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Quantity vs. Quality: Exclusion By Platforms With Network Effects

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Abstract

This paper provides a simple model of platforms with direct network effects, in which users value not just the *quantity* (i.e. number) of other users who join, but also their *average quality* in some dimension. A monopoly platform is more likely to exclude low-quality users when users place more value on average quality and less value on total quantity. With competing platforms, the effect of user preferences for quantity is reversed. Furthermore, exclusion incentives depend in a non-trivial way on the proportion of high-quality users in the overall population and on their opportunity cost of joining the platform relative to low-quality users. The net effect of these two parameters depends on whether they have a stronger impact on the gains from exclusion (higher average quality) or on its costs (lower quantity).

Keywords: multi-sided platforms, network effects, exclusion, quality and quantity.

JEL Classifications: L1, L2, L8

1 Introduction

An important part of many real-world multi-sided platform (MSP) strategies are non-price "governance rules," which regulate access *to* and transactions *on* the platforms (cf. Boudreau and Hagiu (2009)). One of the most common MSP governance rules is the restriction of access on at least one side, resulting in the exclusion of some customers who *are* willing to pay the plaform's access and/or transaction fees. For example: videogame console manufacturers such as Microsoft, Sony and Nintendo restrict access to a select set of game developers and exclude many others (by including security chips in their consoles), even though the latter would also be willing to pay the per game royalties levied by the manufacturers[1]; some romantic matchmaking sites like eHarmony carefully screen and reject a sizeable fraction of applicants who would be willing to pay their membership

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[1] For a detailed analysis of videogame platform strategies, see chapter 5 in Evans Hagiu and Schmalensee (2006) and Hagiu and Halaburda (2009).

fees (cf. Piskorski, Halaburda and Smith (2008)); Apple routinely excludes certain application developers from its highly popular iPhone store; some social networks restrict membership to only a fraction of the users who would be willing to join; etc.

The economics and strategy literature on multi-sided markets to date has devoted most of its attention to pricing strategies (e.g. Armstrong (2006), Caillaud and Jullien (2003), Parker and Van Alstyne (2005), Rochet and Tirole (2003) and (2006)) and although some recent papers have started to tackle certain design issues (cf. Hagiu and Jullien (2009), Parker and Van Alstyne (2008)), there has been very little formal work on MSP governance rules and the factors that drive platforms to restrict access beyond what they can achieve through pricing alone.

This paper aims to start filling this gap. It builds a simple model of profit-maximizing, one-sided platforms with *direct* network effects and formalizes the choice of exclusion policies. The reason for which exclusion is a necessary strategic instrument in my model is rather straightforward: trading off the quantity in favor of the average "quality" of participating agents. Indeed, in contrast with most of the existing literature on platforms with direct or indirect network effects, my model assumes that each user cares about the average quality of other participating users. What is less obvious and constitutes a substantive contribution of the paper is elucidating the impact of several key factors on platforms' exclusion incentives. Is exclusion more or less necessary when high-quality users are more scarce in the overall population? Is exclusion more or less necessary when high-quality users have a higher opportunity cost of joining the platform? As it turns out (and perhaps contrary to common intuition), the answers to both questions can go either way. This is because a higher proportion of high-quality users has two opposing effects on the desirability of exclusion. On the one hand, the gains from exclusion are lower because the resulting increase in average quality is lower. On the other hand, the costs of exclusion are also lower since it entails losing less "quantity". The broader strategic implication is that the correlation between quality and willingness to pay for (or cost of) joining the platform is a key factor affecting the platform's incentives to exclude, but its effect is non-monotonic.

The quality vs. quantity tradeoff emphasized in my model is related to the "lemons market failure" first studied by Akerlof (1970). The key difference is that here a profit-maximizing platform plays a role similar to that of a "public authority" (cf. Boudreau and Hagiu (2009)) imposing a form of non-price regulation (restriction of access) in order to reduce the negative impact of low-quality users. Since network effects are determined by a combination of quality and quantity, the platform has a clear incentive to use an additional strategic instrument other than pricing in order to achieve the "right mix".

