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Self-Matching as a Retailer's  
Multichannel Pricing Strategy

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Multichannel retailing has created several new strategic choices for firms. With respect to pricing, an important decision is whether to offer a “self-matching policy.” Self-matching allows a multichannel retailer to offer the lowest of its online and in-store prices to consumers. In practice, we observe considerable heterogeneity in self-matching policies: there are firms that offer to self-match and firms that explicitly state they will not match prices across channels. Using a game-theoretic model, we investigate the strategic forces behind the adoption (or non-adoption) of self-matching across a range of competitive scenarios, including a monopolist, a mixed duopoly comprised of a multichannel retailer competing with a pure e-tailer, as well as two competing multichannel retailers. Even though self price matching is likely to reduce a retailer’s profits, with some consumers paying the lower price, we uncover two novel mechanisms that can make self-matching profitable in a duopoly setting. Specifically, self-matching can dampen competition, both online and in-store, and its effectiveness in this respect depends on the decision-making stage of various consumers and the heterogeneity of their preference for the online vs. store channels. Surprisingly, self-matching strategies can also be profitable when stores face consumers using smartphones to discover online prices. Our findings provide insights for managers on how and when self-matching can be an effective pricing strategy to embrace.

Key words: Price Self-matching, Multichannel Retailing, Pricing Strategy

1. Introduction

Many, if not most, major retailers today employ a multichannel business model - they offer products in physical stores and online. These channels tend to attract different consumer segments and allow retailers to support different buying behaviors and preferences. Concomitantly, consumers are becoming more savvy in utilizing the various channels during the buying process: researching products, evaluating fit, comparing prices and purchasing (Neslin et al. 2006).
Retailers need to attend to all elements of the marketing mix as they strive to maximize profits. Not surprisingly, pricing has always been an important strategic variable for them to “get right”. When firms were predominantly single-channel, this meant figuring out the most effective in-store price to set for their merchandise. However, having embraced a multi-channel selling format, pricing decisions have become much more complex for these retailers to navigate. Not only do they need to price the products in their physical stores, they need to set prices for products in their online outlets, and consider how the prices across the various channels should relate to one another. This complexity in devising a comprehensive multi-channel pricing strategy is front and center for retailers today, as evidenced by the myriad of commentaries in the retail trade press. For instance, according to Forrester Research (Mulpuru (2012)), “…Given that the majority of retail sales are expected to be influenced by the Web to some degree, it becomes imperative for eBusiness professionals in retail to adopt cross-channel best practices, particularly related to pricing,” and Retail TouchPoints (Fiorletta (2013)) notes that, “Amplified price transparency due to the instant availability of information via the web and mobile devices has encouraged retailers to rethink their omnichannel pricing strategies.”

A critical decision retailers face with respect to managing cross channel pricing is whether to implement a self-matching pricing policy. In contrast to competitive matching policies studied extensively (and reviewed in §2), self-matching is only relevant for multichannel retailers. With such a policy, the retailer commits to charging consumers the lower of its online and in-store prices for the same product when consumers produce appropriate evidence.¹ Commonly, this policy features in-store to online self-matching, allowing consumers to pay typically lower online prices for in-store purchases.²

When we examine several industry contexts, we observe considerable heterogeneity in the adoption of self-matching policies across retailers. For example, RadioShack, Best Buy, Target, Staples and Toys “R” Us price-match their online channels in-store, whereas Walmart, Urban Outfitters and Sports Authority explicitly state that they will not match their prices across channels.³ Interestingly, we also have significant heterogeneity in self-matching across industries. Consider consumer electronics retailing where both major players, Best Buy and RadioShack, choose to offer self-matching, whereas in discount retailing, Target adopts self-matching, and Walmart does not.

¹ A webpage printout, or a mobile screenshot of the webpage usually suffices as appropriate evidence.
² Policies allowing self-matching in the other direction, i.e., allowing web customers to match store prices, are rarely observed in practice. This is likely due to several reasons, first it is more difficult to furnish evidence online for the lower store price. Second, as our analysis will show, the online price is typically lower than the store price in equilibrium; hence an online to in-store matching is unlikely to ever arise.
³ See Appendix C for examples of self-matching policies from retailer websites.
The main objective of our paper is to understand the forces governing the decision to implement (or refrain from implementing) a self-matching pricing policy and to shed light on when to expect different patterns whereby multi-channel retailers in a given category: all self-match, some self-match while others don’t, and none self-match. To this end, we examine how a self-matching pricing policy can be used as a strategic tool by multichannel retailers in a range of competitive settings: a monopoly; a mixed duopoly in which a multichannel retailer competes with an e-tailer; and a duopoly with two multi-channel retailers. With the model we seek to uncover the mechanisms underlying the effectiveness of this pricing practice and address the following specific questions:

1. When do multichannel retailers choose to self-match prices across their channels? How do customer characteristics and the nature of competition influence a retailer’s desire to adopt a self-matching pricing policy?

2. What are the mechanisms underlying self-matching? Does self-matching reduce the price levels or does it support higher prices?

3. Are retailers always better or worse off having access to price matching as a strategic tool?

4. Does greater consumer price transparency across channels, e.g., when consumers use smart devices, make self-matching less likely to be offered? Is a self-matching retailer necessarily worse off as more consumers become smartphone-enabled?

To investigate these questions, we develop a model that captures the implications of self-matching for consumers and firms. Specifically, consumers are heterogeneous along a number of important dimensions: their channel preferences, their stage in the decision process, and their preference over retailers. With regard to channel preferences, we allow for “store-only” consumers who have a strong preference to purchase in-store where they can ‘touch and feel’ products to evaluate their fit and obtain the product instantly, among other reasons. Another set of consumers, which we call “channel agnostic”, don’t have a strong preference from which channel they purchase. Next, we distinguish between consumers who are well aware of the exact product they want to purchase (“Decided”) and consumers who only recognize the need to purchase from a category and require a visit to a store to shop around and find the specific version or model that best fits their needs (“Undecided”). Finally, consumers have horizontal taste preferences for retailers.

Firms are located at the ends of a Hotelling linear city, with consumer location on the line indicating retailer preference. Firms first choose a self-matching price policy, and subsequently set price levels for both store and online channels simultaneously. We examine the subgame perfect equilibria and evaluate firms’ decisions under various competitive scenarios (monopoly, mixed duopoly, and multi-channel duopoly).

Our analysis reveals several underlying mechanisms that affect the profitability of self-matching in equilibrium. The combination of these effects and the tradeoffs between them are, to the best
of our knowledge, novel and have not been previously identified. The presence of a self-matching policy implies that consumers who prefer to shop in-store, and who were able to research the specific product they want online, visit the store knowing the online price. Such consumers pay the lower of the two prices, leading to a channel arbitrage effect that reduces the profits of a self-matching retailer.

While this effect on its own would suggest that self-matching is a bad idea for the firm, and indeed would not be implemented by a monopolist, we uncover two additional competitive mechanisms that can make self-matching beneficial. First, under competition there is a tendency to lower online prices in an attempt to attract channel agnostic consumers. By offering to self-match, a retailer in effect extends this lower price to decided store consumers who know which product they want and can look-up the online price prior to initiating their trip. The retailer is however induced to raise the online price in order to mitigate the effect of extending the price break to store consumers, resulting in less intense competition. Since prices across retailers are strategic complements, the rival who does not self-match raises its online price as well. We term this the online competition dampening effect of self-matching.

When facing a multichannel rival, a retailer lowers store prices in order to compete for decided consumers who know the product they want, and make a decision on which store to visit. But this price reduction will also be enjoyed by undecided consumers who start their purchase process by visiting the store of the retailer closest to their taste preference. The self-matching strategy allows the retailer to charge these two segments different prices – decided consumers can retrieve the online price through the self-matching policy while the undecided consumers pay the higher store price. We term this the store competition dampening effect, and we identify when it helps the retailer extract more surplus from the undecided consumer than is lost by offering the decided consumers the online price in store.

From a profitability standpoint, there is the question of whether the availability of self-matching makes retailers worse off, like in a prisoners’ dilemma, whereby a firm is compelled to adopt this policy because its rival has, yet both would be better off if they were able to commit to refraining from the practice. Our analysis helps show that self-matching is not necessarily harmful and, in fact, both retailers can be better off by offering to self match when the competition dampening effects dominate the negative channel arbitrage effect.4

Finally, we examine how the presence of smartphone-equipped “smart” consumers, who can retrieve online price information when in the store, affects the incentives to implement a self-matching policy. Intuitively, when more consumers are able to retrieve the lower online price

4 We do not model any consumer backlash issues when firms offer different prices across channels and that may affect their reputation. While such considerations may provide an added incentive to price match across channels, our focus is on showing that self-matching may arise endogenously due to strategic reasons.
the channel arbitrage effect is more pronounced. However, we find that an increase in “smart”
consumers can result in higher online prices and greater retailer profits when combined with a
self-matching policy due to the interplay between channel arbitrage and competition dampening
effects.

We next review the literature (§2), set up the model (§3), and proceed to analyze equilibrium
strategies and outcomes (§4). We then consider several extensions of the base model (§5), and
finally conclude by discussing managerial implications and future research (§6).

2. Literature Review

We draw from two separate streams of past research, focused on multichannel retail and on compet-
titive price-matching in a single channel. Research in multichannel retailing has typically assumed
that firms either set the same or different prices across channels, without examining the incentives
to adopt a self-matching policy. Liu et al. (2006), for example, show that when price consistency
across channels is assumed, bricks-and-mortar retailers may refrain from opening an online arm
as this allows them to deter the entry of an e-tailer. Zhang (2009) considers separate prices per
channel and studies the interaction between the retailer’s decision to also operate an online arm
and whether to advertise its store prices. Ofek et al. (2011) study retailers’ incentives to offer in-
store sales assistance when also operating an online channel, while taking into account the higher
likelihood of returns from online purchases. Aside from ignoring self-matching pricing policies, this
literature has not considered or modeled heterogeneity in consumers’ decision making process,
which plays an important role in their channel choice in practice.

A developing literature examines the effects of free-riding (Shin 2007) or show-rooming on retailer
competition (Mehra et al. 2013), including suggestions that in-store to online price-matching across
retailers can serve as a potential strategy to combat show-rooming. However, there has not been
a careful modeling and evaluation of whether and when such policies can be effective; particularly
in a competitive context.

Competitive price-matching is an area that has been well-studied. This literature has generally
focused on retailers’ incentives to match the prices of their competitors in a single channel, typ-
ically bricks-and-mortar, setting. Salop (1986) argued that when retailers price-match each other
this leads to higher prices than otherwise, as the firms no longer have an incentive to engage in
price competition; effectively implying a form of tacit collusion (Zhang 1995). Competitive price-
matching has also been found to intensify competition because it encourages consumer search
(Chen et al. 2001). Other research in competitive price-matching has explored the impact of hassle
costs (Hviid and Shaffer 1999), its role as a signaling mechanism for certain aspects of a firm’s
product or service (Moorthy and Winter 2006, Moorthy and Zhang 2006), the interaction with
product assortment decisions (Coughlan and Shaffer 2009), and the impact of product availability (Nalca et al. 2010).

By contrast, self-matching policies are a phenomenon that is only relevant for multi-channel retailers. Recent trends in retailing make self-matching an important phenomenon to study. First, across a variety of product categories, the nature of competition is evolving, from retailers carrying the same products from multiple brands to manufacturers that establish their own retail stores e.g., Apple, Microsoft and Samsung. Second, retailers are also moving towards establishing strong private label brands across a range of categories and price points, or building exclusive product lines to avoid price wars with competitors (Mattioli 2011, Bustillo and Lawton 2009). For instance, about 56% of the products sold by health products retailer GNC are exclusive or GNC branded, and electronics retailers like Brookstone and Best Buy are also increasingly focusing on private label products. These trends accentuate the relevance of self-matching relative to competitive price matching as the products retailers carry become more differentiated.

3. Model
We develop a model to capture the essential features of the self-matching price phenomenon, involving retailing firms and consumers; we describe them in turn below.

3.1. Retailers
In our competitive model, we have two retailers in the same category situated at the endpoints of the unit consumer interval; one of these retailers will always be multichannel while the other can either be a pure e-tailer or a multichannel retailer. The retailers thus offer horizontally differentiated non-overlapping products of value $v$ across both channels. Since the products carried by retailers are not identical, they do not have the option of offering competitive price matching guarantees. For example, both Gap and Aeropostale sell apparel and operate in the same categories but the items themselves are not the same and reflect the designs and logos of each of these retailers. We model a two-stage game in which the retailers must first decide on self-matching price policies, and then on prices.$^5$

We denote by $SM_i = 0$ the decision of retailer $i$ not to self-match and by $SM_i = 1$ the decision to self-match, so that the game includes four possible subgames - $(0, 0), (1, 1), (1, 0)$ and $(0, 1)$ representing $(SM_1, SM_2)$, the selection of each firm to self-match or not. In each subgame, $p_j^k$ denotes the price set by retailer $j \in \{1, 2\}$ in channel $k \in \{on, s\}$, where $on$ stands for the online or internet channel, and $s$ stands for the physical store channel. With self-matching, consumers that retrieve the match pay the lowest of the two channel prices. All costs are normalized to zero.

$^5$ In the equilibrium analysis that follows, we find that retailers never set lower prices in-store than online. Hence, the only relevant matching policy is the store-to-online self-match.
3.2. Consumers

In order to capture important features of the consumer shopping process in multichannel shopping environments, we model consumers as being heterogeneous along multiple dimensions described below.

