Structural GARCH: The Volatility-Leverage Connection

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Introduction
BAC Leverage and Realized Volatility

![Graph showing BAC Leverage and Realized Volatility from 1998 to 2012](image-url)
Leverage and Equity Volatility

- Crisis highlighted how leverage and equity volatility are tightly linked
- “Leverage Effect” has been around - e.g. Black (1976), Christie (1982) - but...
- A dynamic volatility model that incorporates leverage directly has remained elusive
This Paper

- GARCH-type model where equity volatility is amplified (non-linearly) by leverage as in structural models of credit
- **Asset** return series from observed equity series
- **Assets** have time-varying volatility at high frequencies
- Statistical test of how leverage affects volatility
- Two applications:
  1. Systemic Risk: SRISK and Precautionary Capital (today)
  2. Leverage Effect (in the paper)
Theoretical Foundation
Structural Models of Credit

- Under relatively weak assumptions on the vol process, structural models say $E_t = f(A_t, D_t, \sigma_{A,t}, \tau, r_t)$
  - $A_t =$ market value of assets
  - $D_t =$ book value of debt
  - $\sigma_{A,t} =$ stochastic asset volatility
- Generic dynamics for assets and asset variance (allow for jumps later):
  \[
  \frac{dA_t}{A_t} = \mu_A(t)dt + \sigma_{A,t} dB_A(t) \\
  d\sigma_{A,t}^2 = \mu_V(t, \sigma_{A,t})dt + \sigma_V(t, \sigma_{A,t}) dB_V(t)
  \]
- $B_A(t)$ and $B_V(t)$ potentially correlated
Equity Returns and Equity Volatility

Introducing the Leverage Multiplier

- Apply Itô Lemma and ignore drift (our model is daily, and daily equity returns ≈ 0):

\[
\frac{dE_t}{E_t} = LM_t \sigma_{A,t} dB_A(t) + \frac{\nu_t \sigma_v(t, \sigma_{A,t})}{2\sigma_{A,t}} dB_v(t)
\]

\[
\approx LM_t \times \sigma_{A,t} \times dB_A(t)
\]

\[
vol_t \left( \frac{dE_t}{E_t} \right) \approx LM_t \times \sigma_{A,t}
\]

- \(LM_t = LM\left( E_t / D_t, 1, \sigma_{A,t}, \tau, r_t \right)\) is the “leverage multiplier”

- \(LM_t\) amplifies asset shocks and volatility

- Two questions:

  1. How much does the higher order term contribute? Not Much
  2. What does \(LM_t\) look like? Robust shape across models
The Leverage Multiplier: Three Basic Properties

Popular Continuous Time Option Pricing Models

Discrete Time: GARCH Option Pricing

1. \( LM(0) = 1 \). Mechanical, since assets = equity
2. Monotonically increasing. More leverage means more risk
3. Concave. Reducing leverage more powerful than increasing leverage
Structural GARCH
Our Specification

- The challenge is choosing the right functional form for $LM_t$
- We use simple transformations of Black-Scholes-Merton (BSM) functions:

$$LM_t(D_t/E_t, \sigma^f_{A,t}, \tau) = \left[ \Delta^{BSM}_t \times g^{BSM}(E_t/D_t, 1, \sigma^f_{A,t}, \tau) \times \frac{D_t}{E_t} \right]^{\phi}$$

$g^{BSM}(\cdot)$ is inverse BSM call function. $\Delta^{BSM}_t$ is BSM delta

- $\phi \neq$ specific option pricing model
- Our parametrization preserves necessary properties of $LM$, but still allows us to change its scale
The Full Recursive Model

Structural GARCH

\[ r_{E,t} = LM_{t-1} \times r_{A,t} \]
\[ = LM_{t-1} \times \sqrt{h_{A,t} \times \varepsilon_{A,t}} \]
\[ h_{A,t} \sim GJR(\omega, \alpha, \gamma, \beta) \]

\[ LM_{t-1} = \left[ \Delta_{t-1}^{BSM} \times g^{BSM} \left( E_{t-1}/D_{t-1}, 1, \sigma_{A,t-1}^f, \tau \right) \times \frac{D_{t-1}}{E_{t-1}} \right]^\phi \]

\[ \sigma_{A,t-1}^f = \sqrt{E_{t-1} \left[ \sum_{s=t}^{t+\tau} h_{A,s} \right]} \]

So parameter set is \( \Theta = (\omega, \alpha, \gamma, \beta, \phi) \)
Estimation Results
Estimation Details