The novelty of my modelling approach is the introduction of a quality parameter which users value positively. In this context, exclusion by the platform takes the natural form of refusing access to all users of quality below a certain threshold - even though some of them would be willing to pay the price of access. This modeling of exclusion is similar to the minimum quality standards studied by Leland (1979), who uses a version of Akerlof (1970)'s model with a continuum of quality types. In that paper however, the quality standards are imposed by a public regulator, whereas here they are set by a profit-maximizing two-sided platform *in addition* to its prices.

While most of my formal analysis relies on a one-sided model with direct network effects, it is straightforward to extend it to a two-sided model in which one side at least cares about the average quality of agents on the other side. To show this, I provide the extension for the monopoly platform case. The precursor to this paper (Hagiu (2009)) contains a full-fledged model of exclusion by competing two-sided platforms.

To the best of my knowledge, there are only three related papers in the multi-sided market literature. Damiano and Li (2007) and (2008) use a model of two-sided platforms in which each side cares about the average quality of agents on the other side. The key difference however is that in their model the quality of users is unobservable to the platforms and the latter can only use prices in order to sort quality: they do not allow platforms to exclude as I do. In fact, in my model the average quality of participating users does not depend on platform prices, which makes exclusion absolutely necessary in order to control it.

In a different vein, Casadesus-Masanell and Halaburda (2010) provide a model of two-sided platforms connecting users with complementary products, such that platforms may have an incentive to limit the number of complements. There are two key drivers of this incentive: i) inducing users to shift towards consuming larger quantities of a smaller number of applications in order to generate stronger *direct* network effects around each application; ii) solving a coordination problem in a context with multiple equilibria. Neither of these two mechanisms is related to the quality vs.

quantity tradeoff I explore here.

The remainder of the paper is organized as follows: the next section focuses on the case of a monopoly platform's choices of price and exclusion level; section 3 studies the price-exclusion equilibria arising with two competing platforms; section 4 concludes.

2 Exclusion by a one-sided platform monopolist

2.1 Basic set-up

I use a linear model with direct network effects. A user of type (θ, q) who joins the monopoly platform derives the following utility:

$$U(\theta, q) = V(\overline{q}) + \alpha(\overline{q}) N - P - \theta c(q)$$

where:

- θ is a horizontal differentiation parameter uniformly distributed on $[0, 1]$
- q is the "quality" of an individual user from the perspective of other users, distributed *independently of* θ with cdf $F(.)$ and density $f(.)$ over $[0, +\infty[$, such that the average quality is finite: $\int_0^\infty q f(q)\, dq < \infty$
- $\theta c(q)$ is to be interpreted as the opportunity cost of joining the platform for a type (θ, q) user, where $c(.)$ is increasing
- $\overline{q}$ is the *average quality* of users who join the platform (see below for the derivation of its expression)
- $\alpha(\overline{q}) > 0$ is the direct network effect parameter, with $\alpha'(\overline{q}) \equiv \frac{d\alpha}{d\overline{q}} \geq 0$
- N is the total number of users who join the platform
- $V(\overline{q})$ is the *standalone* utility derived by users from joining the platform: $V'(\overline{q}) \equiv \frac{dV}{d\overline{q}} \geq 0$

- P is the access price charged by the platform

Thus, users are differentiated in two dimensions, θ and q, independently distributed of each other, and their opportunity cost of joining the platform - $\theta c(q)$ - depends on both characteristics. In particular, the function $c(.)$ can be increasing or decreasing. When $c(.)$ is decreasing (increasing), high-quality users have a higher (lower) opportunity cost of using the platform's service than low-quality users.

Users' preference for quality of other users is captured by the fact that $\alpha(\overline{q})$ and $V(\overline{q})$ are non-decreasing functions of $\overline{q}$, the average quality of participating users.

