



Improving Store Liquidation

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Abstract

Store liquidation is the time-constrained divestment of retail outlets through an in-store sale of inventory. The retail industry depends extensively on store liquidation, not only as a means for investors to recover capital from failed ventures, but also to allow managers of going concerns to divest stores in efforts to enhance performance and to change strategy. Recent examples of entire chains being liquidated include Borders Group in 2012, Circuit City in 2009, and Linens 'n Things in 2008; the value of inventory sold during these liquidations alone is \$3B. The store liquidation problem is related to but also differs substantially from the markdown optimization problem that has been studied extensively in the literature. This paper introduces the store liquidation problem to the literature and presents a technique for optimizing key decision variables, such as markdown, inventory, and store closing decisions during liquidations. We show that our approach could improve net recovery on cost (i.e., the profit obtained during liquidations stated as a percentage of the cost value of liquidated assets) by 2 to 7 percentage points in the cases we examined. The paper also identifies ways in which current practice in store liquidation differs from the optimal decisions identified in the paper and traces the consequences of these differences.

1 Introduction

Store liquidation, defined as the as the time-constrained divestment of retail stores through an in-store sale of inventory, is a critical aspect of the retail industry for both defunct and going

concerns. Going concerns use store liquidation to divest sets of stores or even entire concepts. Struggling and failed retailers liquidate thousands of stores and billions of dollars of inventory each year. For example, as Borders Group entered bankruptcy in early 2011, it held \$638.5M dollars of inventory.¹ Circuit City and Linens ‘n Things held \$1.5B and \$795.4M, respectively, of inventory prior to entering bankruptcy. Given the size of these liquidations, even a small improvement in net recovery on cost—i.e., the profit obtained during a liquidation stated as a percentage of the cost value of inventories liquidated—can be substantial. For example, a 1% improvement in Circuit City’s net recovery on cost would have amounted to \$15M.

Store liquidation has important implications for firms and investors. Bankruptcy and, thus, liquidation are common in retailing: for the United States alone, Capital IQ records 2,013 retailer bankruptcy announcements over the decade beginning in 2000. Further, Gaur et al. (2013) find that 3.4% of all public retailers during the past 20 years were liquidated in bankruptcy. Store liquidations operated by asset disposition firms like Gordon Brothers Group (GBG) and Hilco Merchant Resources, such as those conducted for Montgomery Ward (Ordonez et al., 2001) and Syms Corporation, the owner of Filene’s Basement (Mattioli, 2011), help stakeholders recoup funds from failed firms and allow investors to shunt capital to other ventures. According to managers at GBG, that firm alone liquidated over \$2B of inventory measured at retail value during 2011.

Improving store liquidation in bankruptcy can also directly increase liquidity within the retail industry, since the funds and terms available to retailers using inventory-based lending, a type of asset-based lending in which the collateral is a retailer’s inventory, depend on the estimated NOLV of the inventory (Foley et al., 2012). Inventory-based lending is an important source of capital in the retail sector (Alan and Gaur, 2012). Example inventory-based loans include a \$3.28B revolver held by Sears Holdings Corporation and a \$1B revolver used by Barnes & Noble.² Of course, the liquidation value of a retailer can also affect its ability to obtain trade credit as well as the terms of the credit it receives; see, for example, Yang and Birge (2011).

Store liquidation is a valuable tool for going concerns: the ability to redeploy resources by effectively liquidating subsets of stores is key for managers. Store liquidations allow firms to generate

¹The inventory figures reported in this paragraph are recorded by Capital IQ.

²These figures are reported in firm quarterly filings and are current as of January 1, 2013.

cash from poorly performing stores and chains, as in the cases of Barnes & Noble’s decision to close roughly 200 stores over the coming decade (Trachtenberg, 2013), of Sears Holdings’ liquidation of over 100 Sears and Kmart stores (Lahart, 2011), and of Home Depot’s closing of its EXPO stores (Zimmerman, 2009). Store liquidation also allows managers to free resources to abet a change in strategy, as in Best Buy’s closing of 50 big box stores to fund a new focus on mobile device stores (Bustillo, 2012). Store liquidation is useful in other situations as well: when Pamida, a department store chain, merged with Shopko (LBO Wire, 2012), a similar firm, managers conducted store liquidations to empty Pamida stores and prepare them for conversion to Shopko stores. Other going concerns that rely on asset disposition firms to close stores include Dick’s Sporting Goods, Forever 21, J.C. Penney, Rite Aid, and Saks Fifth Avenue.³

The store liquidation problem differs from the markdown optimization problem that has been studied extensively in the literature. First, unlike in the markdown optimization problem, retailers have to close (i.e., stop operating their stores) in the store liquidation problem. The decision of when to close a store and, if needed, move the merchandise to another store is an integral part of the liquidation problem. The decision of which stores to open on a particular day adds a number of binary decision variables to the optimization problem and is a function of demand levels, store operating costs, and inter-store transfer times and costs.

Second, consumers behave differently during a store liquidation than at a store under normal operation. Hence, there is considerably more demand uncertainty during store liquidation than during normal store operations or even in markdown optimization. It is hard to predict ex-ante how consumers will react to a liquidation event, and the reaction can differ substantially from one store in a liquidation event to another. As the liquidation progresses, the level of demand uncertainty goes down. Consequently, in identifying optimal liquidation approaches, one needs to explicitly incorporate these phenomena—demand uncertainty and forecast updating.

Third, liquidations involve “quirks”—special features and constraints that characterize each event and often individual stores within the same liquidation event. For example, there might be greater flexibility on when some stores in a chain can be shut down because of the lease agreement

³For more examples, see the client list posted by Hilco Merchant Resources at <http://www.hilcomerchantresources.com/ClientList.asp>.

with the store’s landlord. Similarly, as we illustrate in examples later in the paper, there might be limitations on changing markdown levels, inventory transfers, or store closings by deal. Any method to optimize store liquidation should be flexible enough to accommodate these unique features associated with each liquidation.

In this paper, we introduce a method for improving the efficiency of store liquidations, i.e., for increasing the net orderly liquidation value (NOLV) of retail stores, with a focus on liquidations conducted by asset disposition firms. The method comprises a dynamic program that informs markdown, inventory, and store closing decisions as well as a demand forecasting model. We provide techniques for estimating the parameters in our model and a heuristic approach to solve the dynamic program. We compare the performance of our method to practice in selected case studies and show that the net recovery on cost improved by 2 to 7 percentage points. Through these applications, we provide novel insights gleaned from the use of our technique. GBG served as the test site and our collaborator for the research in this paper; we partnered with GBG on liquidating over \$3B of inventory.

This paper is organized as follows. The next section provides background on the process of store liquidation. §3 discusses how our work relates to prior literature. §4 presents the full dynamic program. §5 introduces our solution methodology, including the modified program and the forecasting model. In §6, we discuss the performance of our methods in practice as well as insights garnered while applying the methods. Our concluding remarks are in §7.

2 The Process of Store Liquidation

From the retail asset disposition firm’s perspective, the first step of any liquidation is “getting the deal.” In the case of a bankruptcy liquidation, the liquidator receives information on store characteristics, inventory, and historical performance from the bankrupt retailer. Typical data include store location and square footage, store-level or category-level inventory in terms of cost and retail value, count, and age, as well as current- and last-year store revenues. The liquidator must then file a bid with the bankruptcy court for the right to liquidate the bankrupt firm’s inventory within the retailer’s extant retail outlets. The bidding process transpires quickly and is often limited

to less than a week. In the case of a going concern liquidation, the asset disposition firm receives similar information and must engage in a sales process—i.e., earning the right to liquidate from the retailer, often through the estimation of net liquidation proceeds and the negotiation of fees.

During a store liquidation, inventory is sold at an increasing discount in a set of retail stores over a finite time period. The length of a liquidation is limited by law for both bankrupt firms and going concerns. The majority of U.S. states constrain the length of all liquidation, distressed inventory, and going-out-of-business sales to protect consumers from firms that might perpetually use liquidation as a marketing tool. See, for example, Ohio Administrative Code Chapter 109:4-3-17, which constrains liquidations to 90 days, and Massachusetts General Laws Part 1, Chapter 93, §28A, which limits going-out-of-business sales to 60 days. Many other jurisdictions impose similar restrictions.

Liquidators may execute a sale for a fixed fee or on an equity basis. In the latter case, the asset disposition firm pays up front for the right to liquidate the inventory. In the equity case, the liquidator may share some portion of the proceeds with the retailer or the retailer's estate. For instance, in advance of the final liquidation of Borders Group during 2011, liquidators agreed to pay Borders' estate 72% of the audited cost value of inventory present at the outset of liquidation plus 50% of the net proceeds of the liquidation (Checkler, 2011). Given the speed of store liquidation, even a small improvement in net recovery on cost can translate into a substantial increase in annualized return on investment in the case of an equity liquidation. For instance, suppose an asset disposition firm improves net recovery from 3%, which is fairly typical, to 5%. If the firm acquires \$100M of inventory at cost and liquidates the inventory over the course of 12 weeks, then the annualized return on investment improves from 13% to 22%.

