lose market share on side 2 (since that

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[13]We use the equilibrium expressions of p_1 and n_1 provided in the proof of Proposition 1 in the appendix.

entails no loss in market share on side 1) and this happens whenever platform B's marginal cost c_1^B decreases because it leads to lower p_1^B, higher n_1^B and higher n_2^B. The condition that N_1 is sufficiently large relative to N_2 simply ensures that the market of unprofitable customers (side 2) is sufficiently small relative to the market of profitable customers (side 1) so that the platforms can make positive profits in equilibrium.

The result is thus straightforward when the two platforms are sufficiently differentiated so that they don't compete too much on side 1. When inter-platform competition on side 1 becomes more intense (i.e. γ increases), it becomes less and less likely that a cost reduction by one platform on side 1 will benefit its rival. This is because each platform is now ambivalent about losing customers on side 2: although that rids it of unprofitable customers, it also decreases its market share of profitable side 1 customers. One can confirm this intuition by noting that the left hand side of (16) is increasing in γ and strictly positive for $\gamma = 1$. Therefore, for γ high enough, a cost reduction by one platform always hurts its rival, as standard intuition suggests.

4.2   Fully two-sided pricing

Let us now turn to the general case, when prices on both sides of the market are flexible. An investment that reduces platform A's marginal cost c_1^A is soft if and only if (recall 7):

$$\left[\left(p_1^{B*} - c_1^B\right)\frac{\partial n_1^B}{\partial p_1^A} + \left(p_2^{B*} - c_2^B\right)\frac{\partial n_2^B}{\partial p_1^A}\right]\frac{dp_1^{A*}}{dc_1^A} + \left[\left(p_1^{B*} - c_1^B\right)\frac{\partial n_1^B}{\partial p_2^A} + \left(p_2^{B*} - c_2^B\right)\frac{\partial n_2^B}{\partial p_2^A}\right]\frac{dp_2^{A*}}{dc_1^A} < 0$$

which means we want at least one of the two terms in-between square brackets to be negative. Assume then that:

$$\left(p_1^{B*} - c_1^B\right)\frac{\partial n_1^B}{\partial p_1^A} + \left(p_2^{B*} - c_2^B\right)\frac{\partial n_2^B}{\partial p_1^A} < 0$$

or, equivalently

$$\left(p_1^{B*} - c_1^B\right) + \left(p_2^{B*} - c_2^B\right)\frac{\partial n_2^B}{\partial p_1^A}\left(\frac{\partial n_1^B}{\partial p_1^A}\right)^{-1} < 0 \tag{19}$$

Just like in the previous example, the main issue is also satisfying the first order conditions. In particular, the first order condition in p_1^B is:

$$\left(p_1^{B*} - c_1^B\right) + \left(p_2^{B*} - c_2^B\right)\frac{\partial n_2^B}{\partial p_1^B}\left(\frac{\partial n_1^B}{\partial p_1^B}\right)^{-1} = -n_1^B\left(\frac{\partial n_1^B}{\partial p_1^B}\right)^{-1} > 0 \tag{20}$$

To simplify things somewhat, assume that[14] $\frac{\partial \hat{n}_2^B}{\partial N_1^B} = -\frac{\partial \hat{n}_2^B}{\partial N_1^A} > 0$ (recall (5)). Then:

$$\frac{\partial n_2^B}{\partial p_1^A}\left(\frac{\partial n_1^B}{\partial p_1^A}\right)^{-1} = \frac{\partial \hat{n}_2^B}{\partial N_1^B}\left(1 + \left|\frac{\partial n_1^A}{\partial p_1^A}\left(\frac{\partial n_1^B}{\partial p_1^A}\right)^{-1}\right|\right) \quad \text{and} \quad \frac{\partial n_2^B}{\partial p_1^B}\left(\frac{\partial n_1^B}{\partial p_1^B}\right)^{-1} = \frac{\partial \hat{n}_2^B}{\partial N_1^B}\left(1 + \left|\frac{\partial n_1^A}{\partial p_1^B}\left(\frac{\partial n_1^B}{\partial p_1^B}\right)^{-1}\right|\right)$$

[14]This holds for example when platforms compete a la Hotelling with no hinterlands on side 2.