To fix ideas, one can think of this as being a model of a paid social network. Aside from the number of other users, each individual user may care about the overall "ambiance" or reputation of the network, which is reflected in the standalone term $V(\overline{q})$. If the interaction with other users is monetary (e.g. professional social networks), then the surplus derived from each interaction/transaction may also depend on the average quality of participating users through the equilibrium price of these transactions.[2]

A user joins the platform if her expected utility is non-negative. Suppose the platform imposes no restrictions on users except for the payment of the access fee P. To determine $\overline{q}$ and N note that, given $(P, N, \overline{q})$, the number of users of quality q who join is $\frac{V(\overline{q})+\alpha(\overline{q})N-P}{c(q)}$. Therefore we have:

$$N = [V(\overline{q}) + \alpha(\overline{q}) N - P] \int_0^\infty \frac{f(q)\, dq}{c(q)}$$

and:

$$\overline{q} = \frac{\int_0^\infty \frac{qf(q)dq}{c(q)}}{\int_0^\infty \frac{f(q)dq}{c(q)}}$$

Because each user cares about the average quality of other users however, the platform may find it profitable to exclude a positive measure of users, *even though* they would be willing to pay the price of admission P. In particular, if the platform decides to exclude some users, it will always start by excluding the lowest quality ones. Thus, in addition to P, I also allow the platform to set $L \geq 0$, the quality threshold of admission, such that only users of quality $q \geq L$ are allowed to join.

[2] A more complete model would contain the micro-foundations of these transactions among users and resulting equilibrium price. As soon as user quality is imperfectly observable, the equilibrium price and net surplus will depend on the average perceived quality. For the purposes of my analysis, this is all that matters.

In this case:

$$N = \lambda(L)\left[V(\overline{q}(L)) + \alpha(\overline{q}(L))N - P\right] \tag{1}$$

where:

$$\lambda(L) \equiv \int_L^\infty \frac{f(q)\,dq}{c(q)}$$

is the fraction of users who are actually allowed to join the platform among those *willing* to join given P and:

$$\overline{q}(L) = E[q \mid L] = \frac{\int_L^\infty \frac{qf(q)}{c(q)}dq}{\lambda(L)}$$

is the average quality of users conditional on the platform's exclusion policy L.

Naturally, $\lambda(L)$ is decreasing and $\overline{q}(L)$ is increasing in L. Note that the "cost function" $c(q)$ affects the fraction of users excluded λ and the average quality of participating users $\overline{q}$ for any given level of exclusion L in a straightforward way, by placing different "weights" on the density function of users' quality.

I assume for simplicity that the platform has zero marginal costs of serving users, therefore its profits are:

$$\Pi^P(P, L) = P \times N(P, L)$$

which it maximizes over (P, L). The focus of the paper is on determining the optimal level of exclusion chosen by the platform, L.

Note that this formulation assumes away implementation costs of the exclusion mechanism - e.g. costs of screening quality or restricting access through technological locks - and focuses instead on the inherent economic tradeoffs which are independent of such costs. Adding implementation costs would have the unsurprising effect of shifting the balance towards less exclusion.

Before proceeding, several observations on the modeling set-up are in order. First, the quality q of users is assumed to only affect their opportunity cost of joining the platform $\theta c(q)$. In reality, it may also affect the utility they derive from their interactions with other users on the platform, i.e. $V(.)$ and $\alpha(.)$ could also depend on q. The reason for this assumption is simplification: most importantly, it implies that the average quality $\overline{q}$ only depends on L and not on the price P. This eliminates the role played by prices in sorting quality, which is the focus of Damiano and Li (2007) and (2008) in a two-sided setting. Instead, I wish to isolate the effects of various exogenous factors on the platform's choice of L. Thus, $c(.)$ is best interpreted as capturing the *net* effect of quality on users' payoffs from joining the platform.

Second, users can observe the platform's price P and exclusion policy L, so that everyone can correctly infer the average equilibrium quality of users who join the platform.