After securing a deal, the asset disposition firm quickly begins to execute the sale, often within a few days. Once the liquidation commences, each store is assigned a supervisor. At the store level, the supervisors work to increase the profitability of the liquidation through inventory placement (to provide a pleasant shopping environment throughout a sale, asset disposition firms collapse a store by moving inventory toward the front of the store and cordoning off the back of the store) and expense management, including inventory shrinkage and payroll. These supervisors are often

recruited from firms subject to a prior going-out-of-business sale and thus tend to have liquidation experience.

Prior exposure to liquidation is important to managers because stores in liquidation behave very differently than stores in normal operation. One way to illustrate this is to compare revenues during liquidation to revenues during normal operation. To do this, we construct liquidation multipliers, i.e., the ratio of revenue earned during the liquidation of a store to the revenue generated by that store over the same period during the prior year. Figure 1 presents box plots of store liquidation multipliers from four liquidations in the apparel, book, household furniture, and jewelry segments. As this plot shows, most stores see a significant increase in revenue due to liquidation. The median store in both the book and household furniture liquidations more than doubled its revenue. Moreover, this revenue perspective understates the physical volume of product sold, since liquidation discounts exceed normal operating discounts.

In current practice, asset disposition firms use central managers to plan a common markdown cadence across all stores. This markdown cadence is chosen based on the managers' prior experience with a given retail segment. Broadly, the markdowns start relatively low—usually around 25%—and increase to approximately 85% over the course of the sale. Figure 2 plots the realized markdown levels over time across six retail chains in different segments. Each store continues to operate until its inventory is sold through or until it turns unprofitable, i.e., when revenues exceed operating costs. Remaining inventory is either sold at a large discount to a jobber or, rarely, is transferred to a nearby store in the same chain. As will be discussed in §6, our method represents a substantial departure from this practice.

Managers operating liquidations typically track inventory at the cost level rather than at the item level. This is due to the coarseness of the information an asset disposition firm receives from a retailer as well as to the need to execute a liquidation quickly. The optimization and forecasting models proposed herein are formulated accordingly. Determining when it is beneficial to incur the costs of managing inventory at the item level is part of our ongoing research.

Further complicating the operation of a store liquidation is the fact that each sale poses unique constraints. As mentioned previously, Massachusetts laws limit liquidations to 60 days whereas

Ohio laws allow longer, 90-day liquidations. Moreover, mall landlords may limit the duration of a liquidation to a period shorter than that mandated by law in order to avoid giving the impression that their property is faltering. Another possible challenge is that a given store may possess inferior loading and storage facilities that hamper the ability to transport large quantities of inventory in or out of the store. Thus, methods for informing asset disposition firms' decisions during a liquidation must be flexible in order to accommodate such restrictions.

We have identified three key operational levers for central managers to improve efficiency during store liquidations: markdowns, inventory transfers, and store closings. Markdowns allow managers to affect the volume and timing of demand as well as the revenue realized from the sale. For instance, at the outset of a sale, liquidators tend to use small discounts to maximize the revenue from high-demand goods. Inventory transfers allow managers to move inventory to stores that are more attractive from a demand or cost perspective but have costs associated with movement. Finally, store closings are used in conjunction with inventory transfers and markdowns to reduce costs by shuttering certain locations in advance of the overall sale time limit.

We introduce a dynamic program that selects the store-level markdowns, transfers, and store closings that maximize the profitability of a store liquidation subject to random demands, where profitability is defined as revenues less operating expenses such as payroll and shipping costs. This model departs from prior works on markdown optimization by explicitly incorporating store operating expenses and recognizing the inventory imbalances across different stores. Thus, the model may use inventory transfers, markdowns, and store closings to mitigate operating costs.⁴ We now provide a brief overview of related research before turning to our formulation of the store liquidation problem.

3 Related Literature

This work builds on the literature on retailing perishable and seasonal goods, which seeks to optimize pricing, inventory, and other key decisions for merchandise in some stage of liquidation.

⁴We note that this store liquidation problem could also be used in other retail contexts. For example, if a retailer could estimate the cost of the shelf space used to liquidate seasonal merchandise, the retailer could apply this model to end-of-season sales, since shelving costs are analogous to store operating costs.

Often, markdown optimization is the focus of these models. For example, Gallego and Van Ryzin (1994) examine the optimal control of prices during the liquidation of a fixed stock of product subject to price-dependent demand. See Bitran and Caldentey (2003) for an excellent review of the markdown optimization literature. Other works focus on inventory: Fisher and Raman (1996) study pre-season and replenishment inventory decisions when selling a seasonal good while Caro and Gallien (2007) examine assortment decisions at a fast-fashion retailer.

Seasonal and perishable goods research that focuses on implementation is particularly germane to this paper. Smith and Achabal (1998) introduce pricing and inventory policies for seasonal goods when demand is a function of price, inventory level, and season. The authors discuss the application of these policies at multiple retail chains. Caro and Gallien (2010) detail work with the fast-fashion retailer Zara on a new process for distributing a fixed amount of inventory across the retailer's stores during a finite selling season. This process includes both a demand forecasting model and an inventory optimization model. As noted by Bitran et al. (1998), most research that incorporates markdowns focuses on strategies rather than formal tools. Exceptions include Bitran et al. (1998) and research by Caro and Gallien (forthcoming), which introduces a markdown optimization model that has been implemented at Zara.

The paper most related to ours is Bitran et al. (1998), which presents a model to maximize the revenue from the liquidation of a single product across a retail chain. A central planner sets a constant price across all stores, and inventory may be transferred among stores. The authors examine the efficacy of their model using a simulation based on data from a Chilean retail chain. This work departs from ours in several respects, including the absence of store operating costs, the requirement that prices are constant across stores, and the assumption of zero lead times for inventory transfers.

In much of the literature on perishable and seasonal goods retailing, demand is treated as exogenous to inventory. As the literature on strategic customers demonstrates, this assumption can be onerous, particularly in the case of retail chain liquidation, where inventory levels drop precipitously and markdowns change constantly. Gallego et al. (2008) and Cachon and Swinney (2009) demonstrate the effects of customers that strategically delay purchases to take advantage of

future markdowns. Dana and Petruzzi (2001) and Su and Zhang (2009) treat consumers that are cognizant of the probability that a good will be in stock if they visit a retailer. Balakrishnan et al. (2004) and Balakrishnan et al. (2008) model customers that use a retailer’s stocking quantity as a signal of the popularity of a good, with a high stocking quantity signaling high popularity.

We argue that each of these consumer strategies affects demand during a retail chain liquidation. At the outset of a sale, consumers may defer purchase in anticipation of a better deal, a decision that may be more likely when stocking quantities are larger. Toward the end of a liquidation, potential customers may elect not to visit a store, assuming that it is “picked over.” Customers that do visit a store might experience broken assortments (Smith and Achabal, 1998) as well as store rearrangement and, thus, may be unable to locate desired goods. Moreover, stocking quantity may act as a billboard that generates demand for customers treating the liquidation as a rummage sale.

In summary, we extend prior works on retailing perishable and seasonal goods by introducing a model that captures several essential features of store liquidation.

4 Liquidation Model

This section introduces a dynamic program for optimizing the profitability of a store liquidation operated by an asset disposition firm, where profitability is defined as revenues less specific operating costs discussed below. The control variables are three key operational decisions made during a store liquidation: markdowns, inventory transfers, and store closings. This program proves intractable for larger problem instances; a modified program is presented in §5.

Let S be the set of stores to be liquidated. Define T to be the ordered set of days over which the stores are liquidated. Let M be the set of potential markdowns across all stores (e.g., inventory may be sold at a 10%, 20%, ..., 90% discount). Store $s \in S$ begins day $t \in T$ with inventory z_{st} , which, in keeping with asset disposition firm practice, is measured at the store level in retail dollars. Store s sells its inventory at markdown $m_{st} \in M$ on day t . Let $\mathbf{z}_t = (z_{1t}, \dots, z_{|S|t})'$, and let $\mathbf{m}_t = (m_{1t}, \dots, m_{|S|t})'$.