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Stability requires $\left|\frac{\partial n_1^A}{\partial p_1^B}\frac{\partial n_1^B}{\partial p_1^A}\right| \leq \left|\frac{\partial n_1^B}{\partial p_1^B}\frac{\partial n_1^A}{\partial p_1^A}\right|$, therefore, in order to satisfy (19) and (20) simultaneously, we must have:

$$p_2^{B*} - c_2^B < 0 \quad \text{and} \quad \left|\frac{\partial n_1^A}{\partial p_1^B}\left(\frac{\partial n_1^B}{\partial p_1^B}\right)^{-1}\right| < \left|\frac{\partial n_1^A}{\partial p_1^A}\left(\frac{\partial n_1^B}{\partial p_1^A}\right)^{-1}\right|$$

Again, this condition cannot be satisfied in the Armstrong (2006) model. Just like in the previous example, we need to have:

$$\frac{\partial n_1^A}{\partial p_1^B} = \frac{\partial n_1^B}{\partial p_1^A} = -\gamma\frac{\partial n_1^A}{\partial p_1^A} = -\gamma\frac{\partial n_1^B}{\partial p_1^B}$$

with $0 < \gamma < 1$[15]. In this case:

$$\frac{\partial n_2^B}{\partial p_1^A}\left(\frac{\partial n_1^B}{\partial p_1^A}\right)^{-1} = \frac{\partial \hat{n}_2^B}{\partial N_1^B}\left(1 + \frac{1}{\gamma}\right) > \frac{\partial \hat{n}_2^B}{\partial N_1^B}\left(1 + \gamma\right) = \frac{\partial n_2^B}{\partial p_1^B}\left(\frac{\partial n_1^B}{\partial p_1^B}\right)^{-1}$$

To obtain this, we posit the following demand functions:

$$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t_1} + \frac{x_1}{2t_1}\left(V_1 + u_1^i\right) \tag{21}$$

$$n_2^i = N_2\left[\frac{1}{2} + \frac{u_2^i - u_2^j}{2t_2}\right] \tag{22}$$

with:

$$u_1^i = \alpha_1 n_2^i - p_1^i \quad \text{and} \quad u_2^i = \alpha_2 n_1^i - p_2^i \tag{23}$$

where:

$$\alpha_1, \alpha_2, t_1, t_2, x_1, V_1, N_2 > 0 \tag{24}$$

We also assume that marginal costs are symmetric and non-negative for both platforms:

$$\begin{aligned} c_1^A &= c_1^B \equiv c_1 > 0 \\ c_2^A &= c_2^B \equiv c_2 \geq 0 \end{aligned} \tag{25}$$

Note that we have:

$$n_2^A + n_2^B = N_2$$

Relegating calculations in the appendix, we have:

Proposition 2 *There exists a range of parameters $(\alpha_1, \alpha_2, t_1, t_2, x_1, V_1, N_2, c_1, c_2)$ for which an investment that slightly decreases platform i's marginal cost c_1^i is soft, i.e. increases the profits of platform $j \neq i$, $i, j \in \{A, B\}$.*∎

The purpose of this example is to demonstrate the theoretical possibility for tying to be soft. The example is too stylized to warrant a full-blown numerical exercise. We have explored qualitatively

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[15]In the Armstrong (2006) model, $\gamma = 1$.

the factors that determine the likelihood and the strength of this effect. We leave it for future research to perform a quantitative analysis in a realistic structural model.

5   Conclusion

We have shown that strategic interactions in two-sided market duopolies are fundamentally different from those in one-sided markets. The possibility of cross-subsidization between the two sides leads to novel and counterintuitive results. In particular, a cost-reducing investment by one firm can both increase the profits of its rivals and be desirable for the firm undertaking the investment. Thus, increasing competitive advantage through cost advantage for one platform may end up benefiting both platforms (in a one sided market, cost reductions by one firm *always* hurt its rivals). The fundamental reason is that in the event one side of the market is subsidized, the platform gaining a cost advantage helps the rival platform by stealing customers on both sides in proportions that it was unprofitable for the latter to serve. This means that a cost-reducing investment can be part of a fat cat strategy under entry accomodation, which is never the case in a one-sided market, as shown by FT.