Third, the platform is assumed to observe the quality of each user prior to admitting them and this quality is set in stone, i.e. I am not studying here user incentives to invest in quality enhancements (before or after the platform sets prices and access policies). Also, note that in this paper the notion of "quality" of users refers to any measurable characteristic/attribute that increases the utility derived by other users. This can therefore be different (more general) than "objective" quality. For example, the professional social network LinkedIn approves third-party applications which are most relevant to professional social networking and may turn down applications which some might regard as "high-quality", if they do not fit the "professional" profile.

Solving (1) for N, one can derive the effective user participation on for the platform:

$$N = \frac{\lambda(L)\left[V(\overline{q}(L)) - P\right]}{1 - \lambda(L)\,\alpha(\overline{q}(L))}$$

The platform therefore solves:

$$\max_{P,L} \frac{\lambda(L)\left[V(\overline{q}(L)) - P\right]P}{1 - \lambda(L)\,\alpha(\overline{q}(L))} = \max_{L} \frac{\lambda(L)\,V^2(\overline{q}(L))}{4\left[1 - \lambda(L)\,\alpha(\overline{q}(L))\right]} \tag{2}$$

Thus, the optimal level of exclusion L is given by maximizing expression (2) with respect to L. Note that this expression is increasing in λ, V and α. There is therefore a clear tradeoff between quality and quantity involved in choosing the optimal exclusion level L. Indeed, an increase in L (the quality threshold for admission) has two opposite effects on profits: a negative effect through a reduction in $\lambda(L)$ - i.e. by decreasing quantity - and a positive effect through an increase in the average quality $\overline{q}(L)$, which in turn increases participation (V and α are non-decreasing in $\overline{q}$).

2.2 Quality vs. quantity

In order to reach a better understanding of how various factors affect the platform's choice of exclusion L and thereby its choice of quality vs. quantity, the model needs to be specified a bit further. In particular, I assume that the distribution of quality types is binary: a fraction λ of users are of quality $q = 1$ (high) and a fraction $(1 - \lambda)$ are of quality $q = 0$ (low). Let then:

$$c(q) = \begin{cases} c & \text{if} \quad q = 0 \\ \\ \frac{c}{\beta} & \text{if} \quad q = 1 \end{cases}$$

where $c, \beta > 0$ and β is to be interpreted as a measure of high-quality users' opportunity cost advantage of joining the platform relative to low-quality users. In particular, if $\beta > 1$ then high-quality users have a lower opportunity cost of participating, all other things being equal (in particular, holding constant the horizontal differentiation parameter θ). On the other hand, if $\beta < 1$ then high-quality users have a higher opportunity cost.

Since there are only two quality levels of users possible (0 and 1), the platform's choice of exclusion is limited to two options: $L = 0$ which means all users who are willing to pay are allowed to access the platform and $L = 1$, which means the platform excludes all users of low quality ($q = 0$).

It is then straightforward to derive the expressions of λ and $\overline{q}$ under the two governance regimes ($L = 0$ - no exclusion; $L = 1$ - access restricted to high-quality users):

$$\lambda(L) = \begin{cases} \frac{\lambda\beta + (1-\lambda)}{c} \equiv \lambda_{NE} & \text{if} \quad L = 0 \\ \\ \frac{\lambda\beta}{c} \equiv \lambda_E & \text{if} \quad L = 1 \end{cases}$$

$$\overline{q}(L) = \begin{cases} \frac{\lambda\beta}{\lambda\beta + (1-\lambda)} \equiv \overline{q_{NE}} & \text{if} \quad L = 0 \\ \\ 1 & \text{if} \quad L = 1 \end{cases}$$

Note that:

- λ_{NE} and λ_E are both increasing in β as expected

- λ_{NE} is increasing in λ if and only if $\beta > 1$, i.e. if and only if high-quality users have a competitive advantage over low-quality users

- $\overline{q_{NE}}$ is increasing in both β and λ as expected: when β increases, the proportion of high quality users who join the platform under the no exclusion regime ($L = 0$) increases, which results in higher average quality. Similarly for λ.