*Retailer Brand Preferences*: Consumers differ in their preferences for retailers, e.g., a consumer might be a more loyal Macy’s visitor but not frequent a competitor like JC Penney’s, because of differences in type of products or service. This aspect of heterogeneity is captured by allowing consumers to be distributed uniformly across a unit segment in the preference space, $x \sim U[0, 1]$. A consumer at preference location $x$ incurs a “misfit cost” $\theta x$ when purchasing from retailer 1 and a cost $\theta(1 - x)$ when purchasing from retailer 2. Note that the parameter $\theta$ does not involve transportation costs, rather it represents horizontal retailer-consumer “misfit” costs which are the same across channels. Misfit costs reflect heterogeneity in taste over differentiated products of similar value, e.g. the collection of suits at Banana Republic compared with those at J.Crew.

*Channel Preferences*: Channel-agnostic (A) consumers (size $\eta$) do not have an inherent preference for either channel and, for a given retailer, would purchase from whichever channel has the lower price. Store-only (S) consumers, a segment of size $(1 - \eta)$, find the online channel insufficient, e.g. due to waiting times for online purchase, risks associated with online purchases (such as product defects) etc. Store-only consumers, as their name implies, purchase only in the store, although they might research products online and obtain online price information after deciding the specific product.

*Decision Stage*: Consumers differ in their decision-making process (DMP) stage, a particularly important aspect of multichannel shopping (Mohammed 2013, Neslin et al. 2006, Mulpuru 2010). Undecided (U) consumers (size $\beta$) need to undertake a shopping trip to the store because they don’t have a clear idea of the exact product they wish to purchase. Decided (D) consumers (size 1) are certain about the product they wish to buy across both retailers, and can thus costlessly search for price information from home. Undecided consumers first visit a retailer’s store, selecting the store closest to their preference location to discover an appropriate product fit. After determining fit, they may purchase the product in-store or return home to purchase online (from either retailer). Categories like apparel, fashion and sporting goods are likely to display demand from undecided consumers, as styles and sizes of products are important factors that change frequently. Since undecided consumers do not know which product they want before visiting a store, they do not have at their disposal all prices while at the store, since keeping track of a large number of products, models and versions in the category would be impractical.

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6 Decided consumers costlessly gather information on prices from the internet or call a retailer for the store price.

7 Undecided consumers in the model do not form inferences on how self-matching influences retailer prices before they visit a store. They are unaware of the exact product they wish to purchase beforehand, thus limiting their ability to
We normalize the travel cost for a consumer’s first shopping trip to be zero. Note that if consumers have no cost to visit multiple stores in person, then we obtain a trivial specification where there is no distinction between the SD and SU consumers. Throughout the paper, we focus on the more interesting case where further shopping trips are costly enough so that SU consumers will not shop across multiple physical stores (see Appendix for conditions on the shopping cost).

In the baseline model, consumers cannot search online prices while in-store, although we examine this interesting possibility in §5, by including a segment of consumers with mobile internet access.

Table 1 depicts the different consumer segments we include in the model. We denote the four segments of consumers as SU, AU, SD and AD, depending on their decision stage and channel preferences; the size of each segment in the population is indicated in the corresponding cell of the table. Each of the four segments are uniformly distributed on a Hotelling linear city of unit length. Table 2 details the notation.

### Sequence of Events

Firms first simultaneously decide upon a self-matching pricing strategy, and then determine their price levels in each channel. Consumers, depending on their type (decided or undecided, store or channel-agnostic, and horizontal preference) make product and channel decisions, and determine which option to choose (including the no-purchase option). Decided consumers aware of the online price prior to visiting the store can ask for a price-match if the online price is lower and the firm has chosen to self-match. Finally, consumers make purchase decisions and firm profits are realized. Figure 1 depicts the sequence of events.

### 3.3. Consumer Utility

We now express the utility consumers derive under different self-matching scenarios. Recall that decided consumers know all prices across both retailers and channels before they make a purchase decision. Undecided channel-agnostic consumers (AU) have the option of returning home and making an online purchase after they figure out the product they want, whereas Undecided store consumers (US) either purchase in-store or make no purchase.

Infer prices under different self-matching configurations. More generally, the product category the consumer wishes to browse is sufficiently large and varied to make forming expectations of prices more costly than visiting the store.
We specify consumer utilities for possible choices of retailer and channel under the self-matching strategies; consumers obtain zero utility when they don’t make a purchase.\(^8\)

When neither firm self-matches, i.e., \((SM_1, SM_2) = (0, 0)\), the decided channel-agnostic (AD) consumer may purchase either online or in-store and from either retailer. The utility such a consumer derives from each of these options is as follows:

\[
\begin{align*}
    u_{1}^{on} &= v - p_{1}^{on} - \theta x, \\
    u_{1}^{s} &= v - p_{1}^{s} - \theta x \\
    u_{2}^{on} &= v - p_{2}^{on} - \theta (1 - x), \\
    u_{2}^{s} &= v - p_{2}^{s} - \theta (1 - x)
\end{align*}
\]

\(^8\) Consumers indifferent between buying and not buying do make purchases, and channel-agnostic consumers indifferent between channels purchase online.
Table 3 Consumer Choice and Information Sets

<table>
<thead>
<tr>
<th>Consumer Type</th>
<th>Choice Set</th>
<th>Information Set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Store</td>
<td>Online</td>
</tr>
<tr>
<td>AD</td>
<td>$u_1^s$ $u_2^s$</td>
<td>$u_1^o$ $u_2^o$</td>
</tr>
<tr>
<td>SD</td>
<td>$u_1^s$ $u_2^s$</td>
<td>$-$ $-$</td>
</tr>
<tr>
<td>AU ($x \leq \frac{1}{2}$)</td>
<td>$u_1^s$ $-$ $u_2^o$ $u_2^o$</td>
<td>Price at store 1, online prices if returns home</td>
</tr>
<tr>
<td>SU ($x \leq \frac{1}{2}$)</td>
<td>$u_1^s$ $-$ $-$ $-$</td>
<td>In-store price at store 1</td>
</tr>
<tr>
<td>AU ($x &gt; \frac{1}{2}$)</td>
<td>$-$ $u_2^s$ $u_2^o$ $u_2^o$</td>
<td>Price at store 2, online prices if returns home</td>
</tr>
<tr>
<td>SU ($x &gt; \frac{1}{2}$)</td>
<td>$-$ $u_2^s$ $-$ $-$</td>
<td>In-store price at store 2</td>
</tr>
</tbody>
</table>

where $v$ is the inherent value of the product, $p_1^o$ and $p_2^o$ are the online prices set by firms 1 and 2 respectively, $\theta$ measures the degree of consumer preferences for retailers, and $x$ is the consumer’s location (in the preference space) relative to firm 1’s position. The decided store-only consumer (SD) does not consider the option of purchasing online, hence only the utility expression for $u_1^s$ and $u_2^s$ are relevant for this consumer type.

Undecided consumers, who don’t know which specific product they need, first visit the retailer closer to their preference location (i.e., visit retailer 1 if $x \leq \frac{1}{2}$, and retailer 2 otherwise). After their shopping trip, the undecided store-only segment (SU) must decide when at the store whether to buy the product that fits their needs or make no purchase; hence only the corresponding $u_i^s$ expression in (1) is relevant for them. Undecided channel-agnostic consumers (AU) can either purchase in the store they first visited and pay the in-store price, or return home and make an online purchase from either retailer (or make no purchase); the utility expressions $u_1^s$, $u_1^o$, $u_2^o$ are relevant for them. Table 3 summarizes the choice set (with a ‘$-$’ indicating a choice is not available to the consumer) and price information available to the various consumer segments.

The Impact of Self-Matching Prices We now examine how self-matching practices by retailers impact consumer utilities. Decided consumers know all prices for the product they want, and if they shop at the store offering self-matching, they can come armed with the online price and request a price match for a specific product. Thus, when both retailers offer a self-matching pricing policy, i.e., under $(SM_1, SM_2) = (1,1)$, decided consumers can either purchase online and pay the online price, or purchase in the store and pay the lowest of the online and store prices.

Decided channel-agnostic consumers (AD) obtain the following utilities from their retailer and channel choices:

\[
\begin{align*}
    u_1^o &= v - p_1^o - \theta x, \\
    u_1^s &= v - \min(p_1^s, p_1^o) - \theta x, \\
    u_2^o &= v - p_2^o - \theta (1 - x), \\
    u_2^s &= v - \min(p_2^s, p_2^o) - \theta (1 - x).
\end{align*}
\]
For SD consumers the only relevant expressions from (2) are \( u_s \); with the notable difference from the no-self-matching \((0,0)\) case that these consumers can end up with \( u_{on} \), as they research prices online for the specific product they are decided about.

Undecided consumers (both AU and SU) face the same utilities under \((1,1)\) as under \((0,0)\), (see 1), since they cannot redeem matching policies when they visit a retailer’s store without making a costly additional set of trips: back home to determine online prices and then returning to a store.

Utilities in the asymmetric subgames \((1,0)\) where only one retailer offers to self-match prices are defined below, with \( u_s^2 \) and \( u_{on}^2 \) defined as follows.

\[
\begin{align*}
  u_s^1 &= \begin{cases} 
    v - p_s^1 - \theta x & \text{if } SM_1 = 0, \\
    v - \min(p_s^1, p_{on}^1) - \theta x & \text{if } SM_1 = 1,
  \end{cases} \\
  u_{on}^1 &= v - p_{on}^1 - \theta x
\end{align*}
\]

4. Analysis

We begin our analysis by considering the benchmark monopoly case, then examine mixed duopoly competition, and finally the multi-channel duopoly setting. All proofs as well as threshold values and constants are in the Appendix. Note that in all cases, we will derive conditions for the market to be covered in the proofs of the results, and our discussion in the text will focus on coverage in equilibrium.

4.1. Benchmark Monopoly

A multichannel monopolist retailer is located at the center of the \([0,1]\) Hotelling linear city, chooses a self-matching policy and sets prices.\(^{10}\) For expository clarity and tractability, throughout the analysis we restrict our attention to focal regions in the parameter space where the equilibrium solution entails all consumers making a purchase.\(^{11}\)

**Proposition 1.** A monopolist cannot increase profits by self-matching prices across channels.

To understand the intuition for why a monopolist can never benefit from self-matching, consider its profit functions. We focus on the case where \( v > \theta \) so that the market is covered.

The monopolist will price to extract the highest possible surplus from each consumer segment. Under \( SM = 0 \), channel-agnostic consumers will purchase online at price \( p_{on}^1 = v - \frac{\theta}{2} \). Store-only consumers will purchase in-store at a price \( p_s^1 = v - \frac{\theta}{2} \). The monopolist will then earn profits of \( \Pi_{SM=0}^1 = (v - \frac{\theta}{2})(1 + \beta) \). We note that prices are the same online and in stores, and the monopolist retailer endogenously chooses to price consistently across its channels.

\(^9\) The other asymmetric equilibrium is obtained by relabeling firms.

\(^{10}\) Note that qualitatively similar results would obtain if the retailer is located at an end point of the city, as is the case in the duopoly model.

\(^{11}\) Formally, we require bounds on \( v \) and \( \beta \), which are detailed in the appendix.
Despite the self-matching policy, the monopolist will choose again to set the same price in both channels as if it were not self-matching, yielding the identical profits of $\Pi_{SM} = (v - \frac{\theta}{2})(1 + \beta)$. It is noteworthy that these profits are the same as a monopolist serving a consumer segment of size $(1 + \beta)$ in a single channel. The retailer is thus indifferent between offering and not offering a self-matching policy, and will not offer it when it entails a minimal implementation cost.

4.2. Duopoly

We next consider the case of a multichannel retailer facing a pure online e-tailer, i.e. a “mixed duopoly market,” evaluating the subgame perfect equilibrium two-stage self-matching policy and pricing game, according to the sequence in Figure 1. We define the function $\Phi_1$ as the proportion of demand obtained by firm 1, when competing in a duopoly with firm 2, with a retailer differentiation parameter of $\theta$. Specifically, $\Phi_1(p_1, p_2; \theta) := \frac{1}{2} + \frac{(p_2 - p_1)}{2\theta}$.

**Mixed Duopoly: Multichannel retailer and e-tailer** The mixed duopoly market structure is becoming more important for a number of multichannel retailers, e.g., several retailers find that Amazon and potentially other e-tailers are their primary rivals. Also, it is a useful setting for understanding key elements of the intuition for the findings in the multichannel duopoly case, since some of the effects apply there as well.