A prominent application is tying, which in a one-sided market is unambiguously tough. By contrast, our analysis implies that it can actually be soft in a two-sided market, which means that tying may in fact benefit both the tying firm and its rival. This can for example mean that when Microsoft is tying Windows Media Player into Windows (which can simply be interpreted as leveraging a distribution cost advantage on the consumer side over Real Networks and other rivals), *everyone* could possibly benefit. Of course, this possibility - in this example or other contexts - could only be confirmed through empirical analysis, which we leave for future research.

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6 Appendix

Proof. $(\frac{dp_A^*}{dc_A} > 0$ in section 2$)$ ∎

Let:

$$H\left(p_A, p_B, c_A\right) \equiv \left(p_A - c_A\right) \frac{\partial n_A}{\partial p_A}\left(p_A, p_B\right) + n_A\left(p_A, p_B\right)$$

Then we can write the first order condition in p_A as:

$$H\left(p_A^*, p_B, c_A\right) = 0$$

for all p_B and c_A. In particular:

$$H\left(p_A^{**}, p_B^{**}, c_A\right) = 0$$

Given that $\frac{\partial n_A}{\partial p_A} < 0$ and assuming the second order condition in p_A is satisfied, we have $\frac{\partial H}{\partial p_A} < 0$ and $\frac{\partial H}{\partial c_A} > 0$. Totally differentiating $H\left(p_A^{**}, p_B^{**}, c_A\right)$ with respect to c_A we obtain:

$$-\frac{\partial H}{\partial c_A} = \frac{dp_A^{**}}{dc_A}\left[\frac{\partial H}{\partial p_A} + \frac{\partial H}{\partial p_B}\frac{\partial p_B^*}{\partial p_A}\right]$$

Meanwhile, total differentiation of $H\left(p_A^*, p_B, c_A\right)$ with respect to p_B yields:

$$\frac{\partial H}{\partial p_B} + \frac{\partial H}{\partial p_A}\frac{\partial p_A^*}{\partial p_B} = 0$$

Evaluating the last two equations in $\left(p_A^{**}, p_B^{**}, c_A\right)$, we obtain:

$$\frac{dp_A^{**}}{dc_A} = \frac{-\frac{\partial H}{\partial c_A}}{\frac{\partial H}{\partial p_A}\left(1 - \frac{\partial p_B^*}{\partial p_A}\frac{\partial p_A^*}{\partial p_B}\right)}$$

Finally, the stability of the Nash equilibrium $\left(p_A^{**}, p_B^{**}\right)$ implies that $1 > \frac{\partial p_B^*}{\partial p_A}\frac{\partial p_A^*}{\partial p_B}$, therefore $\frac{dp_A^{**}}{dc_A} > 0$.

∎

Micro-foundations leading to the demand system (15)

We assume that on side i there is a Hotelling segment of consumers with linear transportation costs t_i, standalone valuation for either platform V_i and additional utility α_i per customer on side j of the same platform, with $i \neq j \in \{1.2\}$. We assume the total mass of these customers is 1 on side 1 and M_2 on side 2. Furthermore, on side 1, each platform faces a downward-sloping demand of "loyal" customers (i.e. who are never interested in the rival platform), with standalone valuation V_1, utility per side 2 customer on the same platform β_1 and transportation costs $\frac{2t_1}{x}$. This yields the following expression of for platform i's demands:

$$n_1^i = \frac{1}{2} + \frac{\alpha_1\left(n_2^i - n_2^j\right) + \left(p_1^j - p_1^i\right)}{2t_1} + \frac{x}{2t_1}\left(V_1 + \beta_1 n_2^i - p_1^i\right)$$

$$n_2^i = M_2\left(\frac{1}{2} + \frac{\alpha_2\left(n_1^i - n_1^j\right)}{2t_2}\right)$$

where $\alpha_1,\ \alpha_2,\ x,\ V_1,\ \beta_1,\ t_1,\ t_2 > 0$.