- Estimate for 82 financials via QMLE; iterate over $\tau \in [1, 30]$
- Equity returns and balance sheet information from Bloomberg
- $D_t$ is exponentially smoothed book value of debt
  - smoothing parameter $= 0.01$, so half-life of weights $\approx 70$ days
- We estimate the model using two approaches for $\sigma_{A,t-1}^f$, then use the highest likelihood:
  1. A dynamic forecast for asset volatility over life of the option
  2. The unconditional volatility of the asset GJR process
Bank of America: Structural GARCH Estimation

$\phi = 1.4 \ (t = 11.4)$
## Parameter Values

### Cross-Sectional Summary of Estimated Parameters

| Parameter | Mean   | Mean t-stat | % with $|t| > 1.64$ |
|-----------|--------|-------------|---------------|
| $\omega$ | 2.7e-06 | 1.70        | 47.2          |
| $\alpha$ | 0.0458 | 3.07        | 86.1          |
| $\gamma$ | 0.0721 | 2.91        | 80.6          |
| $\beta$  | 0.9024 | 80.08       | 100           |
| $\phi$   | 0.9834 | 4.00        | 73.6          |

- $(\omega, \alpha, \gamma, \beta)$ are standard GJR parameters - for assets, not equity
- Average $\tau = 8.34$
- Leverage matters
Application: SRISK
SRISK
How much would a financial firm need to function normally in another crisis?

- Acharya et. al (2012) and Brownlees and Engle (2012)
- Three steps:
  1. GJR-DCC model using firm equity and market index returns
  2. Expected firm equity return if market falls by 40% over 6 months \[\equiv \text{LRMES}\]
  3. Combine LRMES with book value of debt to determine capital shortfall in a crisis
- The crisis in this case is a 40% drop in the stock market index over 6 months
The Role of Leverage?
Thought Experiment with Structural GARCH

- Firm experiences sequence of negative equity (asset) shocks
- Level of leverage goes up rapidly
- Leverage multiplier increases, equity vol amplification higher
- Painfully obvious in the crisis, so build into SRISK
Bank of America
Capital Shortfall: 2006-2011
Precautionary Capital
Defining Precautionary Capital

e.g. How much additional equity would a bank need, today, to be 90% sure they won’t need bailout money in a future crisis?

- **SRISK**: how much capital would a firm need in a financial crisis to return to a equity/asset ratio of \( k \)%?
- **Precautionary Capital**: How much capital do we have to add to the firm *today* so that we can have a level of certainty, \( \alpha \), that the firm meets a capital requirement of \( k \)% in a crisis?
- Uses the quantiles of the future return distribution
- We set \( k = 2\% \) and vary \( \alpha \)
Primary Takeaway in a Nutshell

- Standard volatility models don’t have a channel for leverage, so adding equity to the firm today won’t reduce future risk
- Structural GARCH: reducing leverage today reduces future risk
  - The effect is further enhanced by the concavity of the $LM$
- Engle and Siriwardane (2014) use this idea to suggest a risk-based total leverage capital requirement
Precautionary Capital: BAC

BAC on 10/1/2008: $E_0 = 173.9$ bn; $D_0 = 1,670.1$ bn

![Graph showing confidence that firm meets capital requirement in crisis vs. size of equity injection. The graph includes two lines representing GJR and Structural GARCH models, with k=0.02.]
What’s Next
Other Applications

- Endogenous Crisis Probability with Structural GARCH
- Estimation of Distance to Crisis
- Endogenous Capital Structure and Leverage Cycles
- Counter-cyclical Capital Regulation
- Model of CDS Volatility
Appendix
Ignore Higher Order Terms

\[
\frac{dE_t}{E_t} = LM_t \sigma_{A,t} dB_A(t) + \frac{\nu_t \sigma_v(t, \sigma_{A,t})}{E_t} \frac{1}{2\sigma_{A,t}} dB_v(t)
\]

How much do the higher order terms contribute?

- Not much. Simple intuition...
- Volatility mean reversion speed \(\ll\) typical debt maturities, so ...
- Total volatility over option is effectively constant
- We verify in paper for variety of option pricing models
Dynamic Forecast vs Constant Forecast

- Estimate two types of models:
  1. Using a dynamic forecast for asset volatility over life of the option
  2. Using unconditional volatility of GJR process
- Then take the model that delivers the highest likelihood
- A few outliers where $\phi$ hits lower bound (exclude from subsequent analysis):
  - SCHW, JNS, LM, BK, BLK, NTRS, CME, CINF, TMK, UNH