We denote the focal multichannel retailer as firm 1 and the online-only e-tailer as firm 2, so only firm 1 has the option of offering a self-matching policy in stage 1 of the game. Subsequently, both firms set prices and compete for demand. When the multichannel retailer does not self-match its prices, i.e., in the subgame $(0, 0)$, store-only consumers will only consider firm 1’s store channel and are captive to this retailer, whereas channel agnostic consumers have the option of shopping across the two retailers’ online sites. Profits for both retailers can be expressed as follows.

$$\Pi_{1,0} = \eta(1 + \beta)\Phi_1(p_{on}^1, p_{on}^2)p_{on}^1 + (1 - \eta)(1 + \beta)p_s^1,$$

$$\Pi_{2,0} = \eta(1 + \beta)(1 - \Phi_1(p_{on}^1, p_{on}^2))p_{on}^2.$$ (3)

Firm 1 serves as an effective monopolist over store-only consumers (both SD and SU segments), who comprise a combined segment of size $(1 - \eta)(1 + \beta)$, and will attempt to extract surplus from them by setting a store price of $p_s^1 = v - \theta$. However, both firms compete for the channel-agnostic consumers, who form a segment of size $\eta(1 + \beta)$. In this case, undecided channel-agnostic consumers located to the right of $x = \frac{1}{2}$ may purchase from the e-tailer, after browsing the entire product category at firm 1’s store, depending on prices. Visiting the multi-channel retailer’s store provides AU consumers with information about the various products in the category and infer fit with the competing e-tailer’s products.
The situation online is similar to firms competing in a horizontally differentiated market comprised of only channel agnostic consumers. Thus, the resulting equilibrium prices reflect the strength or retailer preferences, with \( p_1^o = p_2^o = \theta \). We will refer to a price of \( \theta \) as the “competitive” price level, or the “Hotelling competitive” price level, to reflect the fact that this would be the price charged in a standard Hotelling duopoly model with one retailer channel.

When the multi-channel retailer chooses to self-match, \((1, 0)\), there emerges an important distinction between the decided and undecided store-only consumers. The SD consumers are able to obtain a price-match from firm 1. However, SU consumers only know which product they desire after visiting the store, and as they lack evidence of a lower online price they pay the in-store price. Thus, even though the two segments of store consumers both obtain the product in store, they pay different prices. The channel-agnostic consumers (both decided and undecided) are unchanged in their behavior under self-matching since they once again have access to lower online prices.

In the proof, we find the market to be covered when \( v > 2\theta \). To gain an intuition for the different consumer segments, consider the profits.

\[
\Pi_{1,0}^1 = \eta (1 + \beta) \Phi_1(p_1^o, p_2^o) p_1^o + (1 - \eta) p_1^o + (1 - \eta) \beta p_1^s,
\]
\[
\Pi_{2,0}^1 = \eta (1 + \beta) (1 - \Phi_1(p_1^o, p_2^o)) p_2^o.
\]

As with the no self-matching case, competition online is expected to put downward pressure on the price levels \( p_1^o \) and \( p_2^o \) in the \((1, 0)\) case. With a portion of store consumers, namely the \((1 - \eta)\) SD consumers, now receiving the online price by invoking the self-match price policy instead of paying the store price, there is a negative effect on the multichannel retailer’s profits (through the second term in (4)). In essence, the retailer faces a channel arbitrage effect when it allows consumers to obtain a price match resulting in the lower price being effectively charged to those who request the match. We might intuitively expect self-matching to be an unprofitable strategy—especially since the SD consumers would not have defected to the e-tailer due to their preference for the store channel and paid the store price.

Although this intuition is correct, we find it to be incomplete in determining whether in equilibrium a self-match price policy will be adopted. In particular, we find another effect, which we term the online competition dampening effect and explain shortly, that it acts to increase profitability when the multi-channel retailer chooses to self-match. The following result reflects the net impact of the two effects, channel arbitrage and online competition dampening, and characterizes the mixed-duopoly equilibrium.
Proposition 2. In a mixed duopoly featuring a multichannel retailer (firm 1) operating a store and an online channel and an e-tailer operating only an online channel (firm 2), the multichannel retailer chooses to adopt a self-matching policy when product valuation is relatively low and differentiation is high. Otherwise, the retailer will not adopt a self-matching policy.

To understand the intuition for Proposition 2 consider first the pricing incentives in the online channel without self-matching: the multi-channel retailer can increase its share of channel-agnostic consumers by cutting price, with prices becoming strategic complements across firms, the e-tailer is induced to follow suit and reduce prices as well.

Now, when the multi-channel retailer decides to self-match, there is a cross-channel externality. Reducing the online price allows the multichannel firm to gain more channel agnostic consumers, but it allows the decided store (SD) customers (who previously paid the store price) to obtain the lower online price by requesting a self-match, leading to channel arbitrage. In a bid to reduce the negative impact of channel arbitrage, the multi-channel retailer that implements a self-match has an incentive to raise its online price relative to the no-self-matching case. Strategic complementarity in prices leads both firms to set higher online prices than under no-self match.\(^\text{(5)}\)

The desire to contain losses from the SD segment of consumers that can invoke the price-match effect effectively acts like a “commitment device” to prevent prices from going all the way down to the competitive level online. The important point to note is that this results not only in the SD consumers who request a self-match paying a higher price but also in the channel-agnostic AD and AU consumers paying a higher price (relative to the competitive online price of \(\theta\) they were paying under no self match). This obviously has a positive effect on profits resulting in what we term the online competition dampening effect of implementing a self-matching price policy. While the competition dampening effect requires the channel arbitrage effect to be present, the former effect can dominate because it extends to two consumer segments (AD and AU), whereas the arbitrage effect applies only to the SD segment.

The trade-off between the channel arbitrage and competition dampening effects depends on the product value (\(v\)) and the degree of differentiation between firms (\(\theta\)). With self-matching, for low enough \(v\) (or equivalently, high \(\theta\)), each retailer is effectively able to maintain an online segment who find the competing firm’s product not compelling enough due to the low ratio of value to retailer preference, which diminishes substitutability across retailers. Note that in both cases, the store prices are higher than the Hotelling competitive level (\(\theta\)).
The self-matching multichannel retailer will price similarly across channels and thus channel arbitrage is low. Online prices increase in $v$ as long as firms are sufficiently differentiated. In this case, the online competition dampening effect dominates and grows in $v$. For large $v$ (or low $\theta$), retailers are no longer sufficiently differentiated from one another, and thus compete more intensely to attract consumers closer in preference to their competitor, intensifying online price competition and reducing the competition dampening effect. However, the negative channel arbitrage effect increases in $v$ as SD consumers redeem the lower online price. For high enough $v$, the negative impact of channel arbitrage dominates. As a result, self-matching emerges as an equilibrium outcome only for low values of $\frac{\theta}{v}$.

Increasing $\eta$, the fraction of channel agnostic consumers, intensifies online rivalry in the mixed duopoly setting as there are more consumers to compete for. The intense competition will reduce the range of values for which retailers can maintain sufficient differentiation in the online channel and hence the range of values for which online competition dampening dominates channel arbitrage. When the majority of consumers are undecided, a high $\beta$, online prices are driven down when the multichannel retailer self-matches as retailers compete more heavily for the growing segment of AU consumers, intensifying the negative channel arbitrage effect. Thus, a higher value of $\beta$ reduces the profitability of self-matching. Overall then, it is easier to sustain self matching for lower values of $\beta$ and $\eta$. Turning attention to the impact of self-matching on the e-tailer, we find the following result.

**Corollary 1.** *In a mixed duopoly, the e-tailer always makes higher profits when the multichannel retailer uses a self-matching policy.*

Thus, a self-matching policy has a positive externality on the e-tailer due to reduced competition in the online channel, which allows the e-tailer to increase prices. This holds even though the e-tailer’s price level is lower than that of the multi-channel retailer because it does not internalize the benefit of raising its price through the store channel, like the multichannel retailer is able to do.

**Multichannel Duopoly** We now turn to the case of two competing multichannel retailers, both of whom need to make a decision on whether to self-match simultaneously, and set prices in the next stage (according to the timeline in Figure 1). We examine each of the possible self-matching policy subgames and conclude with the result highlighting the conditions under which self-matching emerges in equilibrium.
No Self-Matching - (0,0): In the (0,0) subgame where neither firm self-matches, both decided and undecided consumers pay the same price if they purchase from the store channel. Similar to the mixed duopoly case, channel-agnostic consumers (both AD and AU segments) will purchase online and store consumers will buy in store. However, the key difference in this case is that SD consumers, who know the product they want and are informed of all prices, have a choice of which retailer’s store to purchase from. Thus, there is now store competition for that segment of consumers, since by reducing store price a retailer can attract more SD consumers. These consumers, sized \((1 - \eta)\), purchase in store but make a decision on which store to visit, factoring in their retailer preference and prices.

On the other hand, SU consumers visit the retailer closest in preference to them to learn about products, as in the mixed duopoly model. They have a choice of either purchasing in the store visited, or not purchasing at all. Recall that these consumers cannot purchase online because of aversion to the online channel, and cannot switch stores because of the travel costs associated with a second store visit. In a sense, these consumers are “captive”; and each retailer effectively has a sub-set \((\frac{\beta}{2})\) of them who visit the store.

The profit functions of firms 1 and 2 can then be written as:

\[
\Pi_{1}^{0,0} = \eta(1 + \beta)\Phi_{1}(p_{1}^{om}, p_{2}^{om})p_{1}^{om} + (1 - \eta)\left(\Phi_{1}\left(p_{1}^{*}, p_{2}^{*}\right) + \frac{\beta}{2}\right) p_{1}^{*}, \\
\Pi_{2}^{0,0} = \eta(1 + \beta)\left((1 - \Phi_{1}(p_{1}^{om}, p_{2}^{om}))p_{2}^{om} + (1 - \eta)\left((1 - \Phi_{1}(p_{1}^{*}, p_{2}^{*})) + \frac{\beta}{2}\right) p_{2}^{*}.
\]

Solving for the equilibrium in the pricing subgame, we find that the online prices are at the competitive level, similar to the mixed duopoly (0,0), with \(p_{1}^{om} = p_{2}^{om} = \theta\). However, the store prices are different from that case because we have an imperfectly competitive market between the two store channels leading to symmetric prices of

\[
p_{1}^{*} = p_{2}^{*} = \begin{cases} 
\theta (1 + \beta), & \frac{v}{\theta} > \frac{3}{2} + \beta \\
\frac{v}{\theta} - \frac{\beta}{2}, & \frac{v}{\theta} \leq \frac{3}{2} + \beta 
\end{cases}
\]

Firms thus charge a higher price in the store than online to extract surplus from SU consumers. These store prices however are lower than under the mixed-duopoly case with no self matching because firms have to compete in the store channel for SD consumers. Note that as the fraction of undecided consumers shrinks, i.e., as \(\beta \rightarrow 0\) under high valuation, the store prices approach the competitive price \(\theta\).
Symmetric Self-Matching - (1,1): When both firms offer self-matching policies, SD consumers retrieve the online price in advance of their store visit and all channel-agnostic consumers have access to the lower online prices as well. So, three segments (SD, AD and AU) of consumers, of total size $(1 - \eta) \times 1 + \eta \times 1 + \eta \times \beta = (1 + \eta \beta)$, effectively pay the lower online price.

SU consumers, who form a segment of size $(1 - \eta) \beta$, cannot invoke the price-match as they do not have a chosen product until they visit the store and they lack evidence of a lower online price. Thus, they are divided equally into two segments each “belonging” to one of the retailers. Retailers’ profits are then:

\[
\Pi_{1,1}^1 = (1 + \eta \beta) \Phi_1(p_{1,1}^{on}, p_{2,1}^{on}) p_{1,1}^{on} + (1 - \eta) \frac{\beta}{2} p_{1,1}^s,
\]

Channel-Agnostic & Decided Store

\[
\Pi_{2,1}^1 = (1 + \eta \beta)(1 - \Phi_1(p_{1,1}^{on}, p_{2,1}^{on})) p_{2,1}^{on} + (1 - \eta) \frac{\beta}{2} p_{2,1}^s.
\]

Undecided Store

In the pricing sub-game, firms set equilibrium online prices $p_{1,1}^{on} = p_{2,1}^{on} = \theta$, as there is no force to prevent prices from dropping to their competitive level, and set store prices $p_{1,1}^s = p_{2,1}^s = v - \theta/2$, as each retailer caters to SU customers who effectively have no other retail options. All decided consumers and undecided channel-agnostic consumers pay the competitive price $\theta$, whereas undecided store consumers pay the store price of $(v - \theta/2)$.

When comparing the price levels across the two cases, (0,0) and (1,1), it is clear that the retailers make the same amount of profits on the channel-agnostic consumers who buy online in both scenarios, since the online price is $\theta$ in both cases. However, in the stores an interesting comparison emerges. If the multichannel retailers don’t offer a self-match policy, and the value of the product is high relative to differentiation, competition over the SD segment is intense and results in lower store prices. Unfortunately for the firms, these lower prices also apply to the SU segment of consumers who purchase in store.

However, when the multi-channel retailers do offer a self-matching policy, the SD segment is no longer a factor in store pricing, as these consumers retrieve the online price. This feature allows the firms to revert back to pricing high in the stores, to the tune of $(v - \theta/2)$, and again obtain high surplus from the SU segment. Of course, the channel arbitrage effect comes into play because SD consumers pay less (by asking for the price match) than in the no self-matching case. We call the ability to avoid price competition across stores by offering a self-match the *store competition dampening effect*. When this effect outweighs channel arbitrage, it can create an incentive to self-match.

Another way to understand the store competition dampening effect is to realize that each firm prefers to charge the captive SU consumers a higher price than SD consumers that have the ability
to choose the rival retailer. However, the firm cannot achieve such price discrimination with a no-matching policy, as both consumer types would purchase in the same channel, the store, and pay the store price. However, with a self-matching policy, the firm can charge different prices to store-bound consumers based on their decision stage by setting a high store price paid by undecided consumers and a lower online price for decided consumers who can redeem the self-match.