Solving this demand system for $\left(n_1^i, n_2^i\right)$ as functions of prices only, we obtain:

$$n_1^i = M_1 - \theta p_1^i + \lambda p_2^j$$

with:

$$M_1 = \frac{t_1 + xV_1 + \beta_1\frac{M_2}{2}}{2t_1}$$

$$\theta = \frac{(1+x)\left(1 - \frac{M_2\alpha_2\alpha_1 + M_2\alpha_2 x\beta_1}{4t_1 t_2}\right) + \frac{M_2\alpha_2\alpha_1 + M_2\alpha_2 x\beta_1}{4t_1 t_2}}{4t_1\left(\frac{1}{2} - \frac{M_2\alpha_2\alpha_1 + M_2\alpha_2 x\beta_1}{4t_1 t_2}\right)}$$

$$\lambda = \frac{\left(1 - \frac{M_2\alpha_2\alpha_1 + M_2\alpha_2 x\beta_1}{4t_1 t_2}\right) + (1+x)\frac{M_2\alpha_2\alpha_1 + M_2\alpha_2 x\beta_1}{4t_1 t_2}}{4t_1\left(\frac{1}{2} - \frac{M_2\alpha_2\alpha_1 + M_2\alpha_2 x\beta_1}{4t_1 t_2}\right)}$$

We need to impose:

$$\frac{1}{2} > \frac{M_2 \alpha_2 \alpha_1 + M_2 \alpha_2 x \beta_1}{4 t_1 t_2}$$

which not surprisingly implies:

$$0 < \lambda < \theta$$

To simplify things, we let then $\gamma = \frac{\lambda}{\theta} < 1$ and $N_1 = \frac{M_1}{\theta}$; $N_2 = \frac{M_2}{\theta}$, since we can factor all profits by θ without changing anything.

So everything is as if we had:

$$n_1^i = N_1 - p_1^i + \gamma p_1^j$$

$$n_2^i = \frac{N_2}{2} + \frac{N_2 \alpha_2}{2 t_2} \left(n_1^i - n_1^j \right)$$

with $\gamma < 1$. $\blacksquare$

Proof. (Proposition 1)   The first order condition ((12) in the text) at the symmetric equilibrium $(p_1^A = p_1^B = p_1$ and $n_1^A = n_1^B = n_1)$ can be written as:

$$p_1 - c_1 + \pi_2 \frac{N_2 \alpha_2}{2 t_2} (1 + \gamma) = n_1 > 0$$

At the symmetric equilibrium:

$$n_1 = N_1 - (1 - \gamma) p_1$$

Plugging in the first order condition above, we obtain:

$$p_1 = \frac{c_1}{2 - \gamma} + \frac{N_1 - \pi_2 \frac{N_2 \alpha_2}{2 t_2} (1 + \gamma)}{2 - \gamma}$$

The condition for a cost reduction by platform A to be profit enhancing for platform B ((11) above) is then:

$$p_1 - c_1 + \pi_2 \frac{N_2 \alpha_2}{2 t_2} \frac{\gamma + 1}{\gamma} < 0$$

Replacing p_1 with its expression as a function of the model parameters, this condition is equivalent to $(16)^{16}$. $\blacksquare$ $\blacksquare$

Proof. (Proposition 2)   Solving (21), (22) and (23) for n_1^i and n_2^i, $i = A, B$, we obtain:

$$n_2^A = \frac{N_2}{2} + \frac{\alpha_2 (2 + x_1) N_2 \left(p_1^B - p_1^A \right) + 2 t_1 \left(p_2^B - p_2^A \right)}{4 t_1 t_2 - 2 \alpha_1 \alpha_2 (2 + x_1) N_2}$$

$$n_2^B = \frac{N_2}{2} + \frac{\alpha_2 (2 + x_1) N_2 \left(p_1^A - p_1^B \right) + 2 t_1 \left(p_2^A - p_2^B \right)}{4 t_1 t_2 - 2 \alpha_1 \alpha_2 (2 + x_1) N_2}$$

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[16] Recall that $\pi_2 < 0$ so that $|\pi_2| = -\pi_2 > 0$.