Using expression (2) and comparing the two governance regimes (exclusion vs. non-exclusion), I conclude that the platform chooses to exclude low-quality users if and only if:

$$\frac{V^2\left(\overline{q_{NE}}\right)}{V^2\left(1\right)} \leq \frac{1-\alpha\left(\overline{q_{NE}}\right)\lambda_{NE}}{1-\alpha\left(1\right)\lambda_E} \times \frac{\lambda_E}{\lambda_{NE}} \tag{3}$$

Note that this condition holds trivially (with equality) when $\lambda = 1$, i.e. when there are no low-quality users.

The following proposition follows directly by using simple functional forms for $V(.)$ and $\alpha(.)$.

Proposition 1 a) *If α is constant and $V(\overline{q}) = \overline{q}$ then the platform excludes low-quality users if and only if:*[3]

$$2\lambda\beta + (1-\lambda) \leq \frac{c}{\alpha} \tag{4}$$

b) If V is constant and $\alpha(\overline{q}) = \alpha_0\overline{q}^2$ then the platform excludes low-quality users if and only if:[4]

$$\frac{c}{\alpha_0} \leq \frac{2\lambda\beta + (1-\lambda)}{1+\frac{1-\lambda}{\lambda\beta}} \tag{5}$$

■

[3] In this case, the necessary and sufficient condition to ensure a well-defined optimization problem is:

$$\lambda\beta + (1-\lambda) \leq \frac{c}{\alpha}$$

[4] In this case, the necessary and sufficient condition to ensure a well-defined optimization problem is:

$$\lambda\beta \leq \frac{c}{\alpha_0}$$

2.3 Interpretation

First, note that in both cases shown in Proposition 1, exclusion is more likely when users place *less* value on *quantity* and *more* value on *quality*. Indeed, in case a), α mesures users' preference for quantity (it is the direct network effect parameter) and the right hand side is decreasing in α so that (4) is less likely to hold when α is higher. By contrast, in case b), α_0 is a measure of users' preferences for quality (the direct network effect parameter is now strictly increasing in average quality) and the left hand side is decreasing in α_0 so that (5) is more likely to hold when α_0 is higher. These two examples capture the fundamental quality vs. quantity tradeoff involved in choosing the optimal level of exclusion. Furthemore, it should be clear that the effects of preferences for quality vs. quantity are robust and generalize to any utility formulation. It is however important to clearly distinguish them from each other in the modelling formulation.

Second, the effect of λ on the likelihood of exclusion is ambiguous, which may seem surprising at first glance. Let us focus on case a). The left hand side is increasing in λ if $\beta > \frac{1}{2}$ and decreasing in λ otherwise. Thus, if $\beta > \frac{1}{2}$ then exclusion is optimal for low λ, i.e. when the proportion of high-quality users in the overall population is sufficiently low. Conversely, if $\beta < \frac{1}{2}$ then exclusion is optimal when the proportion of high-quality users is sufficiently high. The reason λ can have opposite effects on the platform's exclusion policy depending on the formulation of user preferences is that there are two channels through which λ affects the change in platform profits when going from no-exclusion ($L = 0$) to a regime with exclusion ($L = 1$). On the one hand - and perhaps most intuitively - a higher λ makes exclusion less attractive by reducing its benefits (smaller increase in average quality) relative to the no-exclusion regime. On the other hand however, a higher λ also decreases the costs of exclusion by reducing the implied loss of quantity $(1 - \lambda)$, which tends to make exclusion relatively more attractive. The net result of these two mechanisms can go either way in general. When β is smaller, the opportunity cost of high-quality users is higher, and the first mechanism becomes dominated by the second: higher-quality users are relatively more scarce, therefore the gain in terms of average quality from exclusion remains important even when λ is already high.