To represent inventory transfers, let x_{rst} be the retail dollar value of inventory that leaves store r

on day t for delivery to store s , and let $\mathbf{x}_{st} = (x_{s1t}, \dots, x_{s|S|t})'$. We assume that inventory transfers depart and arrive prior to demand. Let $\tau_{rs}(x)$ be the cost of transferring inventory of value x from store r to store s . Let l_{rs} be the transfer time in days for inventory shipped from store r to store s . We note that this time includes not only transportation time but also time that inventory sits in storage at the destination stores. In practice, we have observed that transportation time alone is often a small portion of the overall transfer time. Further, let $\mathbf{X}_t = (\mathbf{x}_{1t}, \dots, \mathbf{x}_{|S|t})$, i.e., the matrix of transfers leaving on day t , where $\mathbf{X}_t \in \Lambda_t$, the set of feasible transfers at time t . Transfers may be restricted by minimum and maximum size, by carrier availability, and by store constraints (e.g., limited loading facilities). Let $\mathcal{X}_t = (\mathbf{X}_1, \dots, \mathbf{X}_{t-1})$, the transfer history on day t .

Define

$$\zeta_{st}(z_{st}, \mathbf{x}_{st}, \mathcal{X}_t) = z_{st} - \sum_{r \in S} x_{srt} + \sum_{r \in S} x_{r,s,t-l_{rs}},$$

the retail value of inventory available for sale at store s on day t , which is equal to the pre-transfer inventory adjusted for all relevant transfers. Let $\boldsymbol{\zeta}_t(\mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t) = [\zeta_{1t}(\cdot), \zeta_{2t}(\cdot), \dots, \zeta_{|S|t}(\cdot)]'$.

Define b_{st} to be a binary variable that takes the value 1 if store $s \in S$ is open on day $t \in T$ and 0 otherwise. We assume that store s incurs operating expenses $o_{st}(b_{st}, z_{st}, \mathbf{x}_{st}, \mathcal{X}_t)$ on day t . Example operating costs include occupancy and payroll expenses, and we assume that operating costs depend on inventory and transfers since stores with more inventory and transfers may require more employees. Let $\mathbf{b}_t = (b_{1t}, \dots, b_{|S|t})'$.

Let Γ_{st} be the set of possible demands at store s on day t , where demands are measured at the retail value of merchandise. Each store experiences a random demand on each day with distribution $F_{st}(y | m_{st}, z_{st}, \mathbf{x}_{st}, \mathcal{X}_t)$ for $y \in \Gamma_{st}$. We note that this formulation accommodates warehouses, which are treated as stores with no demand. We assume a relationship between demand and both price and inventory to capture several aforementioned dynamics, including the promotional role of inventory, the impact of broken assortments and store rearrangement, as well as other strategic consumer behaviors. For brevity, let Ψ_t be the Cartesian product of Γ_{st} over all $s \in S$, and let $\mathbf{y}_t = (y_{1t}, \dots, y_{st})'$. Define $G_t(\mathbf{y}_t | \mathbf{m}_t, \mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t)$ as the joint distribution of demand across all stores for $\mathbf{y}_t \in \Psi_t$.

Using the above definitions, the current period cost across all stores on day t is

$$C_t(\mathbf{b}_t, \mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t) = \sum_{s \in S} \left[\sum_{r \in R} \tau_{sr}(x_{srt}) + o_{st}(b_{st}, z_{st}, \mathbf{x}_{st}, \mathcal{X}_t) \right].$$

The revenue across all stores can be written in terms of the store status indicator, the markdown, the demand, and the inventory available for sale as

$$R_t(\mathbf{y}_t, \mathbf{b}_t, \mathbf{m}_t, \mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t) = \sum_{s \in S} b_{st} (1 - m_{st}) \min [y_{st}, \zeta_{st}(z_{st}, \mathbf{x}_{st}, \mathcal{X}_t)].$$

Then the optimal profit on day $|T|$, the final day of liquidation, is expected revenue less cost:

$$L_{|T|}(\mathbf{z}_{|T|}, \mathcal{X}_{|T|}) = \max_{\mathbf{b}_{|T|}, \mathbf{m}_{|T|}, \mathbf{X}_{|T|}} \left\{ \int_{\Psi_{|T|}} R_{|T|}(\mathbf{y}_{|T|}, \mathbf{b}_{|T|}, \mathbf{m}_{|T|}, \mathbf{z}_{|T|}, \mathbf{X}_{|T|}, \mathcal{X}_{|T|}) dG_{|T|}(\mathbf{y}_{|T|} | \mathbf{m}_{|T|}, \mathbf{z}_{|T|}, \mathbf{X}_{|T|}, \mathcal{X}_{|T|}) - C_{|T|}(\mathbf{b}_{|T|}, \mathbf{z}_{|T|}, \mathbf{X}_{|T|}, \mathcal{X}_{|T|}) \right\},$$

over $\mathbf{b}_{|T|} \in \mathbb{B}^{|S|}$, $\mathbf{m}_{|T|} \in M^{|S|}$, and $\mathbf{X}_{|T|} \in \Lambda_{|T|}$.

Given the profit function for day $|T|$, we now employ backward induction to construct the profit function for periods $\{t : t \in T, t < |T|\}$:

$$L_t(\mathbf{z}_t, \mathcal{X}_t) = \max_{\mathbf{b}_t, \mathbf{m}_t, \mathbf{X}_t} \left\{ \int_{\Psi_t} [R_t(\mathbf{y}_t, \mathbf{b}_t, \mathbf{m}_t, \mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t) + L_{t+1}((\zeta_t(\mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t) - \mathbf{y}_t) \vee \mathbf{0}_{|S|}, \mathcal{X}_{t+1})] dG_t(\mathbf{y}_t | \mathbf{m}_t, \mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t) - C_t(\mathbf{b}_t, \mathbf{z}_t, \mathbf{X}_t, \mathcal{X}_t) \right\},$$

where \vee is the join operator, i.e., for two vectors of size n , $a \vee b = [\max(a_1, b_1), \dots, \max(a_n, b_n)]'$, and where $\mathbf{0}_{|S|}$ is a vector of zeros with size $|S|$. The maximization is solved over $\mathbf{b}_t \in \mathbb{B}^{|S|}$, $\mathbf{m}_t \in M^{|S|}$, and $\mathbf{X}_t \in \Lambda_t$.

It is straightforward to add constraints to this general formulation in order to fit the requirements of a given liquidation. For instance, if markdowns must be non-decreasing, then the program would be modified to include the constraint set $\mathbf{m}_t \geq \mathbf{m}_{t-1} \forall \{t : t \in T, t > 1\}$. Or, if closed stores must remain closed, we would add the constraint set $\mathbf{b}_t \leq \mathbf{b}_{t-1} \forall \{t : t \in T, t > 1\}$.

The state space, which, in the general case, reflects the potential current and in-transit inventories of each store, grows rapidly with problem size. Thus, the problem exhibits the curse of dimensionality and becomes intractable for reasonable instances. To address this issue, §5 introduces a heuristic that has been implemented for large-scale store liquidations. We now turn to this solution methodology.

5 Solution Methodology

In this section, we present a solution methodology for use by an asset disposition firm during a store liquidation. This method, which has been applied by practitioners at GBG, comprises a relaxation of the store liquidation dynamic program as well as a demand forecasting model. We begin with the modified optimization model.

The modified model relaxes the dynamic program by solving it in the absence of recourse, i.e., by committing to all decisions at the outset of the planning horizon. Because of the structure of the store liquidation problem, a mixed-integer program that makes pricing, transfer, and closing decisions concurrently is a nonlinear mixed-integer program that does not reduce to a readily solved form (see, for example, Grossmann (2002) and Bonami et al. (2008)) and that does not easily incorporate the idiosyncratic constraints imposed by instances of the store liquidation problem. We thus decompose the problem into a subproblem and a master problem. The subproblem is a mixed-integer program with bilinear constraints that optimizes profit over inventory transfers and store closings subject to a fixed markdown schedule and specific assumptions. The master problem searches for the markdown schedule that maximizes the objective value of the subproblem. This relaxation is in the spirit of the fourth heuristic considered by Bitran and Caldentey (2003), which also separates pricing and inventory decisions. We start with the subproblem.

5.1 Relaxation Subproblem: Inventory Transfers and Store Closings

The subproblem requires a number of simplifying assumptions for tractability. The most significant are highlighted in this paragraph; others are discussed as they are introduced. First, the subproblem assumes that there is a single opportunity to transfer inventory. This assumption is largely in keeping with practice, since managers are often reluctant to attempt too many transfers due to the complexity of doing so, an observation mirrored by Bitran et al. (1998). Second, it assumes that the transfer time between all stores is a constant.⁵ Finally, it assumes the relationship between demand and inventory can be modeled using a specific functional form that is discussed below. For exposition, the formulation herein models the case when transfers begin on the first day, representing the situation in which the model is solved immediately prior to making transfer decisions. However, the model is readily modified to treat transfers on an arbitrary day.

As in §4, S is the set of stores, T is the set of liquidation days, and M is the set of possible markdowns. We modify the operating and transfer cost definitions as follows. Let o_{st} be the cost of operating store s on day t , which is assessed when the store is open and does not depend on the store's inventory. Let τ_{rs} be the cost of moving one retail dollar of inventory from store $r \in S$ to store s . Further, suppose l is the transportation lead time between all stores. We note that the markdown in store s on day t retains the notation m_{st} but is not a decision variable in the subproblem; let \mathbf{m} be a matrix containing $m_{st} \forall s \in S, \forall t \in T$.