$$n_1^A = \frac{t_1 + V_1 x_1 + \frac{\alpha_1 N_2}{2}}{2t_1} + \frac{\alpha_1 (2 + x_1)\left(p_2^B - p_2^A\right)}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2}$$

$$+ \frac{4t_1 t_2 + \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2} \times \frac{p_1^B}{2t_1} - \frac{4t_1 t_2 (1 + x_1) - \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2} \times \frac{p_1^A}{2t_1}$$

$$n_1^B = \frac{t_1 + V_1 x_1 + \frac{\alpha_1 N_2}{2}}{2t_1} + \frac{\alpha_1 (2 + x_1)\left(p_2^A - p_2^B\right)}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2}$$

$$+ \frac{4t_1 t_2 + \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2} \times \frac{p_1^A}{2t_1} - \frac{4t_1 t_2 (1 + x_1) - \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2} \times \frac{p_1^B}{2t_1}$$

We need to impose the following condition in order for this demand system to be well-defined:

$$4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2 > 0 \tag{26}$$

In order to simplify notation for the calculation of the pricing equilibrium, let:

$$\gamma_1 \equiv \frac{\alpha_1 (2 + x_1)}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2} \quad ; \quad \gamma_2 \equiv \frac{\alpha_2 (2 + x_1) N_2}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2}$$

$$\delta \equiv \frac{2t_1}{4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2} \quad ; \quad \varepsilon \equiv \frac{4t_1 t_2 + \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{2t_1 \left[4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2\right]}$$

$$N_1 \equiv \frac{t_1 + V_1 x_1 + \frac{\alpha_1 N_2}{2}}{2t_1} \quad ; \quad 1 + u \equiv \frac{4t_1 t_2 (1 + x_1) - \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}{4t_1 t_2 + \alpha_1 \alpha_2 x_1 (2 + x_1) N_2}$$

Clearly $u > 0$ since we must have $4t_1 t_2 - 2\alpha_1 \alpha_2 (2 + x_1) N_2 > 0$. We can now write:

$$n_1^A = \frac{N_1}{2} + \gamma_1 \left(p_2^B - p_2^A\right) + \varepsilon p_1^B - \varepsilon (1 + u) p_1^A$$

$$n_1^B = \frac{N_1}{2} + \gamma_1 \left(p_2^A - p_2^B\right) + \varepsilon p_1^A - \varepsilon (1 + u) p_1^B$$

$$n_2^A = \frac{N_2}{2} + \gamma_2 \left(p_1^B - p_1^A\right) + \delta \left(p_2^B - p_2^A\right)$$

$$n_2^B = \frac{N_2}{2} + \gamma_2 \left(p_1^A - p_1^B\right) + \delta \left(p_2^A - p_2^B\right)$$

Recall that platform profits are:

$$\Pi^A = \left(p_1^A - c_1^A\right) n_1^A + \left(p_2^A - c_2^A\right) n_2^A$$

$$\Pi^B = \left(p_1^B - c_1^B\right) n_1^B + \left(p_2^B - c_2^B\right) n_2^B$$

We start from a symmetric situation with $c_1^A = c_1^B = c_1$ and $c_2^A = c_2^B = c_2$ and consider a slight decrease

in c_1^A. Using the demand expressions above, the first order conditions in prices are:

$$\left(p_1^A - c_1^A\right) \times \varepsilon \left(1 + u\right) + \left(p_2^A - c_2^A\right) \gamma_2 = \frac{N_1}{2} + \gamma_1 \left(p_2^B - p_2^A\right) + \varepsilon p_1^B - \varepsilon \left(1 + u\right) p_1^A \tag{27}$$

$$\left(p_1^A - c_1^A\right) \times \gamma_1 + \left(p_2^A - c_2^A\right) \delta = \frac{N_2}{2} + \gamma_2 \left(p_1^B - p_1^A\right) + \delta \left(p_2^B - p_2^A\right) \tag{28}$$

$$\left(p_1^B - c_1^B\right) \times \varepsilon \left(1 + u\right) + \left(p_2^B - c_2^B\right) \gamma_2 = \frac{N_1}{2} + \gamma_1 \left(p_2^A - p_2^B\right) + \varepsilon p_1^A - \varepsilon \left(1 + u\right) p_1^B \tag{29}$$

$$\left(p_1^B - c_1^B\right) \times \gamma_1 + \left(p_2^B - c_2^B\right) \delta = \frac{N_2}{2} + \gamma_2 \left(p_1^A - p_1^B\right) + \delta \left(p_2^A - p_2^B\right) \tag{30}$$