It is interesting to note that in a (1,1) multichannel duopoly, the online competition dampening effect is absent. Recall that in the mixed duopoly, the self-matching multichannel firm would price higher online to mitigate channel arbitrage, and it was able to price higher online because it faced no store competition. But with two competing multichannel retailers that offer self-matching policies, if one firm tries to deviate and price higher than $\theta$ online, the other retailer has an incentive to keep the price low as it can attract not only AD and AU consumers on the margin from its competitor, but also some SD consumers (who now have a choice from which store to buy) – this keeps prices at the competitive level online.

Store competition dampening occurs in a multichannel duopoly but not in a mixed duopoly, since in the latter setting, the multichannel firm faced no competition for SD consumers. Hence, these consumers were captive and could be charged the monopoly store price in the absence of a price match; so there was no “sacrifice” of a lower price to the SU segment. In a multichannel duopoly, on the other hand, firms compete in the store channel for SD consumers who are not captive, and can choose to visit the competing store. The result is a lower store price, absent the price match, which applies to the SU segment as well. The price match effectively allows price discrimination between the SU and SD segments in store, so that greater profits can be obtained from the SU segment.

**Asymmetric Self-Matching - (1,0):** We next explore the subgame in which multichannel retailer 1 offers a self-matching policy while multichannel retailer 2 does not, i.e., the (1,0) subgame. By symmetry (or relabeling), similar results follow in the (0,1) subgame. Observe that a SD consumer who visits retailer 1’s store can purchase there and pay the lower of the online and store price, i.e., $p_{1n}^* = \min(p_{1n}^0, p_{1s}^s)$. However, if the SD consumer visits retailer 2’s store instead, she faces a price of $p_{2s}^s$, and cannot obtain the online price in the store (as retailer 2 does not self-match). Moreover, by offering self-matching, and as long as its online price satisfies $p_{1n}^0 < p_{2s}^s$, firm 1 attracts some SD consumers who are closer in preference to the competing retailer 2 but who choose to visit 1’s store in anticipation of paying the lower online price through a self-match. Thus, a self-match in this case can serve as a mechanism to steal demand from the competitor. Note that for SD consumers, the store price of retailer 1, i.e., $p_{1s}^s$ is irrelevant (since they retrieve the price match), and the retailer can set a price level to capture the highest possible surplus from the SU consumers who are closer to 1’s location.
The fraction of store-only consumers who shop at retailer 1’s store channel is given by \( \Phi (p_{on}^1, p_{s}^2) \), and firm 1 obtains a potentially larger share of the store-only segment of consumers. Each retailer secures a \( \frac{1}{2} \) proportion of SU consumers. We obtain the following profit functions:

\[
\Pi_{1,0}^1 = \eta (1 + \beta) \Phi (p_{on}^1, p_{on}^2) p_{on}^1 + (1 - \eta) \Phi (p_{on}^1, p_{s}^2) p_{on}^1 + (1 - \eta) \frac{\beta}{2} p_{s}^1, \]

\[
\Pi_{2,0}^1 = \eta (1 + \beta) (1 - \Phi (p_{on}^1, p_{on}^2)) p_{on}^2 + (1 - \eta) \left( (1 - \Phi (p_{on}^1, p_{s}^2)) + \frac{\beta}{2} \right) p_{s}^2.
\]

Again, we determine the equilibrium strategies and outcomes of the second-stage pricing subgame, and find that the price levels chosen by the firms are higher than the Hotelling competitive price of \( \theta \), and critically depend on the valuation and degree of differentiation. For sufficiently high ratio of value to retailer differentiation, we find that firms set prices:

\[
\begin{align*}
p_{on}^1 &= \theta \left( \frac{2}{3} + \frac{1 + \beta}{3(1 + \eta \beta)} \right) \\
p_{on}^2 &= \theta \left( \frac{5}{6} + \frac{1 + \beta}{6(1 + \eta \beta)} \right) \\
p_{s}^1 &= v - \frac{\theta}{2} \\
p_{s}^2 &= p_{on}^2 + \beta \frac{\theta}{2}
\end{align*}
\]

It is interesting to note that equilibrium online prices in this asymmetric self-matching case are greater than those set in the other subgames \((0,0)\) and \((1,1)\). This results from the additional positive impact of the online competition dampening effect introduced in the mixed duopoly model. Firm 1 suffers from the channel arbitrage effect, as SD consumers now claim its lower online price when they purchase in the store. Although firm 1 steals some store demand from firm 2 by enabling a lower price in its store through self-matching, the channel arbitrage effect on all SD consumers with \( x < \frac{1}{2} \) is substantial, and the firm has an incentive to charge a higher online price to mitigate this impact. Since prices across firms in the online channel are strategic complements, this effect in turn leads to higher online prices set by the competing retailer 2. Thus, the presence of channel arbitrage resulting from self-matching and the accompanying action of the retailer to alleviate it by increasing online prices enables dampening of online price competition in the asymmetric self-matching equilibrium.

Note how the situation here differs from the case when both firms self-match. Under \((1,1)\), SD consumers can redeem the online price at both retailers’ stores, which forces online prices down to their competitive level \( \theta \). By contrast, in the asymmetric equilibrium \((1,0)\), SD consumers can only redeem the match from firm 1. Firm 2 will price higher in the store relative to firm 1’s online price, as both SD consumers and its captive segment of SU consumers pay its store price, whereas firm 1 fully segments out its store consumers through the self-matching policy which is invoked by the SD consumers (while the SU consumers pay the store price). Consequently, firm 1 need not set its online price as low as \( \theta \), and can price higher online to mitigate channel arbitrage. The
Table 4  Effects of self-matching for firm 1 in a multichannel duopoly

<table>
<thead>
<tr>
<th>Effect and Description</th>
<th>Formal Definition</th>
<th>Subgames Present if Competitor Matches</th>
<th>Dominates for v/θ level</th>
<th>Present in Mixed Duopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Channel Arbitrage</td>
<td>$\Pi_{1,SM2}^{1(SD)} - \Pi_{1,SM2}^{0(SD)}$</td>
<td>(1,0)</td>
<td>✓</td>
<td>Intermediate ✓</td>
</tr>
<tr>
<td>(D) Online Competition</td>
<td>$\Pi_{1,SM2}^{1(A)} - \Pi_{1,SM2}^{0(A)}$</td>
<td>(1,0)</td>
<td>×</td>
<td>Small ✓</td>
</tr>
<tr>
<td>(S) Store Competition</td>
<td>$\Pi_{1,SM2}^{1(SU)} - \Pi_{1,SM2}^{0(SU)}$</td>
<td>(1,1)</td>
<td>✓</td>
<td>Large ×</td>
</tr>
</tbody>
</table>

Online competition dampening effect is positive when only one of the firms, but not both, chooses to self-match in a multichannel duopoly.

Just as in the symmetric self-matching case, the store competition dampening effect is also present under (1,0), as self-matching by firm 1 allows it to avoid the need to lower the store price to compete intensely with firm 2 over the SD segment and instead allows it to price high to extract as much surplus as possible from its captive SU segment.

The results detailed in this section are based on the equilibria in the pricing sub-game, which is conditional on the self-matching policy chosen earlier. The pricing equilibria that emerge depend on the strength of the various effects we have identified on firms’ profits that result from self-matching at different parameter values. Table 4 presents a summary of the different effects induced by self-matching. We now examine the full equilibrium results of the game beginning with the price-matching strategy choices.

4.3. Self-Matching Policy Equilibria in a Multichannel Duopoly

According to the sequence of events in Figure 1, both firms determine their self-matching pricing policy in the first stage, and then set price levels simultaneously. For a self-matching policy configuration to emerge in equilibrium, it must be the case that neither firm would be better off by unilaterally deviating to offer a different policy. Proposition 3 details the equilibrium conditions and the resulting choices of self-matching policies.

**Proposition 3.** In a duopoly featuring two multichannel retailers, both operating store and online channels, we find the following:

(a) Self matching policies are determined by the following mutually exclusive regions based on product value, degree of differentiation and the size of the undecided consumer segment.
**Asymmetric Equilibrium:** One firm will offer to self-match prices while the other will not when product values are relatively low or firm differentiation is high.

**Symmetric non-matching equilibrium:** Neither firm will self-match prices when product values and firm differentiation are at intermediate levels.

**Symmetric matching equilibrium:** Both firms will self-match prices when product values are high or firm differentiation is low.

(b) An increase in the number of undecided consumers will grow the asymmetric equilibrium region and shrink the symmetric equilibrium regions.

The above result indicates that all three types of combined strategies can emerge in equilibrium depending on the nature of the product, competitive interaction and consumer segmentation. Indeed, no single price matching strategy consistently dominates, and three main regions are associated with each of the equilibrium outcomes.

To understand the intuition behind the emergence of the different equilibria, we examine how the focal firm’s best response functions evolve as \( v \) changes and translate this into equilibria in Figure 2. The first arrow depicts firm 1’s best response if firm 2 does not match. The second arrow depicts firm 1’s best response if firm 2 matches. The dominant effects for firm 1 are listed underneath the arrows in bold. The final arrow shows the emergent equilibria.

Specifically, we wish to investigate what affects the focal firm’s incentives to self-match when its competitor chooses to adopt or not to adopt a self-matching policy. To determine the answer, we focus on the role the various effects—channel arbitrage, online competition dampening, and store competition dampening—play based on the product value \( v \) and the size of the undecided consumer segment \( \beta \).

We start with the case whereby firm 2 decides not to offer a self-match pricing policy. When \( v \) is low, firm 1 earns very little additional profits from being able to charge SU consumers the price that extracts the most surplus from them \((v - \theta_2)\) by self-matching, i.e., the store competition dampening effect is weak. However, the firm does benefit from online competition dampening if it chooses self-matching, as it affects the AD and AU segments, and this positive effect outweighs the negative channel arbitrage in this region; this leads firm 1 to choose self-matching at low values of \( v \). As \( v \) grows, firm 1 is able to charge higher store prices to capture more surplus from the SU segment, i.e., store competition dampening is more pronounced. By contrast, the positive impact of online competition dampening ceases to grow in \( v \) as firms reach a point when they are increasingly similar in the eyes of online consumers (i.e., the disutility from preference location, \( \theta x \), plays less of a role in their utility relative to the product value \( v \)). The negative impact of channel arbitrage then outweighs the positive impact of competition dampening, and the firm no longer finds it profitable to self-match as a best response.
At high $v$ levels, firms reach a point where they are no longer sufficiently differentiated in the market for store-only consumers.\textsuperscript{13} If the focal firm does not self-match, it will compete intensely with its competitor in the store channel over the SD segment. However, with a self-matching policy, the focal firm can price high in store and extract more surplus from US consumers. The focal firm attracts SD consumers by offering them the lower online price upon their request for a self-match. The positive impact of such store competition dampening outweighs the negative impact of channel arbitrage, and the focal firm finds it profitable to self-match. This result is diametrically different from the mixed-duopoly case, where for high $v$ values the multichannel retailer is worse off when self-matching. The key difference between these cases is that only in a multichannel duopoly is there competition in the store channel, which makes store competition dampening relevant.

We now turn our attention to the case when firm 2 decides to offer a self-matching policy, and examine how focal firm 1’s best response varies. Recall that when both firms self-match, the online competition dampening effect is nullified and ceases to exist. However, the channel arbitrage and store competition dampening effects will still be present. Since Firm 2 is self-matching, its actions will result in online competition dampening only if Firm 1 does \textit{not} self match; this creates a force for Firm 1 to refrain from self-matching. At low $v$ values, prices are fairly similar across channels, hence both arbitrage and store dampening are negligible, so there is not a strong force driving Firm 1 to self-match. As $v$ grows, so does the positive profit impact of store competition dampening on focal firm 1.\textsuperscript{14} Channel arbitrage and online competition dampening cease to increase in $v$ because the firms compete intensely when product valuation is high relative to the degree of retailer preference. Beyond a certain valuation threshold, the positive impact of store competition dampening that comes with self-matching will thus outweigh the negative impact of arbitrage, leading firm 1 to reverse its strategy and choose self-matching.

\textit{Strategic Substitutes or Complements:} When we examine the best responses and combine them to obtain equilibrium strategies, we observe that at low product valuation, the self-matching strategies act like \textit{strategic substitutes}, so that a firm will chose the strategy opposite to that of its competitor. As $v$ grows, the negative impact of channel arbitrage outweighs the positive impact of online competition dampening, and neither firm will offer to self-match. Not self-matching becomes a dominant strategy for each firm, independent of the competitor’s strategy. As $v$ continues to grow, we enter a region where multiple equilibria can exist, and the self-matching game becomes a

\textsuperscript{13} Because of the captive SU consumers for each retailer, competition in the store channel is less intense than online. That is why the threshold value of $v$ that makes differentiation less relevant is different across the two channels.

\textsuperscript{14} Although Firm 2 is partially mitigating store competition by self-matching, Firm 1 still must price lower in stores because of the need to attract SD consumers who pay the same price as SU consumers. When Firm 1 also self-matches, it relieves itself of the need to consider SD consumers when pricing in the store channel and can raise price to extract maximal surplus from the SU segment.
coordination game, where neither firm prefers the asymmetric outcome. If firm 2 self-matches, the price that firm 1 can charge SD consumers if it does not match is driven down (because they effectively get an online price from Firm 2); this makes the store competition dampening effect stronger and drives Firm 1 to self-match as well. Conversely, if Firm 2 does not self-match, then for firm 1, the negative impact of channel arbitrage from self-matching outweighs the positive impact of store competition dampening. Hence, neither retailer prefers to deviate from a symmetric configuration—either both retailers offer to self-match, or neither one does, so the self-matching strategies are \textit{strategic complements}.