Demand is modeled in the following fashion in the subproblem. We decompose demand into a set of discrete demand scenarios, where each scenario specifies a base level of demand for each store on each day. As an input to the subproblem, this base level of demand incorporates the effect of markdowns and beginning inventory levels but does not account for the effect of inventory transfers. We discuss the construction of these demand scenarios from historical data in §5.3. Let \mathbf{d} be a demand scenario that specifies a demand d_{st} for each store on each day; i.e., \mathbf{d} is a matrix of size $|S| \times |T|$. Let D be the set of possible demand scenarios, and let $\phi(d)$ be a probability measure on D . To model the relationship between demand and inventory, we introduce a parameter, g_s , which is defined as the percentage increase or decrease in post-transfer demand (in terms of dollars

⁵In our experience, this assumption is not overly unrealistic, since transportation itself requires only a day or two while the inventory often sits in storage at the destination store for at least a week.

of retail inventory) associated with a \$1 inbound or outbound inventory transfer. This relationship mirrors empirical observations while maintaining tractability; we discuss g_s further in §5.3.

There are two sets of decision variables in the subproblem. As before, b_{st} , $s \in S$ and $t \in T$, is a binary variable that takes the value 1 if store s is open on day t and 0 otherwise. Let \mathbf{b} be a matrix of size $|S| \times |T|$ with elements b_{st} . The second set of decision variables, x_{rs} , represents the dollars of retail value inventory transferred from store r to store s at the outset of the first time period. Let \mathbf{x} be a matrix containing $x_{rs} \forall r \in S$ and $\forall s \in S$.

Let z_s be the pre-transfer beginning inventory at store s on the first day of T . Let ζ_{dst} be the post-transfer beginning inventory in store s on day t under demand scenario d . Let ξ_{dst} be lost sales, i.e., the amount by which demand exceeds supply in store s on day t under demand scenario d . Let $\gamma_{st} \geq 0$ be the scaling of demand at store s at time t as a result of inventory transfers.

The subproblem is $\Theta(\mathbf{m}, D, \phi(\cdot)) =$

$$\begin{aligned}
& \text{Maximize}_{\mathbf{b}, \mathbf{x}, \xi} && \sum_{d \in D} \phi(d) \sum_{s \in S} \sum_{t \in T} [(1 - m_{st})(\gamma_{st} d_{st} - \xi_{dst})] \\
& && - \sum_{s \in S} \sum_{t \in T} c_{st} b_{st} - \sum_{r \in S} \sum_{s \in S} \tau_{rs} x_{rs} \\
\text{Subject to:} &&& \gamma_{st} = 1 && \forall s \in S, \forall t < l \\
&&& \gamma_{st} = 1 + g_s \left[\sum_{r \in S} x_{rs} - \sum_{r \in S} x_{sr} \right] && \forall s \in S, \forall t \geq l \\
&&& \sum_{r \in S} x_{sr} \leq z_s && \forall s \in S \\
&&& \zeta_{ds1} = z_s - \sum_{r \in S} x_{sr} && \forall d \in D, \forall s \in S \\
&&& \zeta_{dst} = \zeta_{ds, l-1} - \gamma_{st} d_{s, l-1} + \xi_{ds, l-1} + \sum_{r \in S} x_{rs} && \forall d \in D, \forall s \in S \\
&&& \zeta_{dst} = \zeta_{ds, t-1} - \gamma_{st} d_{s, t-1} + \xi_{ds, t-1} && \forall d \in D, \forall s \in S, \forall t \notin \{1, l\} \\
&&& \xi_{dst} \geq b_{st} (\gamma_{st} d_{st} - \zeta_{dst}) + (1 - b_{st}) \gamma_{st} d_{st} && \forall d \in D, \forall s \in S, \forall t \in T \\
&&& b_{st} \leq b_{s, t-1} && \forall s \in S, \forall t > 1 \\
&&& b_{st} \in \mathbb{B} && \forall s \in S, \forall t \in T \\
&&& x_{rs} \geq 0 && \forall r \in S, \forall s \in S \\
&&& \xi_{dst} \geq 0 && \forall d \in D, \forall s \in S, \forall t \in T.
\end{aligned}$$

The objective is expected revenue less store operating costs and transfer costs. We note here that incorporating markdown as a decision variable would drastically increase the nonlinearities in this problem, not just in the objective function but also within the constraints, since demand would become a function of markdown. The first two constraints specify the effect of transfers on demand. The third constraint ensures that a store does not transfer more than its beginning inventory.

The fourth constraint calculates first-period inventory as a result of transfers. The fifth constraint calculates inventory in period l as a result of transfers. In the special case $l = 1$ (i.e., no lead time), the fourth and fifth constraints collapse to $\zeta_{ds1} = z_s - \sum_{r \in S} x_{sr} + \sum_{r \in S} x_{rs} \forall d \in D, \forall s \in S$.

The sixth constraint links inventory between periods. The seventh constraint calculates unsatisfied demand. The final constraint dictates that closed stores remain closed (while this constraint is not mandated by the general formulation introduced in §4, it is a common feature of store liquidations, so we incorporate it into our statement of the heuristic). Thus, the subproblem calculates the optimal inventory transfers and store closings for fixed per-store markdown schedules subject to the aforementioned assumptions. This formulation easily accommodates instance-specific constraints (e.g., store 1 is not allowed to close before day 30 due to an agreement with the store's landlord) may be added to the subproblem.

5.2 Relaxation Master Problem: Markdowns

To treat the selection of a markdown schedule, we construct a master problem that models the relationship between price and demand. To do so, the master problem creates the demand scenarios used by the subproblem, where the scenarios are a function of markdowns and beginning inventory. The construction of these demand scenarios is detailed in §5.3. The master problem then maximizes the objective value of the subproblem over the set of possible markdowns.

To reflect the fact that the master problem creates the subproblem's demand scenarios, we introduce the notation $D(\mathbf{m})$, which is the set of demand scenarios subject to a given markdown

schedule, \mathbf{m} . The probability measure on this demand set is $\phi(\cdot | \mathbf{m})$. The master problem is

$$\begin{aligned} & \text{Maximize}_{\mathbf{m}} \quad \Theta(\mathbf{m}, D(\mathbf{m}), \phi(\cdot | \mathbf{m})) \\ & \text{Subject to:} \quad \mathbf{m} \in M. \end{aligned}$$

We note that restrictions such as non-increasing markdowns are codified in M and that consumer reactions to markdowns are modeled in $D(\mathbf{m})$ and $\phi(\cdot | \mathbf{m})$. For our research, this model has been implemented using MATLAB for the master problem and IBM ILOG CPLEX for the subproblem. In our implementation, we solve the master problem via parallelized enumeration augmented by a straightforward branch and bound rule: when evaluating a particular markdown cadence, if the global maximum profit is greater than the realized profit on day t plus the full retail value of all remaining inventory, then the markdown cadence in question and all markdown cadences that are identical through day t must not be optimal.

5.3 Demand Forecasting

The majority of the parameters for both the dynamic program and the modified problem are usually provided to asset disposition firms or are readily obtained, such as store operating costs, beginning inventories, and transportation costs. Nonetheless, the distribution of demand may be difficult to determine, particularly because stores in liquidation often behave radically differently than stores in normal operation. In this section, we present a statistical model for forecasting demand during a store liquidation. This forecasting model has been fit on historical data provided by GBG comprising 43 retail chain liquidations since 2006, involving 2,516 stores and approximately \$7.5B of inventory at retail value.

The literature offers formal tools for forecasting demand in the case of product liquidation, including those proposed by Smith and Achabal (1998) and Bitran et al. (1998). Our forecasting model differs from prior research for two key reasons. First, as discussed in §2, inventory is managed by retail value rather than by count during a store liquidation. Second, as illustrated in Figure 1, demand during a store liquidation can depart significantly from demand during normal operations. Therefore, our method incorporates information not only about a store’s prior performance but

also about how consumers have responded to prior liquidations.