We can use (29) and (30) to determine platform B's best response prices $\left(p_1^B, p_2^B\right)$ as a function of platform A's prices $\left(p_1^A, p_2^A\right)$:

$$p_1^B = \frac{1}{4\delta\varepsilon\left(1 + u\right) - \left(\gamma_1 + \gamma_2\right)^2} \times \left\{ \begin{array}{c} 2\delta\frac{N_1}{2} - \left(\gamma_1 + \gamma_2\right)\frac{N_2}{2} \\ + \left[2\delta\varepsilon\left(1 + u\right) - \left(\gamma_1 + \gamma_2\right)\gamma_1\right]c_1^B + \delta\left(\gamma_2 - \gamma_1\right)c_2^B \\ + \left[2\delta\varepsilon - \left(\gamma_1 + \gamma_2\right)\gamma_2\right]p_1^A + \delta\left(\gamma_1 - \gamma_2\right)p_2^A \end{array} \right\}$$

$$p_2^B = \frac{1}{4\delta\varepsilon\left(1 + u\right) - \left(\gamma_1 + \gamma_2\right)^2} \times \left\{ \begin{array}{c} 2\varepsilon\left(1 + u\right)\frac{N_2}{2} - \left(\gamma_1 + \gamma_2\right)\frac{N_1}{2} \\ + \varepsilon\left(1 + u\right)\left(\gamma_1 - \gamma_2\right)c_1^B + \left[2\varepsilon\delta\left(1 + u\right) - \left(\gamma_1 + \gamma_2\right)\gamma_2\right]c_2^B \\ + \varepsilon\left[\gamma_2\left(1 + 2u\right) - \gamma_1\right]p_1^A + \left[2\varepsilon\delta\left(1 + u\right) - \gamma_1\left(\gamma_1 + \gamma_2\right)\right]p_2^A \end{array} \right\}$$

Therefore, prices are strategic complements across platforms ($\frac{\partial p_i^k}{\partial p_j^l} > 0$ for all $i, j \in \{1, 2\}$ and $k \neq l \in \{A, B\}$) if and only if the following 5 conditions hold:

$$\left\{ \begin{array}{c} 4\delta\varepsilon\left(1 + u\right) - \left(\gamma_1 + \gamma_2\right)^2 > 0 \\ 2\delta\varepsilon - \left(\gamma_1 + \gamma_2\right)\gamma_2 > 0 \\ \gamma_1 - \gamma_2 > 0 \\ \gamma_2\left(1 + 2u\right) - \gamma_1 > 0 \\ 2\varepsilon\delta\left(1 + u\right) - \gamma_1\left(\gamma_1 + \gamma_2\right) > 0 \end{array} \right. \tag{31}$$

Taking the derivative in c_1^A of both sides of the equality in (27), (28), (29) and (30) above, we obtain the following system of 4 equations with 4 unknowns $(\frac{dp_1^A}{dc_1^A}, \frac{dp_2^A}{dc_1^A}, \frac{dp_1^B}{dc_1^A}, \frac{dp_2^B}{dc_1^A})$:

$$\frac{dp_1^A}{dc_1^A} \times 2\varepsilon\left(1 + u\right) + \frac{dp_2^A}{dc_1^A} \times \left(\gamma_1 + \gamma_2\right) - \frac{dp_1^B}{dc_1^A} \times \varepsilon - \frac{dp_2^B}{dc_1^A} \times \gamma_1 = \varepsilon\left(1 + u\right)$$

$$\frac{dp_1^A}{dc_1^A} \times \left(\gamma_1 + \gamma_2\right) + \frac{dp_2^A}{dc_1^A} \times 2\delta - \frac{dp_1^B}{dc_1^A} \times \gamma_2 - \frac{dp_2^B}{dc_1^A} \times \delta = \gamma_1$$