Finally, at high product valuations, the strong impact of store competition dampening leads to self-matching becoming a dominant strategy regardless of what the competitor chooses.

We now turn our attention to part \((ii)\) of Proposition 3, which deals with how the equilibrium regions are affected by \(\beta\), the size of the undecided consumer segment. Consider the case of focal firm 1’s best response when firm 2 does not match. According to the proposition, Firm 1 has more of an incentive to offer a self-matching policy as \(\beta\) increases, implying that the \(v\)-region to get a \((1,0)\) equilibrium expands. The intuition is this: as the size of the undecided segment increases, the firms stand more to gain from online competition dampening (because of the greater size of the AU segment). Thus, if Firm 1 is self-matching, Firm 2 will refrain from doing so (because when both self-match the effect is nullified). For large \(v\), note that Firm 1 can charge a fairly high store price even if it does not self-match. The store price Firm 1 can charge if it does not match is increasing in \(\beta\), which leads to an arbitrage effect from SD consumers to be increasing in \(\beta\). At the same time, the highest price Firm 1 can charge SU consumers if it does match is the monopoly price for
that segment, which is independent of $\beta$, so any increase in the store competition dampening effect is only driven by an increase in the size of the undecided segment. As a result, store competition dampening grows at a slower rate than arbitrage as $\beta$ increases, which yields the result that Firm 1 is less likely to self-match as $\beta$ increases for large $v$, implying that the $(1,1)$ equilibrium region shrinks as $\beta$ increases.

4.4. The Profitability of Self-Matching

We have thus far analyzed how firms decide whether to adopt self-matching policies and characterized the strategies that can be sustained in equilibrium. Here, we wish to examine the profit impact of having self-matching available as a strategic option relative to the case without such an option. The key issue we seek to understand is whether firms are compelled by competitive forces to adopt self-matching, even though it might make them worse off and result in lower profits in equilibrium than were self-matching not an option. In other words, does implementing self-matching end up being a prisoners’ dilemma type outcome or a profit enhancing practice? Proposition 4 outlines the profitability of firms under different equilibrium configurations.

**Proposition 4.** The profit implications of the self-matching mechanism, compared to the baseline case where self-matching is not available as an option, are as follows:

(a) In the asymmetric equilibrium: only the firm offering to self-match earns greater profits.

(b) In the symmetric self-matching equilibrium: both firms earn higher profits when product valuation is high. However, they both earn lower profits when product valuation is intermediate.

The result above reveals that firms can potentially earn greater profits from self-matching. In particular, both firms are better off having this pricing practice when product valuation is high. We also find that at intermediate product values, self-matching can occur due to competitive forces, and can result in a prisoners’ dilemma type situation where both firms would be better off had they been able to commit to not self-matching prices. Figure 3 shows the equilibrium regions that emerge in the $v_\theta \leftrightarrow \beta$ space based on Proposition 4.\(^\text{15}\) The dashed line shows the region where firms are playing a prisoner’s dilemma in self-matching.

To understand why a prisoner’s dilemma may emerge in equilibrium, we must understand why a firm would want to deviate from $(0,0)$ when profits are greater in $(0,0)$ than in $(1,1)$. Consider firm 1’s incentive to deviate from $(0,0)$ to $(1,0)$ by self-matching. We find that in addition to earning additional profits from SU consumers who pay a higher store price, it can lower online prices to steal demand of SD consumers from its competitor. Firm 1 would suffer from channel arbitrage, but this effect would not dominate the other effects; thus, in such a case $\Pi_1(1,0) > \Pi_1(0,0)$. Now,\(^\text{15}\)In the figure, we focus on the Pareto-dominant equilibrium when multiple equilibria are present.
we can also see how \((1,0)\) would not be an equilibrium if firm 2 has an incentive to deviate, i.e. if \(\Pi_2(1,1) > \Pi_2(1,0)\) which happens in the case of intermediate product valuation.

However, when valuations are high, we find that both firms choose to self-match and can have higher profitability in doing so. This occurs because with increasing \(v\), store competition dampening eventually overtakes the negative impact of channel arbitrage.

Overall, we find that the availability of self-matching as a strategy has the potential to enhance profits for at least one firm, and can also do so for both competing firms across a range of product valuations, thus highlighting its importance as a strategic option for the multi-channel retailer.

5. Extensions
The base model analyzed in §4 focused on developing an understanding of the mechanisms underlying the effectiveness of self-matching and the conditions for firms to implement it in equilibrium. Here, we aim to examine additional variables that may be of relevance for retailers as they contemplate whether to offer a self-matching pricing policy as well as to relax some of the assumptions previously made, with a view towards increasing the range of applicability of the findings. We consider three extensions: allowing for “smart-device” consumers to capture the possibility that some store-only consumers have access to online prices; letting store-only consumers purchase online at a cost; and letting retailers offer a “buy now, pick up in store” option. Proofs of results in this section are provided in Appendix B.

5.1. Impact of “Smart-Device” Enabled Consumers
The baseline model characterized undecided consumers as not knowing what specific product they wanted until they visit a store to evaluate which item from the many available best fit their needs.
They could not invoke a self-matching policy because they were in store at the time they made their final decision and we assumed that there was no way for them to access the internet to produce evidence of a lower online price. Here we recognize the increasing importance of mobile devices to alter this dynamic and examine the implications on self-matching policies. Intuitively, one might expect that the greater the proportion of consumers that carry smart devices and take the trouble to check online when in the store, the less profitable self-matching should be (because of the increased threat of cross-channel arbitrage). We show that this need not be the case.

Suppose that a fraction $\mu$ ($0 < \mu < 1$) of consumers has access to the internet while shopping in the store. We refer to these consumers as “smart” to reflect the notion that with the aid of internet-enabled smartphone devices these consumers can easily obtain online price information while in the store. Undecided store-only smart consumers will invoke a self-matching policy if the online price offered by a firm is lower than its store price. Note that undecided channel agnostic smart consumers will effectively behave in an identical fashion to the channel agnostic ones we have already modeled.

We model undecided smart consumers as visiting a store, and having access to and evidence of both retailers’ online prices while in the store. However, for store-only smart consumers, choosing the competing retailer would involve making a trip to the other store, which is accompanied by high travel costs, and in our framework these costs are high enough to discourage store-to-store comparison shopping.\footnote{Undecided store-only consumers with smart devices are like decided consumers in the sense that they have access to the online prices of both retailers after they find the product they desire. However, they are different because they do not have the option of deciding which store to visit ex-ante. So, having a higher proportion of smart consumers is not equivalent to having more decided consumers, and has a more nuanced effect.} To see how the existence of smart consumers impacts firms’ strategies, consider the profits retailers earn if they both offer to self-match:

$$\Pi_{1,1}^{1,1} = (1 + \eta \beta) \Phi_1(p_{1n}^{on}, p_{2n}^{on}) p_{1n}^{on} + (1 - \eta) \frac{\beta}{2} ((1 - \mu) p_{1s}^{s} + \mu p_{1n}^{on}),$$

$$\Pi_{2,1}^{1,1} = (1 + \eta \beta) (1 - \Phi_1(p_{1n}^{on}, p_{2n}^{on}))(1 - \eta) \frac{\beta}{2} ((1 - \mu) p_{2s}^{s} + \mu p_{2n}^{on}).$$

Solving for the equilibrium reveals that retailers will set $p_{1n}^{on} = p_{2n}^{on} = \theta \left( (1 - \mu) + \mu \frac{1 + \beta}{1 + \eta \beta} \right)$ and $p_{1s}^{s} = p_{2s}^{s} = v - \frac{\theta}{2}$. Note that the online prices are increasing in $\mu$. We detail how smart consumers impact retailers’ equilibrium incentives to self-match in Proposition 5 below.

**Proposition 5.** When two multichannel retailers compete for consumers, some of whom use a smart device in store to obtain online price information, the following can occur:

(a) As the fraction of smart consumers increases, the region of self-matching equilibria increases for low product values but shrinks for high product values.

(b) Retailer profits can increase in the fraction of smart consumers.
The conditions for symmetric self-matching policies to emerge in equilibrium for high product values become more stringent as $\mu$ grows. Namely, as $\mu \to 1$, the symmetric self-matching region for high $v$ shrinks in size to zero. This happens because the existence of smart consumers in this case greatly erodes the positive store competition dampening effect of self-matching, as there are less SU consumers that will still pay the high store price while more consumers pay the much lower online price; thus reducing firms’ incentives to self-match. On the other hand, smart consumers enhance the online competition dampening effect, which allows firms to price higher online when offering to self-match. At low $v$ values this latter effect is dominant.

Thus, and somewhat counterintuitively, we observe that the existence of smart consumers need not decrease the profitability of a self-matching retailer (see Appendix for details of the profit enhancing case). On the contrary, smart consumers can motivate retailers to charge higher online prices, increasing the profitability of self-matching policies. Therefore, we find that given current technology trends, horizontally differentiated retailers would find it worthwhile to more carefully examine whether self-matching is an appropriate strategic option.

5.2. Store-Only Consumers Purchasing Online
Throughout the analysis, we assumed that store consumers have high channel-misfit costs when shopping online, which is why they never shop via that channel. We now relax this assumption so that the store-only consumers face a channel-misfit cost of $s$ when shopping through the online channel, and may consider purchasing online if the price is sufficiently low and offsets the channel misfit costs. We restrict attention to the case when these misfit costs are low compared to the product value, so that there is a non-trivial tradeoff.\(^\text{17}\)

Both firms self-matching is the unique equilibrium when the product value is high relative to the costs faced by store consumers when shopping in the online channel (proof in Appendix).

In this case, the channels become more effective substitutes in the eyes of store-only consumers. This result points to the robustness of Proposition 3, as in both cases high $v$ results in symmetric self-matching policies in equilibrium $(1,1)$. The only difference is that the firms will charge $p^*_j = p_j^\text{on} + s$ instead of $p^*_j = v - \frac{\theta}{2}$, and hence $p_j^\text{on} > \theta$, as they would have incentive to increase online prices, which now determine how high the firms can price in store.

5.3. Connection with Buy Online, Pick-Up In-Store Service
Some retailers implement policies that allow consumers to place an online order using a credit card, and pick-up the product in store. This option combines the convenience of online shopping with some advantages of the retail store environment, and often offers the ability to obtain merchandise

\(^{17}\)With costs above a threshold, in equilibrium consumers would not consider buying online, which could be ensured with a misfit cost close to $v$. 
immediately. These policies are closely connected to self-matching, if the product is in-stock at the store when the consumer buys it online. Here, decided store-only (SD) consumers will be able to claim the online price by purchasing the product online for in-store pick-up (even without a self-match policy), while undecided store-only buyers cannot take advantage of this option since they only make a decision on products after they are able to evaluate a set of choices in the store (and they would not be willing to incur the costs of going home, ordering online, and coming back to the store for pick up). Therefore, there is a clear parallel between a decided consumer who chooses this option and the decided consumer under self-matching in the baseline model, and results from the baseline analysis carry over in this case.

6. Discussion, Limitations and Conclusion

The self-matching pricing policy has become an important element of overall cross-channel strategy for multi-channel retailers, and is used in a variety of markets, including sporting goods, apparel, electronics, discount retail and home furnishings. Our paper makes the first attempt to model this strategic pricing policy and understand how a company’s self-matching decision is determined by consumers’ buying processes and the competitive landscape.

We developed a framework with multichannel retailers competing against e-tailers (mixed duopoly) or other multichannel retailers (multichannel duopoly). Firms in the model choose whether to offer a self-matching pricing policy in the first stage, and then set price levels in the second stage. The retailers’ products are horizontally differentiated, with consumers having heterogeneous preferences over retailers. Consumers are also heterogeneous along two additional dimensions, decision-stage and channel preference, leading to three dimensions of heterogeneity captured in our framework. Thus, our framework explicitly incorporates a wide variety of decision processes for consumers enabled by the multichannel setting.

Self-matching clearly results in some consumers obtaining the lower online price when they would otherwise pay the higher store price, an effect we termed channel arbitrage. We then uncovered two additional mechanisms that only come into play under competition, by which self-matching can enhance firm profits. In several cases these effects can overcome the adverse effects of channel arbitrage. First, we found that self-matching can lead to online competition dampening, an effect that arises because a self-matching firm has less of an incentive to reduce online prices to the competitive level in an asymmetric equilibrium. By offering a self-matching policy, the firm can minimize the damage from channel arbitrage. Second, we found that self-matching allows a multi-channel retailer to achieve store competition dampening, an effect that arises when two multichannel retailers compete intensely for a segment of store consumers. Competition forces them to lower store prices and lose on their respective captive segments. Yet by offering a self-matching policy,
a retailer in effect separates out the competitive decided segment, letting that segment obtain the online price, and charging high store prices from undecided store customers. Self-matching is thus profitable when the positive effects of competition dampening overcome the negative effects of channel arbitrage. We find that the profitability of self-matching is determined by product value (relative to retailer preference), the nature of the competition as well as consumer heterogeneity.

Beyond the baseline model, we considered several extensions, one of which explicitly modeled a setting with smart-device enabled (or “smart”) consumers, who can look up online prices while in store, an increasingly prevalent phenomenon. We find that self-matching can, under certain conditions, increase retailer profitability as the proportion of smart consumers increases. This trend among consumers can prove to be an important issue for retailers to consider when making pricing policy decisions going forward.