During a store liquidation, managers track sales performance in terms of multipliers, which reflect the revenue lift due to the store liquidation. Let Revenue_{st} be the liquidation revenue of store s on day t . Let Last-Year Revenue_{st} be the revenue of store s during normal operations one year prior to day t . Then the multiplier on day t at store s is

$$\text{Multiplier}_{st} = \frac{\text{Revenue}_{st}}{\text{Last-Year Revenue}_{st}}.$$

It is useful to define the multiplier over specific time periods. For instance, the multiplier at store s for an entire liquidation is

$$\text{Liquidation Multiplier}_s = \frac{\sum_{t \in T} \text{Revenue}_{st}}{\sum_{t \in T} \text{Last-Year Revenue}_{st}},$$

while the multiplier for the first week of a sale is

$$\text{First-Week Multiplier}_s = \frac{\sum_{t=1}^7 \text{Revenue}_{st}}{\sum_{t=1}^7 \text{Last-Year Revenue}_{st}}.$$

Further, the liquidation multiplier can be constructed at the chain level:

$$\text{Chain Liquidation Multiplier} = \frac{\sum_{s \in S} \sum_{t \in T} \text{Revenue}_{st}}{\sum_{s \in S} \sum_{t \in T} \text{Last-Year Revenue}_{st}}.$$

To illustrate the challenge of forecasting demand during a store liquidation, consider the variation in store performance exhibited in Figure 1. During the liquidation of the 59 jewelry stores depicted, certain stores performed roughly the same as they did in the prior year while many other stores more than tripled their prior year's performance. The variance in multipliers demonstrates that stores in liquidation may behave differently than their prior performance or their peers' results would indicate.

We construct the demand forecasting model, which forecasts multipliers, as a function of store characteristics as well as current economic conditions. The store-level variables are beginning inventory, markdowns, store square footage, last-year sales, liquidation duration, and retail segment

(e.g., apparel store or bookstore). When the model is fit during a liquidation, we add realized store liquidation revenues.

In order to account for local economic conditions, the model also employs data on the demographics of the store's neighborhood, which is determined by the store's ZIP code. The United States Census Bureau provides the demographic data at the ZIP code level. Data from the 2000 census are median household income, number of households, average household size, and number of houses for sale. Data from the 2007 economic census are the number of local business establishments as well as the average payroll of these businesses.

To capture the state of the broader economy, the model uses the Thomson Reuters / University of Michigan consumer sentiment index. This index is used in favor of other indicators of economic activity because of its frequent (monthly) release. Other measures, such as the economic activity data released by the U.S. Bureau of Economic Analysis, yield similar insights about the relationship between liquidation demand and broader economic conditions but are released too infrequently for forecasting in practice. When estimating the forecasting model, we assign to a store the value of the consumer sentiment index that was released in the month prior to the month in which the store's liquidation commenced; this ensures data availability when forecasting upcoming store liquidations.

We propose a random effects model where the clusters are individual retail liquidation events. For instance, when Borders Group filed bankruptcy, it closed a set of stores beginning on February 19, 2011. The outcome of Borders' bankruptcy process required the liquidation of the remaining Borders stores; this latter sale began on July 22, 2011. Our forecasting model treats these two liquidations as separate clusters even though they are part of the same retail chain. The average number of stores per cluster is 58.51 with a minimum of 15 and a maximum of 371.

The model may be fit to an arbitrary time period, e.g., the first day of liquidation, the second week of liquidation, or the overall liquidation. The variables that depend on time period—Multiplier, Beginning Inventory, Average Markdown, Remaining Days Open, and Last-Year Sales—

are then measured over the specified time period. The model for store s in liquidation event v is:

$$\begin{aligned}
\ln(\text{Multiplier}_{vs}) = & + \beta_1 \ln(\text{Beginning Inventory}_{vs}) + \beta_2 \ln(\text{Average Markdown}_{vs}) \\
& + \beta_3 \ln(\text{Store Square Footage}_{vs}) + \beta_4 \ln(\text{Remaining Days Open}_{vs}) \\
& + \beta_5 \ln(\text{Last-Year Sales}_{vs}) + \boldsymbol{\delta}'(\text{Local Economic Variables}_{vs}) \quad (1) \\
& + \kappa \ln(\text{Consumer Sentiment}_v) + \text{Event Random Effect}_v \\
& + \text{Segment Fixed Effect}_v + \epsilon_{vs}.
\end{aligned}$$

If the model is fit during a liquidation, when sales over some set of days are available, we add a term to reflect obtained sales information, $\beta_6 \ln(\text{Current Sales}_{vs})$. This variable is the sum of revenue over the specified set of days.

This forecasting model is used to generate the demand distributions required by both the full dynamic program and the modified program. In the case of the relaxation subproblem, where we need an approximation of the relationship between demand and inventory, we let β_1 represent the percent change in demand that results from a given percent change in inventory. Therefore, for store s , g_s , the relationship between demand and inventory within the relaxation subproblem, is equal to $\frac{\beta_1}{z_s}$, where z_s is the store's pre-transfer beginning inventory. When creating demand scenarios for the subproblem, the master problem treats the effect of markdowns as deterministic. That is, the demand scenarios are generated by omitting the term $\beta_3 \ln(\text{Average Markdown}_{s\omega})$. The markdown effect dictated by β_3 is then added to the demand scenarios as each markdown cadence is evaluated.

To illustrate the accuracy of the forecasting method, we cross-validate the model by estimating the coefficients on data from 42 of the 43 retail chains and then forecasting liquidation multipliers for the omitted chain. Figure 3 graphs store-level forecast first-week multipliers against actual first-week multipliers. Figure 4 graphs store-level forecast liquidation multipliers against actual liquidation multipliers, where the forecast incorporates data from the first week of the liquidation. In both figures, we omit the largest 2% by actual first-week and liquidation multipliers for visualization purposes. The coefficients of determination for these two cross-validations are 72% and

87%, respectively.

As these forecasting results indicate, sales during the first week of liquidation convey a great deal of information about sales throughout the liquidation, even though first-week liquidation revenues are on average only 12.23% of overall revenues. This pattern has been observed in other retail contexts. For example, Fisher and Raman (1996) note that demand for fashion products early in a selling season is highly predictive of demand throughout the entire selling season.

Although the store-level forecasts benefit greatly from a small amount of realized sales information, the pre-liquidation forecasts are still useful. Figure 5 compares the accuracy of chain-level liquidation multiplier forecasts generated via this model to expert forecasts for 13 retail chain liquidations recorded by GBG (historical forecasts for the other liquidations in our database were not available). Across the sample of 13 liquidations, the forecasting model generates a 37% reduction in absolute deviation from actual multipliers in comparison to expert forecasts. Specifically, the cumulative absolute deviation of the expert forecasts is 2.71 while the cumulative absolute deviation of the statistical forecasts is 1.70.

As this section illustrates, it is possible to generate statistical forecasts of store-level multipliers, which are a proxy for demand, both in advance of and during a store liquidation. These forecasts may be used in conjunction with the heuristic program as a decision aid. While we focus on applying these forecasts to improve operational decisions, we note that our research partner also has employed this forecasting model to improve performance during the pre-liquidation bidding and sales process. In the next section, we discuss the performance of this solution methodology.

6 Applications

In this section, we discuss results and insights gleaned through the application of our methods to recent store liquidations. The examples are chosen to highlight each of the three key operational levers—inventory transfers, markdowns, and store closings—while also illustrating how our method accommodates the quirks that are inherent to each liquidation event. We discuss the monetary benefit of applying our methods to augment current liquidation practice (see §2 for an overview of current practice). We also provide general insights for practitioners operating store liquidations.

6.1 Application: Markdowns

The first example is the liquidation of stores owned by a branded apparel manufacturer that also sold its merchandise through other outlets, primarily department stores. The company’s stores contributed a small portion of its revenues (less than 5%) and were located near department stores that also sold the company’s products. The apparel manufacturer’s managers elected not to allow inventory transfers during liquidation, since transferring large amounts of inventory into their own stores risked harming their relationship with nearby department stores. Further, because of the apparel manufacturer’s leases with the malls in which its stores were located, there was no opportunity to close stores early. Thus, the focus in this case was on markdown optimization.

Using historical information collected during the liquidation of 84 of the company’s stores, we fit our forecasting model (Equation 1) to estimate store-level demand distributions on a daily basis as a function of markdowns, inventory levels, and the other forecasting variables. We then use these distributions to simulate the performance of both the actual managerial decisions and the relaxed model’s decisions, assuming in both cases that markdown schedules are fixed prior to the sale⁶ and that transfers and early closings are prohibited. We thus conservatively find that the benefit of markdown optimization in this context is a 5.6% increase in revenue at the chain level, which equates to a 7 percentage point increase in net recovery on the cost value of the inventory sold. Further, the formal method outperformed the actual decisions during 89% of the 100,000 liquidations simulated, even in the absence of recourse.

Figure 8 graphs the actual managers’ decisions against the relaxed model’s revenue maximizing decisions aggregated to the chain level. This figure illustrates two patterns that we have seen across many of the cases we have studied. First, the model generally recommends lower markdowns than managers use in practice toward the *end of a liquidation*, a finding in keeping with the results obtained by Smith and Achabal (1998). By using lower markdowns, the model tends to assume leftover inventory; in this case, the model leaves slightly over 12% of the chain’s inventory unsold on average. Intriguingly, adding 100% sell-through constraints to the optimization model generates markdowns that are remarkably similar to those chosen by managers at the asset disposition firm,

⁶We fix markdown decisions from the formal tool because the managers’ obtained decisions also do not vary.

as Figure 8 illustrates. Broadly, it appears that liquidators exhibit a “sell-through bias.” Our industry collaborators argue that this is due to the perception of their clients—both merchants and courts—that unsold inventory demonstrates poor performance.