$$\frac{dp_1^B}{dc_1^A} \times 2\varepsilon\left(1 + u\right) + \frac{dp_2^B}{dc_1^A} \times \left(\gamma_1 + \gamma_2\right) - \frac{dp_1^A}{dc_1^A} \times \varepsilon - \frac{dp_2^A}{dc_1^A} \times \gamma_1 = 0$$

$$\frac{dp_1^B}{dc_1^A} \times \left(\gamma_1 + \gamma_2\right) + \frac{dp_2^B}{dc_1^A} \times 2\delta - \frac{dp_1^A}{dc_1^A} \times \gamma_2 - \frac{dp_2^A}{dc_1^A} \times \delta = 0$$

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Taking the difference between the first and third equalities, then between the second and fourth, we obtain:

$$\left(\frac{dp_1^A}{dc_1^A} - \frac{dp_1^B}{dc_1^A}\right) \times \varepsilon\,(3 + 2u) + \left(\frac{dp_2^A}{dc_1^A} - \frac{dp_2^B}{dc_1^A}\right) \times (2\gamma_1 + \gamma_2) = \varepsilon\,(1 + u)$$

$$\left(\frac{dp_1^A}{dc_1^A} - \frac{dp_1^B}{dc_1^A}\right) \times (\gamma_1 + 2\gamma_2) + \left(\frac{dp_2^A}{dc_1^A} - \frac{dp_2^B}{dc_1^A}\right) \times 3\delta = \gamma_1$$

Similarly, summing first and third, then second and fourth, we also have:

$$\left(\frac{dp_1^A}{dc_1^A} + \frac{dp_1^B}{dc_1^A}\right) \times \varepsilon\,(1 + 2u) + \left(\frac{dp_2^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A}\right) \times \gamma_2 = \varepsilon\,(1 + u)$$

$$\left(\frac{dp_1^A}{dc_1^A} + \frac{dp_1^B}{dc_1^A}\right) \times \gamma_1 + \left(\frac{dp_2^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A}\right) \times \delta = \gamma_1$$

Solving the two systems above, we finally obtain:

$$\frac{dp_1^A}{dc_1^A} - \frac{dp_1^B}{dc_1^A} = \frac{3\delta\varepsilon\,(1 + u) - \gamma_1\,(2\gamma_1 + \gamma_2)}{3\delta\varepsilon\,(3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)}$$

$$\frac{dp_2^A}{dc_1^A} - \frac{dp_2^B}{dc_1^A} = \frac{\gamma_1\varepsilon\,(3 + 2u) - (\gamma_1 + 2\gamma_2)\,\varepsilon\,(1 + u)}{3\delta\varepsilon\,(3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} = \frac{\varepsilon\,[\gamma_1\,(2 + u) - \gamma_2\,(2 + 2u)]}{3\delta\varepsilon\,(3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)}$$

$$\frac{dp_1^A}{dc_1^A} + \frac{dp_1^B}{dc_1^A} = \frac{\delta\varepsilon\,(1 + u) - \gamma_1\gamma_2}{\delta\varepsilon\,(1 + 2u) - \gamma_1\gamma_2}$$

$$\frac{dp_2^A}{dc_1^A} + \frac{dp_2^B}{dc_1^A} = \frac{\gamma_1\varepsilon\,(1 + 2u) - \gamma_1\varepsilon\,(1 + u)}{\delta\varepsilon\,(1 + 2u) - \gamma_1\gamma_2} = \frac{\gamma_1\varepsilon u}{\delta\varepsilon\,(1 + 2u) - \gamma_1\gamma_2}$$

We then immediately get the two terms we are interested in:

$$\frac{dp_1^A}{dc_1^A} = \frac{1}{2}\left[\frac{3\delta\varepsilon\,(1 + u) - \gamma_1\,(2\gamma_1 + \gamma_2)}{3\delta\varepsilon\,(3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} + \frac{\delta\varepsilon\,(1 + u) - \gamma_1\gamma_2}{\delta\varepsilon\,(1 + 2u) - \gamma_1\gamma_2}\right] \tag{32}$$

$$\frac{dp_2^A}{dc_1^A} = \frac{1}{2}\left[\frac{\varepsilon\,[\gamma_1\,(2 + u) - \gamma_2\,(2 + 2u)]}{3\delta\varepsilon\,(3 + 2u) - (\gamma_1 + 2\gamma_2)(2\gamma_1 + \gamma_2)} + \frac{\gamma_1\varepsilon u}{\delta\varepsilon\,(1 + 2u) - \gamma_1\gamma_2}\right] \tag{33}$$

We want both of these terms to be positive.