An interesting implication for consumer advocates also stems from our research; particularly with respect to the call for consistency across channels. This call has led retailers to strive for an omni-channel paradigm (Neslin et al. 2006, Dahlhoff and Kireyev 2012), suggesting that consumers be offered a similar if not identical experience across the many different channels they may use to interact with the retailer. Self-matching policies, by design, offer retailers the flexibility of setting different prices across channels, while affording consumers the possibility of a consistent experience, presumably in line with the omni-channel philosophy. Although self-matching policies appear to offer consistency across channels, our model predicts that the discrepancy between actual prices paid by different segments of consumers may actually grow, resulting in differential welfare gains across consumer segments. Therefore, advocating price consistency, depending on how retailers implement it, may surprisingly reduce welfare not only for certain types of consumers but potentially even diminish overall consumer surplus.

Our model yields a number of results that are empirically testable. First, retailers offering to self-match will have a larger online to store price discrepancy relative to those that do not self-match. Second, we should expect to find asymmetric self-matching equilibrium configurations in markets for relatively low-valued items (or highly differentiated retailers), with the store price being higher than the online price in such cases. Third, as the penetration of smart-devices among consumers increases, online prices set by retailers offering to self-match are expected to increase and be closer to the in-store prices, i.e., we should observe more coordination across channels.

Although we believe this to be the first research rigorously examining the idea of a self-matching pricing strategy, the present paper has several limitations that future research could tackle. First, we do not model competitive price matching policies. Such policies have been extensively explored in the literature and it would be interesting to examine whether they complement or substitute for self-matching policies in settings where competing retailers sell identical products. Second, although
we have incorporated a variety of consumer decision making processes and preference dimensions, it would be useful to consider a richer model of consumer decision making, for example where consumers could visit a retailer’s store, then decide whether to visit a second based on expectations of price as well as the benefits they may obtain. Such an effort would connect with the search literature, and it would be interesting to examine whether self-matching leads to more search and larger consideration sets. Finally, while we expect the mechanisms detailed here to apply to the case where there are \textit{ex-ante} differences among retailers (based on costs, or loyalty etc.) beyond horizontal differentiation, there might be additional insights obtained in modeling the more general case.

Broadly, our findings suggest that although a self-matching policy may at first appear to be an unprofitable but necessary evil resulting from a prisoners’ dilemma type situation, it has more subtle and positive competitive implications. Indeed, self-matching can be profitably used as a strategic choice and result in higher profits for all firms in the industry. Multichannel retailers should therefore treat their price self-matching decisions as an important element of their overall cross-channel strategy, taking into account their products and consumer characteristics, as well as the competitive landscape.

Appendix A: Proofs of Propositions

Proof of Proposition 1

First, we determine both interior and corner solutions without price matching. At price \( p \), consumer demand is \( D(p) = \min(2\frac{v-p}{\theta}, 1) \). For an interior optimal price, we have the FOC \( \frac{\partial D(p)}{\partial p} \bigg|_{p^*} = 0 = -\frac{v}{\theta} + \frac{v-p}{\theta} \Rightarrow p^* = \frac{v}{\theta} \). The condition for an interior solution is \( v < \theta \). When we have a corner solution, i.e. under \( v > \theta \), the monopolist sets a price of \( p^* = v - \frac{\theta}{2} \). In the rest of the proof, we focus on the case when the markets are covered, i.e. \( v > \theta \).

The monopolist’s profit is determined as follows:

\[
\begin{align*}
\Pi_{1}^{SM=0} &= \eta p_{1}^{on} + (1-\eta)p_{1}^{s} + \beta(\eta p_{1}^{on} + (1-\eta)p_{1}^{s}), \\
\Pi_{1}^{SM=1} &= \eta p_{1}^{on} + (1-\eta)\min(p_{1}^{on},p_{1}^{s}) + \beta(\eta p_{1}^{on} + (1-\eta)p_{1}^{s}),
\end{align*}
\]

so the demand from all segments is equal to 1.\textsuperscript{18}

\textsuperscript{18}To see this, consider the case of a AD consumer. A consumer located at \( x \) purchases online if \( v - p_{1}^{on} - \theta \mid x - \frac{1}{2} \mid \geq 0 \). In other words, consumers located at \( x \geq \frac{1}{2} - \frac{v-p_{1}^{on}}{\theta} \text{ for } x \leq \frac{1}{2} \text{ and at } x \leq \frac{1}{2} + \frac{v-p_{1}^{on}}{\theta} \text{ for } x \geq \frac{1}{2} \) purchase and the remainder do not, leading to a total demand of \( 2 \left( \frac{v-p_{1}^{on}}{\theta} \right) \) for the monopolist from AD consumers. For an interior
To solve for prices, the consumer located farthest from the monopolist must be indifferent between purchasing or not. This yields \( v - p_1^s - \frac{\theta}{2} = 0 \) and \( v - p_1^{on} - \frac{\theta}{2} = 0 \) in the case of no self-matching policy. Prices are then \( p_1^s = p_1^{on} = v - \frac{\theta}{2} \). Similarly, for when the retailer self-matches, we solve \( v - p_1^s - \frac{\theta}{2} = 0, \) \( v - \min(p_1^{on}, p_1^s) - \frac{\theta}{2} = 0 \) and \( v - p_1^{on} - \frac{\theta}{2} = 0 \). Regardless of whether or not SD consumers choose to redeem the self-matching policy, the multichannel retailer will set identical prices across channels, equal to those had it not self-matched: \( p_1^s = p_1^{on} = v - \frac{\theta}{2} \). As the profits under the two conditions are equal, so the monopolist will always prefer \( SM = 0 \), which weakly dominates \( SM = 1 \).

**Proof of Proposition 2**

First, we note that the derivatives of the proportion function \( \Phi_1 \) to be \( \frac{\partial \Phi_1(p_1, p_2)}{\partial p_1} = -\frac{1}{2\theta} \) and \( \frac{\partial \Phi_1(p_1, p_2)}{\partial p_2} = \frac{1}{2\theta} \).

Suppose that a multichannel retailer competed with an online-only e-tailer. Assume \( v > 2\theta \) to ensure that all markets are fully covered. Assume \( v < 4\theta \) and \( \beta > \frac{1}{4}(2 - \theta - 5) \) to ensure that the multichannel firm has positive online sales. Under \((0, 0)\) the firms earn profits

\[
\Pi_1^{0,0} = \eta(1 + \beta)\Phi_1(p_1^{on}, p_2^{on})p_1^{on} + (1 - \eta)(1 + \beta)p_1^s, \quad \Pi_2^{0,0} = \eta(1 + \beta)(1 - \Phi_1(p_1^{on}, p_2^{on}))p_2^{on}
\]

Taking the FOCs with respect to the prices, we solve:

\[
\frac{\partial \Pi_1^{0,0}}{\partial p_1^{on}} = \eta(1 + \beta)\left[\Phi_1(p_1^{on}, p_2^{on}) + p_1^{on}\frac{\partial \Phi_1(p_1^{on}, p_2^{on})}{\partial p_1^{on}}\right] = 0
\]

\[
\frac{\partial \Pi_1^{0,0}}{\partial p_1^s} = (1 - \eta)(1 + \beta) > 0, \text{implying a corner solution.}
\]

We obtain the corresponding FOCs for firm 2 and solve for the equilibrium corresponding to the best responses of both firms. All channel-agnostic consumers will purchase online, whereas store-only consumers will buy from the multichannel firm’s store. The firms will set competitive prices online, \( p_1^{on} = p_2^{on} = \eta \), and firm 1 will set monopoly price in-store \( p_1^s = (v - \theta) \). That is, we obtain an interior solution for online pricing, but a corner solution for the store price where the multichannel firm maximizes profits from all captive SU consumers.

Under the \((1, 0)\) subgame of competition between a self-matching multichannel retailer with an e-tailer, firms earn profits

\[
\Pi_1^{1,0} = \eta(1 + \beta)\Phi_1(p_1^{on}, p_2^{on})p_1^{on} + (1 - \eta)(p_1^{on} + \beta p_1^s), \quad \Pi_2^{1,0} = \eta(1 + \beta)(1 - \Phi_1(p_1^{on}, p_2^{on}))p_2^{on}
\]

As under \((0, 0)\), channel-agnostic consumers will purchase online and store-only consumers will purchase from the multichannel firm’s store. Decided store-only consumers redeem the matching policy, and pay the online price, whereas undecided store-only consumers fail to redeem the policy and pay the in-store price. The firms will set prices \( p_1^{on} = \theta + \frac{4\theta(1-n)}{3\eta(1+\beta)}, \) \( p_2^{on} = \theta + \frac{2\theta(1-n)}{3\eta(1+\beta)}, \) \( p_1^s = v - \theta \) for \( v > 2\theta + \frac{4\theta(1-n)}{3\eta(1+\beta)} \). Once again, solution to exist (the market is not completely served) it must be the case that \( 2\left(\frac{v - p_1^{on}}{\theta}\right) < 1 \), or \( p_1^{on} > v - \frac{\theta}{2} \).

However, solving the optimization problem for the monopolist (maximizing \( 2\left(\frac{v - p_1^{on}}{\theta}\right)p_1^{on} \)) will yield a price of \( \frac{v}{2} \), and \( \frac{v}{2} > v - \frac{\theta}{2} \) if only if \( v < \theta \). A similar logic follows for the other segments. Hence, the condition \( v > \theta \) ensures that the monopolist serves the entire market.
there is an interior solution in online pricing for \( v \) sufficiently large, and a corner monopoly solution for the store price. The threshold for \( v \) is derived from the condition that in equilibrium \( p_1^{on} < v - \theta \) for an interior solution. That is, the online price charged by firm 1 cannot exceed the monopoly price for SD consumers, or equivalently, \( v - p_1^{on} - \theta > 0 \), ensuring that the SD consumer farthest away from store 1 purchases in equilibrium for the market to remain covered. For \( v \leq 2\theta + \frac{4(1-\eta)}{3\eta(1+\beta)} \), this condition fails, and firms will set prices \( p_1^{on} = v - \theta, p_2^{on} = v/2, p_1^s = v - \theta \) which corresponds to a corner solution.

Now we substitute prices into profits for the appropriate \( v \) and identify the parameter ranges for which \( \Pi_{1,0} > \Pi_{0,0} \) to see when firm 1 would prefer to self-match. Suppose that \( \beta > \frac{4-7\eta}{3\eta} \). Then, \( \Pi_{1,0} > \Pi_{0,0} \) if \( v < z_1 = \theta \left( \frac{7}{3} + \frac{8(1-\eta)}{9\eta(1+\beta)} \right) \). Otherwise, if \( \beta \leq \frac{4-7\eta}{3\eta} \), then \( \Pi_{1,0} > \Pi_{0,0} \) if \( v < z_2 = 3\theta \). Hence, there exists a \( z_0 = \min\{z_1, z_2\} \), such that for \( v < z_0 \), the multichannel retailer will prefer to self-match.

To show the second part of the proposition, we explore how the threshold \( z_0 \) changes as the model parameters change. First, note that \( \frac{\partial z_2}{\partial \theta} > 0 \), but \( \frac{\partial z_2}{\partial \eta} = 0 \), and \( \frac{\partial z_2}{\partial \beta} = 0 \). So if \( \beta \geq \frac{4-7\eta}{3\eta} \) which is possible if \( \eta \leq \frac{4}{7} \), then the incremental profits from self-matching to the multichannel firm grow as \( \theta \) increases (as the threshold for self-matching to emerge in equilibrium becomes less stringent in \( v \)), but remain unaffected by changes in \( \beta \) and \( \eta \). For \( \beta > \frac{4-7\eta}{3\eta} \), note that \( \frac{\partial \Pi_2}{\partial \theta} > 0 \), \( \frac{\partial \Pi_2}{\partial \eta} < 0 \), and \( \frac{\partial \Pi_2}{\partial \beta} < 0 \), implying that incremental profits from self-matching grow as \( \theta \) increases, but shrink as \( \eta \) and \( \beta \) increase. Also, note that we require \( v \) to be small for self-matching to emerge. Hence, as \( v \) grows and approaches \( z_0 \) the incremental profits from self-matching shrink.

**Proof of Corollary 1**

A comparison of the e-tailer’s profits, \( \Pi_{1,0} - \Pi_{0,0} \) reveals that it earns greater profits when the multichannel retailer offers a self-matching policy. To see this, note that the e-tailer’s price under \( (1,0) \) is \( p_2^{on} = \theta + \frac{2\eta(1-\eta)}{3\eta(1+\beta)} \) which is greater than \( \theta \), the price it would charge under \( (0,0) \). Also in \( (1,0) \), the e-tailer’s price is less than \( p_1^{on} = \theta + \frac{4(1-\eta)}{3\eta(1+\beta)} \), the online price charged by the multichannel retailer. Under \( (1,0) \) the e-tailer sets a higher price, and earns a greater fraction of demand than under \( (0,0) \). As a result, its profits are greater.