Second, the formal method prescribes larger markdowns at the outset of a liquidation than liquidators use in practice. We speculate that this result is due to the tendency for managers to seek to “capitalize” on the highest quality inventory in a store by selling it at a low discount. In contrast, we recommend using slightly larger discounts early in a sale in order to benefit from the substantial volume of traffic at the outset of a liquidation event.

6.2 Application: Store Closings

Our next example is a going-out-of-business sale conducted for a chain of discount department stores. This application illustrates the benefits of accurate demand forecasting in the presence of store closing constraints. The liquidation was limited to 60 days, and nine of the chain’s roughly forty stores were subject to a lease constraint that rent would have to be paid through the 60th day of the sale if the stores were occupied on the 46th day.⁷ Critically, landlords had to be notified two weeks in advance of a store’s closing. Initially, the operating plan was to run these nine stores through day 60 of the liquidation. Two weeks into the liquidation, the decision support model indicated that a combination of inventory transfers and steeper markdowns at certain stores would allow the nine stores to close in advance of the 46th day of the liquidation. The model provided enough lead time that all landlords could be notified ahead of the deadline.

Operating those nine stores from day 46 to day 60 would have added \$1.5M of operating expenses, which was roughly 160% of the projected revenue from days 46 to 60 at these stores. In fact, the costs were 141% of the actual revenues from days 31 through 45. Since revenues tend to decrease over the course of a store liquidation, it is likely that operating (or just leasing) the stores for the next 15 days would have been highly unprofitable. This example shows the importance of forecasting and reacting to demand in the face of constraints imposed by a retailer’s extant contracts. Further, this application highlights the value of pricing at the store level rather than at

⁷The 46th day of the liquidation was the first day of a month, and the stores’ leases had “in for one, in for fifteen” clauses.

the chain level.

In order to illustrate how the optimal markdowns change as the number of days a store stays open decreases, Figure 8 includes the optimal markdown schedule for a shorter version of the liquidation discussed in §6.1. As the figure shows, the optimal markdowns for the abridged liquidation, which is shortened by 15 days, are higher at both the outset and the end of the sale. Even though the liquidation duration is decreased by 35%, the optimal markdowns are still substantially lower than the sell-through and actual markdowns. The resulting revenue is 12% lower on average.

6.3 Application: Inventory Transfers

The final example application treats the liquidation of a major retail chain involving 400 stores holding around \$600 million of inventory. GBG liquidated this chain as a joint venture with other asset disposition firms. Because of the reporting structure established for the joint venture, GBG did not have direct control over markdowns and closings. Nonetheless, in this case, GBG managers were able to transfer inventory among stores.

Three weeks into the store liquidation, we solved the relaxed program using the latest available information (i.e., including early liquidation sales) to determine inventory transfers. Using our model, we generated a transfer schedule for roughly \$20 million of inventory. The transfers commenced during the fourth week of the liquidation.

To estimate the impact of a transfer on the sales of any given store, we observe how the post-transfer revenues of an affected store perform relative to the average revenue of a comparison group. We construct a set of comparison stores for each affected store by selecting ten stores with similar sales during the pre-transfer period that were not subject to inbound or outbound transfers.⁸ Specifically, we sort all outlets by pre-transfer revenues and then create a transfer store’s comparison group by selecting the five non-transfer stores above and below the transfer store. We note that only the date at which the inventory leaves a donor store is recorded in our data. That is, we do not know the date on which the merchandise arrived or was displayed in the receiving store. Therefore, the post-transfer period includes the transfer time.

⁸The results do not change when stores are matched on inventory.

Figure 6 illustrates one inventory transfer. The focal store received \$396,075 of inventory at retail value. This figure shows both the average difference and the cumulative difference between the revenues of the focal store and the average revenues of its comparison stores. The vertical line indicates the date when inventory left the donor store. When the transfer is executed, the focal store has generated revenues similar to those of the comparison stores. By the end of the sale, the focal store has generated roughly \$80,000 of incremental revenue in comparison to its peer group.

Applying this method to all transfers yields an estimate of the net impact of the transfers. Figure 7 charts the potential effect of transfers on sales as the cumulative difference between the revenue of a store subject to a transfer and the average revenue of its peer group amassed between the execution of the transfer and the end of the sale. The lines of fit, both of whose slopes are significant at the 90% level, indicate that the benefits of the inbound inventory transfers outweighed the costs of removing inventory from the donor stores. Using this straightforward regression analysis and incorporating the operating expenses of the transfers, we estimate that the profit increase was over 2.5% of the retail chain liquidation's overall revenues. Further, net recovery on the cost value of the inventory increased 2 percentage points.

This application demonstrates the importance of reading and reacting to demand at the outset of a store liquidation. By matching supply and demand, managers sell merchandise at lower markdowns and thus increase revenues. This example also illustrates the importance of transfer lag to the dynamic program and the modified problem. Figure 6 shows that it takes a number of days for the transferred inventory to begin affecting the focal store's sales. Since markdowns increase over time, large transfer lags can nullify the benefit of inventory transfers.

In this section, we discussed example applications of our methods for increasing the efficiency of retail chain liquidation. We argue that these applications, which span disparate retail segments, provide evidence of the benefit of these techniques. Further, these applications provide general insights for managers operating store liquidations, demonstrating: (i) the importance of reading and reacting to demand at the outset of a liquidation, (ii) the potential benefits of applying markdown optimization and leaving inventory unsold, (iii) that practitioners should employ larger markdowns at the outset of a sale and smaller markdowns at the end of a sale, and (iv) how a scientific

approach to store liquidation can help managers respond to contractual limitations. We now turn to our closing remarks.

7 Conclusion

Store liquidation is important for firms and investors, affecting everything from retailer performance to how retailers are financed and how investors are compensated (Foley et al., 2012). Further, store liquidation is fundamental to innovation in the retail sector, since extracting value from defunct stores and firms is a key step in the process of creative destruction (Schumpeter, 2008). In this paper, we introduce methods for increasing the efficiency of store liquidations operated by retail asset disposition firms, and we thus extend management science techniques to a consequential problem that has not yet been addressed by the literature. These methods were developed through a collaboration with GBG, a prominent liquidator, during the liquidation of over \$3B of inventory.

Although the literature has addressed markdowns and inventory transfers in the context of liquidating seasonal or fashion goods, the store liquidation problem differs significantly, both mathematically and managerially. First, during a store liquidation, managers must make not only pricing and inventory decisions but also store closing decisions. Second, customers behave differently during a store liquidation than they do during normal operations, which makes demand during a store liquidation difficult to predict. Third, each store liquidation may involve idiosyncratic constraints, which are often due to legal and lease requirements. Finally, when asset disposition firms operate liquidations in practice, they work with inventory at the dollar rather than unit level.

We introduce a dynamic program that generates an operating plan for an asset disposition firm by jointly optimizing markdowns, inventory transfers, and store closings. Since the dynamic program is intractable for reasonable problem instances, we introduce a mixed-integer programming relaxation of the dynamic program that can be solved readily using commercial optimization software. This relaxation is designed to easily incorporate the types of constraints that retail asset disposition firms face, e.g., the requirement that stores in one state close by 60 days while stores in another state close by 90 days. We also introduce a demand forecasting model for supplying demand distributions to the dynamic program and the modified program. The relaxed problem and

the demand forecasting model constitute a practical solution methodology that has been applied during a number of recent store liquidations.

We demonstrate the effectiveness of this solution methodology using three recent applications. In each of these contexts, we show that the method provided a significant improvement over prior practice. Moreover, we discuss general insights for managers in the store liquidation industry. It is our hope that these techniques will be beneficial to practitioners at asset disposition firms that operate retail chain liquidations.

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References

- Alan, Y. and Gaur, V. (2012), ‘Operational investment and capital structure under asset based lending’.
- Balakrishnan, A., Pangburn, M. S. and Stavrulaki, E. (2004), “’stack them high, let ’em fly’”: Lot-sizing policies when inventories stimulate demand’, *Management Science* **50**(5), 630–644.
- Balakrishnan, A., Pangburn, M. S. and Stavrulaki, E. (2008), ‘Integrating the promotional and service roles of retail inventories’, *Manufacturing and Service Operations Management* **10**(2), 218–235.
- Bitran, G. and Caldentey, R. (2003), ‘An overview of pricing models for revenue management’, *Manufacturing and Service Operations Management* **5**(3), 203–229.
- Bitran, G., Caldentey, R. and Mondschein, S. (1998), ‘Coordinating clearance markdown sales of seasonal products in retail chains’, *Operations Research* **46**(5), 609–624.
- Bonami, P., Biegler, L. T., Conn, A. R., Cornuéjols, G., Grossmann, I. E., Laird, C. D., Lee, J., Lodi, A., Margot, F., Sawaya, N. and Wächter, A. (2008), ‘An algorithmic framework for convex mixed integer nonlinear programs’, *Discrete Optimization* **5**(2), 186–204.