Finally, we also have to determine the symmetric equilibrium $p_i^A = p_i^B = p_i$ and $n_i^A = n_i^B = n_i$ for $i = 1, 2$ and $c_1^A = c_1^B = c_1$. It suffices to use the four first order conditions above (27, 28, 29, 30) in order to obtain:

$$p_1 \times \varepsilon\,(1 + 2u) + p_2 \times \gamma_2 = \frac{N_1}{2} + c_1 \times \varepsilon\,(1 + u) + c_2 \times \gamma_2$$

$$p_1 \times \gamma_1 + p_2 \times \delta = \frac{N_2}{2} + c_1 \times \gamma_1 + c_2 \times \delta$$

which we can immediately solve to get:

$$p_1 - c_1 = \frac{-\delta\varepsilon u}{\delta\varepsilon\,(1 + 2u) - \gamma_1\gamma_2}c_1 + \frac{N_1\delta - N_2\gamma_2}{2\,[\delta\varepsilon\,(1 + 2u) - \gamma_1\gamma_2]}$$

$$p_2 - c_2 = \frac{\gamma_1 \varepsilon u}{\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2} c_1 + \frac{N_2 \varepsilon (1 + 2u) - N_1 \gamma_1}{2 [\delta \varepsilon (1 + 2u) - \gamma_1 \gamma_2]}$$

We can now write all the conditions that need to be satisfied in order for an equilibrium in which a slight reduction in c_1^A increases B's profits while prices are strategic complements across platforms to exist.

Using the expressions of n_i^k ($i \in \{1, 2\}$, $k \in \{A, B\}$) as functions of $(p_1^A, p_2^A, p_1^B, p_2^B)$, the softness condition is:

$$[\varepsilon (p_1 - c_1) + \gamma_2 (p_2 - c_2)] \frac{dp_1^A}{dc_1^A} + [\gamma_1 (p_1 - c_1) + \delta (p_2 - c_2)] \frac{dp_2^A}{dc_1^A} < 0$$

which can be rewritten as:

$$(p_1 - c_1) \left(\varepsilon \frac{dp_1^A}{dc_1^A} + \gamma_1 \frac{dp_2^A}{dc_1^A} \right) + (p_2 - c_2) \left(\gamma_2 \frac{dp_1^A}{dc_1^A} + \delta \frac{dp_2^A}{dc_1^A} \right) < 0 \tag{34}$$

At the same time, just like in the previous example, we need to make sure that the equilibrium n_1 is positive:

$$n_1 = \frac{1}{2} + \frac{x_1}{2t_1} \left(V_1 + \alpha_1 \frac{N_2}{2} - p_1 \right) > 0 \tag{35}$$

Finally, platforms must make positive profits in equilibrium:

$$(p_1 - c_1) \left[\frac{1}{2} + \frac{x_1}{2t_1} \left(V_1 + \alpha_1 \frac{N_2}{2} - p_1 \right) \right] + (p_2 - c_2) \frac{N_2}{2} > 0 \tag{36}$$

Together with (24), (25), (26), (31), (32) and (33), (34), (35) and (36) complete the set of necessary conditions.

We have then used Mathematica to show that this system of necessary conditions can be satisfied for a range of initial parameter values (ε, δ, u, γ_1, γ_2, N_1, N_2, c_1, c_2) or of the primitive parameters (α_1, α_2, t_1, t_2, x_1, V_1, N_2, c_1, c_2). One such solution is:

$$(\alpha_1, \alpha_2, t_1, t_2, x_1, V_1, N_2, c_1, c_2) = \left(1, 1.001, \frac{23}{16}, 2, \frac{1}{160}, 3, \frac{127}{128}, 106, 1.1 \right)$$

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