Precisely, suppose \( v \leq 2\theta + \frac{4(1-\eta)}{3\eta(1+\beta)} \). Then \( \Pi_{1,0} = \frac{2(1+\beta)(1-\eta)}{3\eta} \) and \( \Pi_{0,0} = \frac{\eta(1+\beta)}{2} \). The difference \( \Pi_{1,0} - \Pi_{0,0} = \frac{(2^2 - 4\eta^2)(1+\beta)}{8\eta(1+\beta)} \) which is positive when \( v > 2\theta \), which is the lower bound required for markets to be fully covered. Now, suppose \( v > 2\theta + \frac{4(1-\eta)}{3\eta(1+\beta)} \). Then \( \Pi_{1,0} = \frac{\eta(n+3\eta)(2+2\eta)}{18\eta(1+\beta)^2} \) and \( \Pi_{0,0} = \frac{\eta(1+\beta)}{2} \). The difference \( \Pi_{1,0} - \Pi_{0,0} = \frac{2(1-\eta)}{3} \left( 1 + \frac{1-\eta}{3\eta(1+\beta)} \right) \) is greater than zero whenever \( \beta > -\frac{1+2\eta}{3\eta} \), which is always the case as \( \beta > 0 \). Hence, the e-tailer always makes higher profits when the multichannel retailer matches.

Below, we refer to \( m \) as the cost of undertaking a second shopping trip for undecided store-only consumers. We derive bounds on \( m \) that ensure the consumer behavior specified in our assumptions.

**Proof of Proposition 3**

First, we consider each subgame separately. Then, we compare the profits from each subgame to derive the bounds for the equilibrium results. The following constraints need to be imposed:

- \( v > \frac{3\beta}{2} \) ensures that all markets are fully covered,
- \( \beta < 5/3 \) ensures that no firm sets such a high online price to earn zero demand from decided consumers in the \( (1,0) \) subgame,
• $v < \theta \left( \frac{9\beta^2 + 48\beta + 25}{360} + \frac{(1+\beta)^2}{36(1+\beta\eta)^2} + \frac{3\beta^2 + 8\beta + 5}{18\beta(1+\beta\eta)} \right)$ ensures that no firm wants to price exclusively for its captive segment of undecided store-only consumers and forego all demand for decided store-only consumers,

• $m > v - \frac{\eta}{2}$ ensures that no undecided store-only consumers switch stores after their first visit, and that no consumer returns home and visits the store a second time to redeem a self-matching policy.

No Matching - (0,0) Channel-agnostic consumers will purchase online. Store-only consumers will buy in-store. All consumers will pay the price set in the channel they buy from. The firms will earn profits

$$\Pi_1^{0,0} = \eta(1 + \beta) \Phi_1(p_1^{on}, p_2^{on}) p_1^{on} + (1 - \eta) \left( \Phi_1(p_1^t, p_2^t) + \frac{\beta}{2} \right) p_1^t,$$

$$\Pi_2^{0,0} = \eta(1 + \beta)(1 - \Phi_1(p_1^{on}, p_2^{on})) p_2^{on} + (1 - \eta) \left( 1 - \Phi_1(p_1^t, p_2^t) \right) + \frac{\beta}{2} p_2^t.$$

Again, we solve for the first order conditions $\frac{\partial \Pi_j^{0,0}}{\partial p_j^t} = 0$ and $\frac{\partial \Pi_j^{0,0}}{\partial p_j^{on}} = 0$ for $j \in \{1, 2\}$, and check for corner solutions. We find an interior solution with equilibrium prices at $p_1^{on} = p_2^{on} = \theta$ and $p_1^t = p_2^t = \theta(1 + \beta)$ for $\frac{\eta}{2} > \frac{3}{2} + \beta$, and a corner solution in in-store prices with $p_1^t = p_2^t = v - \frac{\eta}{2}$ for $\frac{\eta}{2} \leq \frac{3}{2} + \beta$. The in-store price is larger than the online price in all cases as firms have an incentive to price higher for their captive segment of store-only consumers. The binding condition for an interior solution requires that all SU consumers purchase in equilibrium. For firm 1, this can be written as $v - p_1^t - \frac{\eta}{2} > 0$ (the utility for the SU consumer farthest away from store 1 is greater than zero). When this condition fails (i.e. $\frac{\eta}{2} \leq \frac{3}{2} + \beta$), we have a corner solution where firms set local monopoly prices $v - \frac{\eta}{2}$ in-store. No other constraints apply and there are no other corner solutions. The equilibrium profits earned by firms are

$$\Pi_1^{0,0} = \Pi_2^{0,0} = \begin{cases} \frac{1}{2} (1 + \beta)(1 + \beta(1 - \eta)), & \frac{\eta}{2} > \frac{3}{2} + \beta \\ \frac{1}{2} [2v(1 - \eta) - t(1 - 3\eta)], & \frac{\eta}{2} \leq \frac{3}{2} + \beta \end{cases}.$$

Symmetric Self-Matching - (1,1) Channel-agnostic consumers will purchase online, and pay the online price. Decided store-only consumers will buy in-store, but will redeem the online price of the store they purchase from. Undecided store-only consumers will buy in the store they first visit and will pay the in-store price. The firms will earn profits:

$$\Pi_1^{1,1} = (1 + \eta\beta) \Phi_1(p_1^{on}, p_2^{on}) p_1^{on} + (1 - \eta) \beta p_1^t,$$

$$\Pi_2^{1,1} = (1 + \eta\beta)(1 - \Phi_1(p_1^{on}, p_2^{on})) p_2^{on} + (1 - \eta) \beta p_2^t.$$

and set prices $p_1^{on} = p_2^{on} = \theta$ online and $p_1^t = p_2^t = v - \theta/2$ in-store. The online price is the familiar competitive price $\theta$ and is an interior solution to the first-order conditions $\frac{\partial \Pi_j^{1,1}}{\partial p_j^{on}} = 0$ for $j \in \{1, 2\}$. Differentiating with respect to in-store prices yields $\frac{\partial \Pi_j^{1,1}}{\partial p_j^t} = (1 - \eta) \frac{\beta}{2} > 0$, implying a corner solution. The firms will set the highest in-store price they can, ensuring all SU consumers purchase, which is $v - \frac{\theta}{2}$. There are no other corner solutions. The equilibrium profits earned by firms are

$$\Pi_1^{1,1} = \Pi_2^{1,1} = \frac{1}{2} \left( \theta(1 + \eta\beta) + \beta(1 - \eta) \left( v - \frac{\theta}{2} \right) \right).$$
Asymmetric Self-Matching - (1,0) Channel-agnostic consumers will purchase online, and pay the online price. Decided store-only consumer will buy in-store, but will redeem the online price (as it will be lower) if they buy from the non-self-matching firm. They will pay the in-store price if they buy from the non-self-matching firm. Undecided store-only consumers will buy from the store they first visit and pay the in-store price. The firms profits are then:

$$\Pi_1^{1,0} = \eta(1 + \beta)\Phi_1(p_1^{on}, p_2^{on})p_1^{on} + (1 - \eta) \left( \Phi_1(p_1^{on}, p_2^{on})p_1^{on} + \frac{\beta}{2} p_1^{on} \right),$$

$$\Pi_2^{1,0} = \eta(1 + \beta)(1 - \Phi_1(p_1^{on}, p_2^{on}))p_2^{on} + (1 - \eta) \left( (1 - \Phi_1(p_1^{on}, p_2^{on})) + \frac{\beta}{2} p_2^{on} \right).$$

The first order conditions can be written as:

$$\frac{\partial \Pi_j^{1,0}}{\partial p_2^{on}} = 0, \frac{\partial \Pi_j^{1,0}}{\partial p_1^{on}} = 0 \text{ for } j \in \{1, 2\}, \text{ and } \frac{\partial \Pi_1^{1,0}}{\partial p_1} = (1 - \eta) \frac{\beta}{2} > 0.$$

In equilibrium, there is an interior solution for online prices and for the in-store price of firm 2, and a corner solution for the in-store price of firm 1, for large \(v\), the firms set online prices \(p_1^{on} = \theta \left( \frac{v}{\theta} + \frac{3 + \frac{1 + \beta}{6(1 + \eta \beta)}}{6(1 + \eta \beta)} \right)\) and in-store prices \(p_1^{in} = v - \frac{\theta}{2}\) and \(p_2^{in} = p_2^{on} + \frac{\beta}{2}\) for \(\frac{v}{\theta} > \left( \frac{4}{3} + \frac{1 + \beta}{6(1 + \eta \beta)} + \frac{\beta}{2} \right)\).

Otherwise, if \(v\) is small, we have a corner solution for \(p_2^{in}\) which yields prices \(p_2^{on} = \theta + \frac{(1 - \eta)(2v - 3\eta)}{2(4 - \eta(1 - \delta))}\) online and \(p_1^{in} = p_2^{in} = v - \frac{\theta}{2}\) in-store. The binding threshold on \(v\) for an interior solution requires that all of firm 2's SU consumers purchase in equilibrium. In other words, \(v - p_2^{on} > \frac{\theta}{2} > 0\). Substituting the interior solution equilibrium in-store price for firm 2 into the inequality shows that the corner solution holds for \(v/\theta \leq \left( \frac{4}{3} + \frac{1 + \beta}{6(1 + \eta \beta)} + \frac{\beta}{2} \right)\).

The equilibrium profits earned by firms are

$$\Pi_1^{1,0} = \frac{v\beta}{2}(1 - \eta) + \frac{t}{9} \left[ 4 - \frac{\beta}{4}(1 - 17\eta) + \frac{(1 + \beta)^2}{2(1 + \eta \beta)} \right],$$

$$\Pi_2^{1,0} = \frac{t}{72} \left[ 9\beta^2(1 - \eta) + \beta(34 - 5\eta) + 29 + \frac{7(1 + \beta)^2}{1 + \eta \beta} \right]$$

for \(\frac{v}{\theta} > \left( \frac{4}{3} + \frac{1 + \beta}{6(1 + \eta \beta)} + \frac{\beta}{2} \right)\). The expression for \(\frac{v}{\theta} \leq \left( \frac{4}{3} + \frac{1 + \beta}{6(1 + \eta \beta)} + \frac{\beta}{2} \right)\) can be obtained similarly by substituting equilibrium prices into the profit functions, and is available upon request.

It is interesting to note that the price discrepancy across channels grows for a self-matching firm. The firm offering to self-match will set monopoly price \(v - \frac{\theta}{2}\) in-store to leverage the demand from its captive segment of undecided store-only consumers. In both (1,1) for all \(v\) and in (1,0) for high \(v\) the online price set by the matching firm is constant in \(v\). Hence, the price discrepancy across channels grows with \(v\). In (1,0) for low \(v\), the matching firm sets online price \(\theta + \frac{(1 - \eta)(2v - 3\eta)}{4 - \eta(1 - \delta)}\) which grows at a rate \(\frac{2(1 - \eta)}{4 - \eta(1 - \delta)}\) in \(v\). This is strictly less than 1, which is the rate of growth of the monopoly price \(v - \frac{\theta}{2}\). So, in all cases, the price discrepancy across channels grows for a self-matching firm.

Equilibrium Analysis A self-matching configuration is an SPNE if no firm has the incentive to unilaterally deviate. Equivalently, for (0,0) to be an SPNE, firm 1 must not have the incentive to deviate to (1,0). For (1,1) to be an SPNE, firm 2 must not have the incentive to deviate to (1,0). For (1,0) or (0,1) to be an SPNE the self-matching firm must not prefer (0,0) and the non-self-matching firm must not prefer (1,1). By comparing the profits at the equilibrium prices defined above, we can construct equilibrium regions.

Let \(\beta' = \sqrt{\frac{25\eta^2 + 448\eta + 256 - 108/11}{8 + 22\eta}} + \frac{\beta}{22}\). The results in proposition 3 focus on the region where \(\beta < \beta'\) for clarity of exposition. In this proof, we provide an extended analysis, including the region where \(\beta \geq \beta'\). Define
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\[ v' = \theta \left( \frac{9\beta^2 + 48\beta + 25}{36\beta} + \frac{(1+\beta)^2}{36\beta(1+\beta)} + \frac{3\beta^2 + 8\beta + 5}{18\beta(1+\beta)} \right) \]

- \[ z_1 = \min \left( \frac{10\beta + 27}{18} + \frac{\beta(1+\beta)}{9(1+\beta)} \cdot \frac{9\beta + 59}{36} + \frac{7(1+\beta)}{36(1+\beta)} \cdot \frac{13}{6} + \frac{8(1-n)}{3(8+n(1+\beta))} \right), \]
- \[ z_2 = \max \left( \frac{9\beta + 59}{36} + \frac{7(1+\beta)}{36(1+\beta)} \cdot \frac{10\beta + 27}{18} + \frac{\beta(1+\beta)}{9(1+\beta)} \right), \]
- \[ z_3 = \frac{3\beta}{18} + \beta - \frac{1+\beta}{9(1+\beta)}. \]

We calculate equilibrium profits and the applicable thresholds under all subgames. Then, for \( \frac{v}{\beta} < z_1 \), the incremental profit from self-matching for firm 1 is positive: \( \Pi_{1,0} - \Pi_{1,0} > 0 \), and firm 2 prefers not to deviate as \( \Pi_{2,1} - \Pi_{2,0} < 0 \), so \( (1,0) \) and \( (0,1) \) are SPNE. For \( z_1 < \frac{v}{\beta} < z_2 \), \( (0,0) \) is the unique equilibrium for \( \beta < \beta' \), as \( \Pi_{1,0} - \Pi_{1,0} < 0 \) while \( \Pi_{1,1} = \Pi_{1,0} > 0 \), and \( (1,1) \) is the unique equilibrium for \( \beta > \beta' \) as \( \Pi_{1,1} = \Pi_{1,0} > 0 \) while \( \Pi_{1,1} = \Pi_{1,0} < 0 \). For \( z_2 < \frac{v}{\beta} < z_3 \), both \( (0,0) \) and \( (1,1) \) are SPNE as \( \Pi_{1,0} - \Pi_{1,0} < 0 \) while \( \Pi_{1,1} = \Pi_{1,0} < 0 \), so no firm prefers to unilaterally deviate from either symmetric setup. For \( \frac{v}{\beta} > z_3 \), \( (1,1) \) is the unique SPNE as \( \Pi_{1,1} = \Pi_{1,0} > 0 \) while \( \Pi_{1,1} = \Pi_{1,0} < 0 \).