- Bustillo, M. (2012), ‘Best Buy Forced to Rethink Big Box’, *Wall Street Journal* p. B1.
- Cachon, G. P. and Swinney, R. (2009), ‘Purchasing, pricing, and quick response in the presence of strategic consumers’, *Management Science* **55**(3), 497–511.
- Caro, F. and Gallien, J. (2007), ‘Dynamic assortment with demand learning for seasonal consumer goods’, *Management Science* **53**(2), 276–292.
- Caro, F. and Gallien, J. (2010), ‘Inventory management of a fast-fashion retail network’, *Operations Research* **58**(2), 257–273.
- Caro, F. and Gallien, J. (forthcoming), ‘Clearance pricing optimization for a fast-fashion retailer’, *Operations Research* .
- Checkler, J. (2011), ‘Judge OKs Borders Liquidation’, *Wall Street Journal* p. B7.
- Dana, J. D. and Petruzzi, N. C. (2001), ‘Note: The newsvendor model with endogenous demand’, *Management Science* **47**(11), 1488–1497.
- Fisher, M. L. and Raman, A. (1996), ‘Reducing the cost of demand uncertainty through accurate response to early sales’, *Operations Research* **44**(1), 87–99.
- Foley, C. F., Raman, A. and Craig, N. C. (2012), *Inventory-Based Lending Industry Note, Harvard Business School Case No. 612-057*, Harvard Business School Publishing, Boston, MA.
- Gallego, G., Phillips, R. and Azahin, O. (2008), ‘Strategic Management of Distressed Inventory’, *Production and Operations Management* **17**(4), 402–415.
- Gallego, G. and Van Ryzin, G. J. (1994), ‘Optimal dynamic pricing of inventories with stochastic demand over finite horizons’, *Management Science* **40**(8), 999–1020.
- Gaur, V., Kesavan, S. and Raman, A. (2013), ‘Retail inventory: Managing the canary in the coal mine’, *Working Paper* .
- Grossmann, I. E. (2002), ‘Review of nonlinear mixed-integer and disjunctive programming techniques’, *Optimization and Engineering* **3**(3), 227–252.
- Lahart, J. (2011), ‘Investors: The Only Thing We Have to Fear Is Sears Itself’, *Wall Street Journal* p. C12.
- LBO Wire (2012), ‘Deal Closure Announcements February 27 - March 2’.
- Mattioli, D. (2011), ‘For Syms, a Final Markdown’, *Wall Street Journal* p. B1.
- Ordonez, J., Bandler, J. and Smith, R. A. (2001), ‘Montgomery ward’s close to hit landlords, rivals’, *Wall Street Journal* p. A3.
- Schumpeter, J. A. (2008), *Capitalism, Socialism and Democracy*, HarperCollins, New York, NY.
- Smith, S. A. and Achabal, D. D. (1998), ‘Clearance pricing and inventory policies for retail chains’, *Management Science* **44**(3), 285–300.

- Su, X. and Zhang, F. (2009), 'On the Value of Commitment and Availability Guarantees When Selling to Strategic Consumers', *Management Science* **55**(5), 713–726.
- Trachtenberg, J. A. (2013), 'B&N to Cut Up to 33% Of Its Retail In a Decade', *Wall Street Journal* p. B1.
- Yang, S. A. and Birge, J. R. (2011), How inventory is (should be) financed: Trade credit in supply chains with demand uncertainty and costs of financial distress.
- Zimmerman, A. (2009), 'Corporate News: Home Depot Shuttters Expo', *Wall Street Journal* p. B3.