Note that for sufficiently large \( \beta \), firms prefer to offer symmetric self-matching policies at intermediate \( v \). This is because as \( \beta \) grows, firm 1 has an incentive to match for lower \( v \) given that firm 2 also matches. As the critical threshold of \( v \) becomes lower, it may cross the threshold at which the other firm no longer prefers to match, yielding an equilibrium where both firms match for intermediate \( v \).

To summarize, for low \( \beta \), as \( v \) increases, there will first be an asymmetric solution, then \( (0,0) \), then both \( (0,0) \) and \( (1,1) \), and then uniquely \( (1,1) \). For high \( \beta \), as \( v \) increases, there will first be an asymmetric solution, then \( (1,1) \), then both \( (0,0) \) and \( (1,1) \), and then uniquely \( (1,1) \).

**Proof of Proposition 4**

A comparison of profits in the \((1,0)\) subgame reveals that \( \Pi_{1,0} > \Pi_{1,0} \) everywhere. A comparison of profits earned by firm 1 in the \((1,1)\) subgame and in the \((0,0)\) subgame reveals that \( \Pi_{1,1} > \Pi_{1,0} \) if \( \frac{v}{\beta} > 3 + \beta = z_4 \), which is strictly greater than \( z_3 \).

**Appendix B: Proofs for Extensions**

**Proof of Proposition 5**

The proof of proposition 5 proceeds just as that the previous proposition, except with an extra parameter \( \mu \) representing fraction of “mobile” consumers, or consumers who can costlessly search for online information while in-store. The \( \mu \) segment will be relevant for undecided store-only consumers, as these will only be able to claim a self-matching policy if they have access to the internet in-store. The non-mobile undecided store-only consumers will not be able to claim a self-matching policy, and will have to pay the in-store price.

We require the following restrictions:

- \( v > \frac{3\beta}{2} + \frac{\beta n(1-n)}{1+\beta n} \) ensures that all markets are fully covered,
- \( \beta < \frac{3-2v}{3} \) ensures that no firm sets such a high online price to earn zero demand from decided consumers in the \((1,0)\) subgame,
- \( \beta > \frac{v}{2} + \beta = z_4 \),
\( v < \theta \left( \frac{1}{\beta} + \frac{4\mu + 16\eta + 3\beta \eta - 4\mu \eta + 2}{12\eta} \right) \) ensures that no firm wants to price exclusively for its captive segment of undecided store-only consumers and forego all demand for decided store-only consumers,

\( m > v - \frac{3\eta}{2} + \mu \theta - \frac{\theta(1 + \beta)}{1 + \beta} \) ensures that no undecided store-only consumers switch stores after their first visit, and that no consumer returns home and visits the store a second time to redeem a self-matching policy.

**No Firms Self-Match - (0,0)** The equilibrium prices under (0,0) emerge just as in proposition 3, as mobile consumers behave just as the rest of the consumers.

**One Firm Self-Matches - (1,0)** Undecided store-only consumers who are mobile will redeem the self-matching policy if they first visit the store that offers the higher profit. Profits are

\[
\Pi_1^{1,0} = \eta(1 + \beta)\Phi_1(p_1^{on}, p_2^{on})p_1^{on} + (1 - \eta) \left( \Phi_1(p_1^{on}, p_2^{on})p_1^{on} + \frac{\beta}{2} \left( (1 - \mu)p_1^{on} + \mu p_1^{on} \right) \right),
\]

\[
\Pi_2^{1,0} = \eta(1 + \beta)(1 - \Phi_1(p_1^{on}, p_2^{on}))p_2^{on} + (1 - \eta) \left( (1 - \Phi_1(p_1^{on}, p_2^{on}))p_2^{on} + \frac{\beta}{2} p_2^{on} \right).
\]

In equilibrium, the firms set online prices \( p_1^{on} \) and \( p_2^{on} \) and in-store prices \( p_1 = v - \frac{\theta}{\beta} \) and \( p_2 = p_2^{on} + \frac{\beta}{2} \) for \( \frac{\eta}{\theta} > \frac{(1 + 2\mu)(1 + \beta)}{12(1 + \beta)} + \frac{2(1 - \mu)}{3} \). Otherwise, \( p_1^{on} = \frac{\theta + 2v + 2\mu - 2\eta \theta + 3\beta \eta \theta - 2\beta \mu \theta + 6\beta \eta \theta - 2\beta \mu \theta}{4(1 + 3\beta \eta)} \) and \( p_2^{on} = \frac{\theta + 2v + 2\mu - 2\eta \theta + 3\beta \eta \theta - 2\beta \mu \theta}{4(1 + 3\beta \eta)} \) and \( p_1 = p_2 = v - \frac{\theta}{\beta} \) in-store.

**Both Firms Self-Match - (1,1)** The firms will earn profits

\[
\Pi_1^{1,1} = (1 + \eta/\beta)\Phi(p_1^{on}, p_2^{on})p_1^{on} + (1 - \eta) \frac{\beta}{2} \left( (1 - \mu)p_1^{on} + \mu p_1^{on} \right),
\]

\[
\Pi_2^{1,1} = (1 + \eta/\beta)(1 - \Phi_1(p_1^{on}, p_2^{on}))p_2^{on} + (1 - \eta) \frac{\beta}{2} \left( (1 - \mu)p_2^{on} + \mu p_2^{on} \right),
\]

and set prices \( p_1^{on} = p_2^{on} = \theta(1 - \mu) + \frac{\mu \theta(1 + \beta)}{1 + \beta \eta} \) online and \( p_1 = p_2 = v - \frac{\theta}{\beta} \) in-store.

**Equilibrium Analysis** To prove existence of the result, we provide an example with \( \eta = 0.2 \) and \( \beta = 0.5 \).

Let

\[
y_0 = \frac{2385\mu - 41(529\mu^2 + 1408\mu + 1936)\eta^2 + 7810}{4004} + \frac{32\mu + 331}{198}, \quad y_1 = \frac{32\mu + 331}{198}, \quad y_2 = \frac{364\mu + 395}{198} + \frac{3}{88(1 - \mu)}.
\]

Comparing profits when \( \frac{\eta}{\theta} < y_0 \), (1,1) is the unique SPNE. For \( y_0 < \frac{\eta}{\theta} < y_1 \), (1,0) and (0,1) are SPNE. For \( y_1 < \frac{\eta}{\theta} < y_2 \), (0,0) is the unique equilibrium. For \( y_2 < \frac{\eta}{\theta} < y_3 \), both (0,0) and (1,1) are SPNE. For \( \frac{\eta}{\theta} > y_3 \), (1,1) is the unique SPNE. To prove the associated proposition, note that \( y_0, y_1, y_2 \) and \( y_3 \) are all increasing in \( \mu \), so that holding constant \( \frac{\eta}{\theta} \), an increase in mobile consumers shrinks the equilibrium region that admits self-matching policies.

**Increasing profits with mobile consumers** In the (1,1) equilibrium for large \( v, \eta = 0.2 \) and \( \beta = 0.5 \), the retailers’ profits are increasing in \( \mu \) if \( \frac{\eta}{\theta} < \frac{\eta}{\theta} + \frac{2\mu}{11} \), which is possible if \( \mu < 0.83 \). Furthermore, the retailers’ profits are larger than when \( \mu = 0 \) if \( \frac{\eta}{\theta} < 5/2 + 4\mu/11 \), which is possible if \( \mu < 0.72 \). This shows that retailers in symmetric self-matching equilibria need not fear the existence of mobile consumers.
Store Consumers Purchasing Online

We require that \( v > s + \frac{\theta}{2} + \theta(1 + \beta)/(1 + \beta \eta) \) for the analysis of the equilibria below. In all cases, profit expressions as a function of prices are identical to those in the proof for proposition 3. The only difference is that now \( v \) is sufficiently large so that firms can no longer price at a monopoly level for in-store consumers, or they may switch to the online channel - which the firms would find unprofitable given the more competitive nature of the online channel.

No Firms Self-Match - (0,0) Solutions to the first order conditions yield that in this subgame the two firms will price \( p_1^{o_n} = p_2^{o_n} = \theta \) and \( p_1^s = p_2^s = \theta(1 + \beta) \) for \( s > \beta \theta \) (an interior solution), and \( p_1^{o_n} = p_2^{o_n} = \frac{\theta(1 + \beta) - s(1 - \eta)}{1 + \beta \eta} \) and \( p_1^s = p_2^s = p_1^{o_n} + s \) for \( s < \beta \theta \) (a corner solution). Note that for \( s < \beta \theta \) we have a corner solution, where the binding constraint \( p_j^s \leq p_j^{o_n} + s \) that ensures store consumers purchase in-store and not online binds for both firms \( j \in \{1, 2\} \). Neither firm wants store consumers to purchase online as the online channel is more competitive because it does not contain a captive segment of consumers. The equilibrium profits earned by firms are

\[
\Pi_1^{0,0} = \Pi_2^{0,0} = \begin{cases} 
\frac{(1 + \beta)}{2(1 + \beta \eta)} [s \beta \eta (1 - \eta) + t(1 + \beta)], & s \leq \beta \theta \\
\frac{1}{2} (1 + \beta) (1 + (1 - \eta)(1 + \beta)), & s > \beta \theta
\end{cases}
\]

Both Firms Self-Match - (1,1) Under symmetric self-matching, the firms will set \( p_1^{o_n} = p_2^{o_n} = \frac{\theta(1 + \beta)}{1 + \beta \eta} \) (interior solution) online and \( p_1^s = p_2^s = p_1^{o_n} + s \) (corner solution) in-store.

The equilibrium profits earned by firms are

\[
\Pi_1^{1,1} = \Pi_2^{1,1} = \frac{1}{2} \left[ s \beta (1 - \eta) + t(1 + \beta)^2 \right].
\]

One Firm Self-Matches - (1,0) In the asymmetric subgame, the firms will price \( p_1^{o_n} = \frac{\theta(1 + \beta)}{1 + \beta \eta} \), \( p_2^{o_n} = \frac{\theta(1 + \beta)}{2(1 + \beta \eta)} \) online (an interior solution), and \( p_1^s = p_1^{o_n} + s \) (a corner solution), \( p_2^s = \frac{\theta(1 + \beta) (2 + \beta \eta)}{2(1 + \beta \eta)} \) (an interior solution) in-store for \( s > \beta \theta \). For \( s \leq \beta \theta \), firm 2 will set \( p_2^{o_n} = \frac{\theta(1 + \beta) - s(1 - \eta)}{1 + \beta \eta} \) and \( p_2^s = \frac{\theta(1 + \beta) - s(1 - \eta)}{1 + \beta \eta} \) (a corner solution). For \( s \) sufficiently small, the relevant constraint is \( p_2^s \leq p_2^{o_n} + s \) for firm 2. The equilibrium profits earned by firms are

\[
\Pi_1^{1,0} = \frac{1}{2} \left[ s \beta (1 - \eta) + t(1 + \beta)^2 \right], \quad \Pi_2^{1,0} = \begin{cases} 
\frac{(1 + \beta)}{2(1 + \beta \eta)} \left[ s \beta \eta (1 - \eta) + t(1 + \beta) - \frac{s^2 \eta (1 - \eta)}{t} \right], & s \leq \frac{\beta \theta}{2} \\
\frac{(1 + \beta)}{8} \left[ 1 + \beta (1 - \eta) + \frac{3(1 + \beta)}{2(1 + \beta \eta)} \right], & s > \frac{\beta \theta}{2}
\end{cases}
\]

Equilibrium Analysis A comparison of equilibrium profits yields that \( \Pi_1^{1,0} - \Pi_2^{1,0} > 0 \) and \( \Pi_1^{1,1} - \Pi_2^{1,1} > 0 \), so \( (1, 1) \) is the unique equilibrium solution.

Appendix C: Self-Matching Policies in Practice

Below we list the self-matching policies of several popular retailers as obtained from their websites on 11/14/2014.
Self-Matching Retailers

**Best Buy:** “We match BestBuy.com prices on in-store purchases”

**RadioShack:** “We’ll match... Radioshack.com”

**Target:** “For matching Target.com or online competitors, the retail price must be shown on your mobile device or by bringing in a printed page showing the current price.”
- [https://corporate.target.com/about/shopping-experience/shop-with-confidence](https://corporate.target.com/about/shopping-experience/shop-with-confidence)

**Staples:** “If you purchase an item from Staples and tell us within 14 days that you found that item at a lower price in our stores or at staples.com, we’ll refund the difference.”

**Toys”R”Us:** “We will match Toysrus.com and Babiesrus.com online pricing in our stores”

Retailers who do not Self-Match

**Walmart:** “We do not honor... internet pricing”

**Urban Outfitters:** “While merchandise offered on-line at UrbanOutfitters.com will usually be priced the same as merchandise offered at our affiliate Urban Outfitters stores, in some cases, Urban Outfitters stores may have different prices or promotional events at different times.”

**Sports Authority:** “Ad Price Match Promise is not valid on sportsauthority.com”

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