Figures

Figure 1: Liquidation Multipliers Across Segments

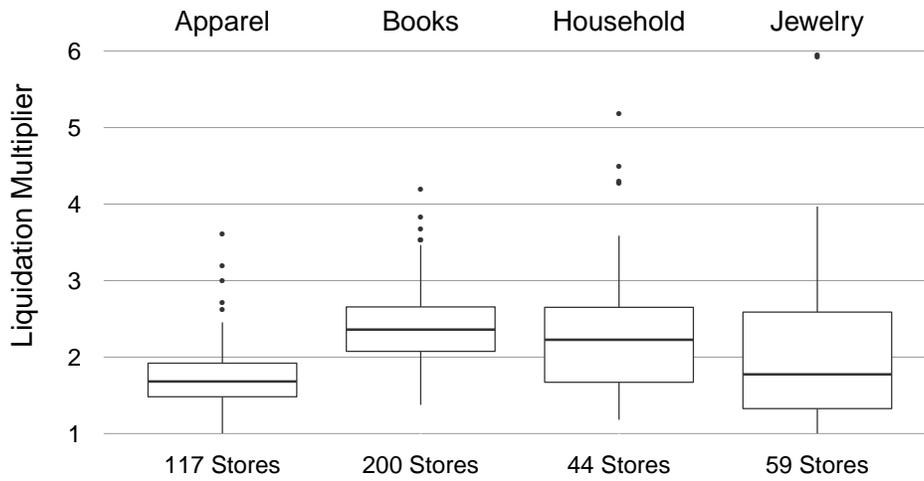


Figure 2: Realized Markdowns for Retailers in Six Segments

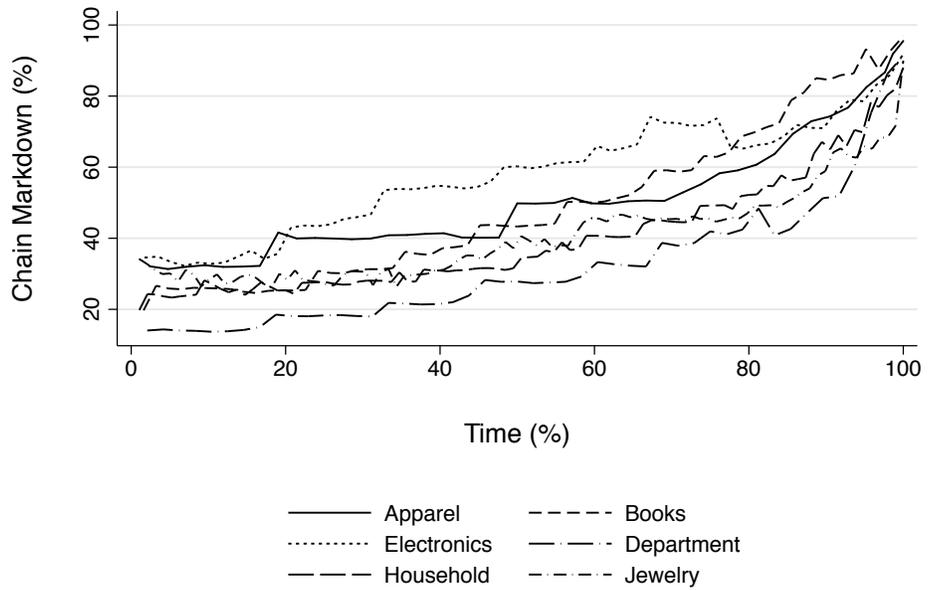


Figure 3: Actual and Forecast First-Week Multipliers Prior to Liquidation

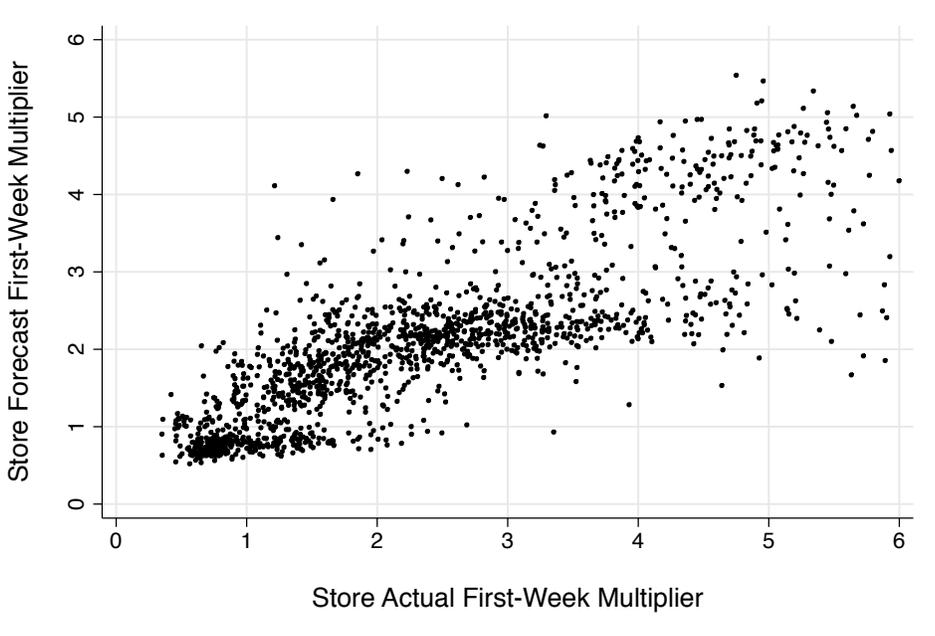


Figure 4: Actual and Forecast Liquidation Multipliers Using First-Week Sales

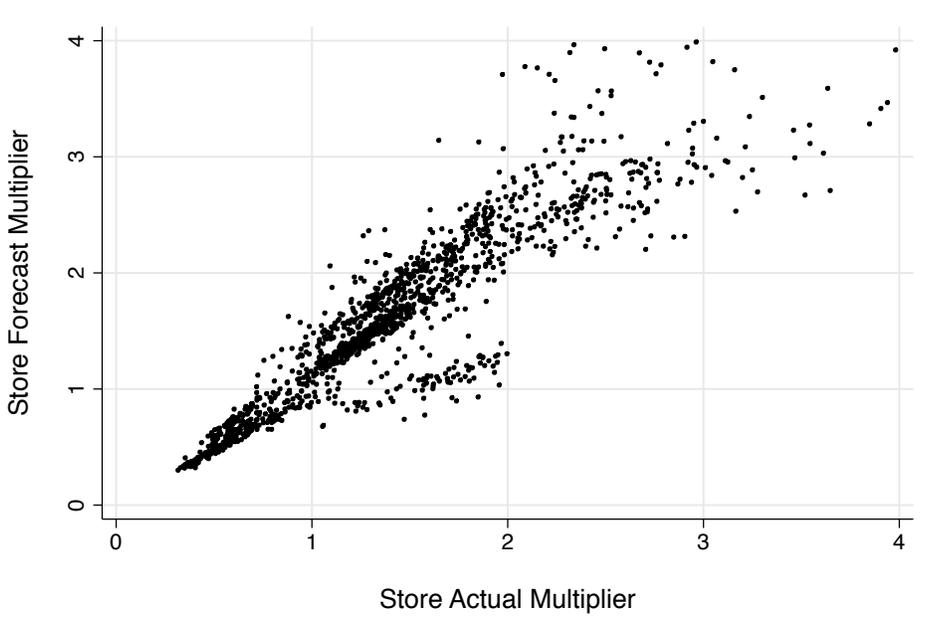


Figure 5: Actual Chain Multipliers Versus Forecast and Planned Chain Multipliers

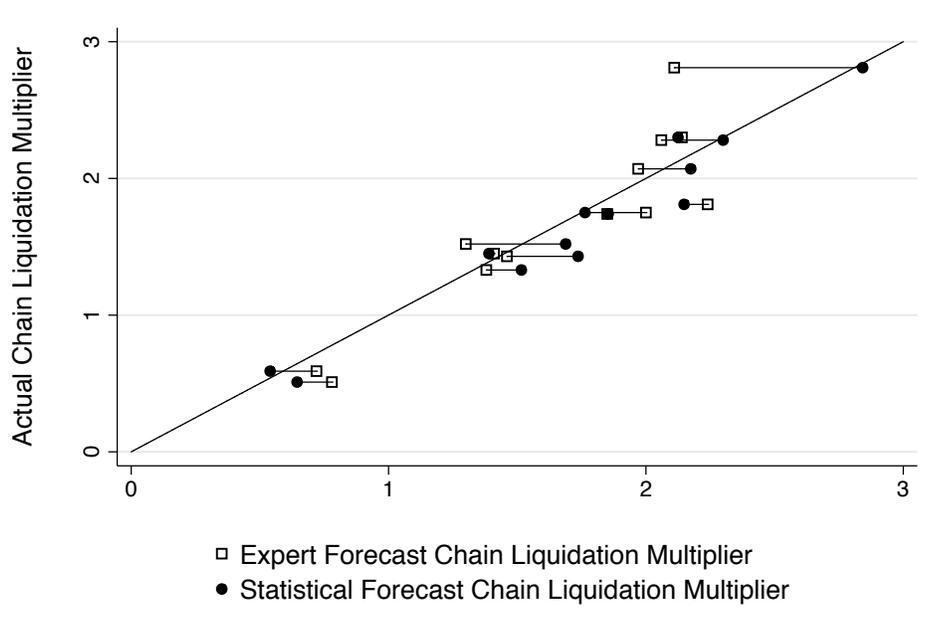


Figure 6: Example Inventory Transfer

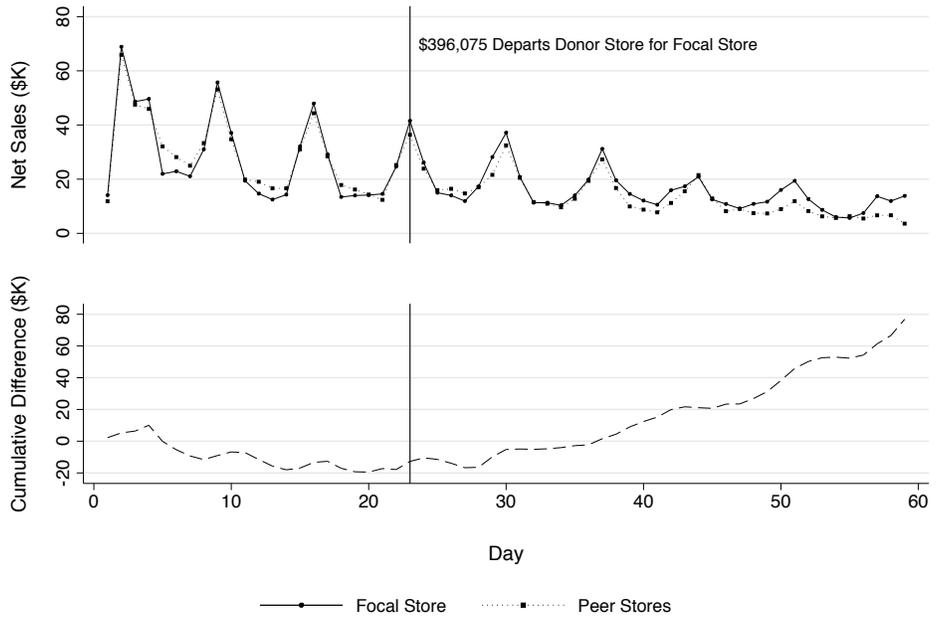


Figure 7: The Effect of Chain-Wide Inventory Transfers

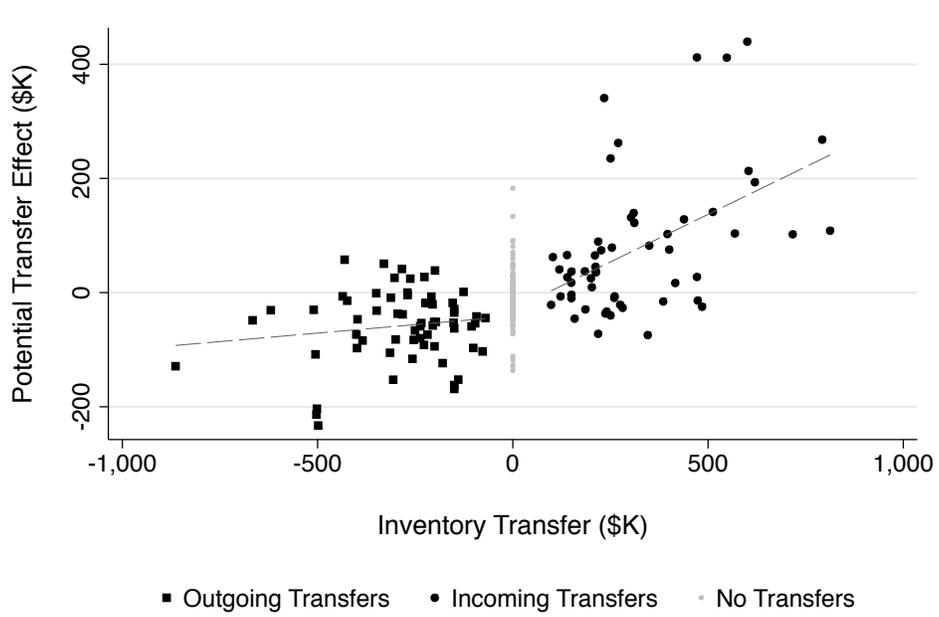


Figure 8: Optimal Markdown Schedule for a Retail Chain