Third, the effect of β - the cost advantage of high-quality users - on the likelihood of exclusion is also ambiguous and relies on a similar tradeoff. A higher β increases the fraction of high-quality users joining the platform for any price P and therefore increases the average quality. This means that the potential gains from exclusion are smaller (there is less to gain in terms of average quality) but so are the costs (exclusion would entail a less onerous quantity sacrifice since there are fewer users of low-quality willing to participate to begin with. The first effect dominates in the version of the model with constant α, whereas the second mechanism prevails in the version with constant V.

2.4 Two-sided platform case

It is straightforward to extend the model above to a two-sided platform with indirect network effects in order to convince ourselves that the quality vs. quantity tradeoff and the analysis of exclusion choice are very similar.

To do so, consider the following simple two-sided model. The two sides of the market are denoted by M (men) and women W (women). Assume that the utility derived by W agents from joining the platform is increasing not only in the number of M agents but also in the *average quality* of the M agents who join the platform. Conversely, assume for simplicity that M agents only care about the number of W agents (this should not be viewed as a necessary consequence of the initial labeling of the two sides as men and women!). In Hagiu (2009) I derive the general expression of platform profits when each side cares about both quality and quantity on the other side and the platform can exclude agents on both sides. Working with this more general expression would be more cumbersome but the main conclusions would remain unchanged. Furthermore, in most of the real-world applications that this model is intended to represent, quality is an issue on only one side of the market, specifically the "producer" side when the two sides are consumers and producers (e.g. videogame consoles; platforms for software applications like iPhone, Facebook, LinkedIn, Symbian).

The respective utilities of M agents and W agents from joining the platform when N_W W agents and N_M M agents join are:

$$U_M(\theta_M, q) = \alpha_M N_W - P_M - \theta_M c(q)$$

$$U_W(\theta_W) = V_W(\overline{q_M}) + \alpha_W(\overline{q_M}) N_M - P_W - \theta_W$$

where:

- θ_M and θ_W are horizontal differentiation parameters (to be interpreted as opportunity costs of joining the platform), both uniformly distributed on $[0, 1]$

- q is the "quality" of an individual M agent from the perspective of W agents, distributed *independently of* θ_M, with cdf $F(.)$ and density $f(.)$ over $[0, +\infty[$, such that the average quality is finite: $\int_0^\infty qf(q)\,dq < \infty$

- $\overline{q_M}$ is the *average quality* of M agents who join the platform (cf. below for its derivation)

- $\alpha_M > 0$ and $\alpha_W(\overline{q_M}) > 0$ are the quantity-related indirect network effect parameters on the two sides and $\alpha'_W(\overline{q_M}) \equiv \frac{d\alpha'_W}{d\overline{q_M}} \geq 0$

- $V_W(\overline{q_M})$ is the standalone utility derived by W agents from joining the platform and $V'_W(\overline{q_M}) \equiv \frac{dV'_W}{d\overline{q_M}} \geq 0$

- P_M and P_W are the access price charged by the platform to the two sides

The assumptions over functional forms are the same as in the one-sided case.

Since W agents care about the average quality of M agents, the platform may find it profitable to exclude the lowest quality M agents by imposing a minimum quality threshold L_M. In this case, the effective participations on both sides of the market (N_W, N_M) are given by:

$$N_W = V_W(\overline{q_M}(L_M)) + \alpha_W(\overline{q_M}(L_M))\,N_M - P_W \tag{6}$$

$$N_M = \lambda_M(L_M)(\alpha_M N_W - P_M) \tag{7}$$

where:

$$\lambda_M(L_M) = \int_{L_M}^{\infty} \frac{f(q)}{c(q)}dq$$

is the fraction of M agents willing to join the platform (given P_M and N_W) who are allowed to join and:

$$\overline{q_M}(L_M) = E[q \mid L_M] = \frac{\int_{L_M}^{\infty} \frac{qf(q)}{c(q)}dq}{\lambda_M(L_M)}$$

is the average quality of M agents conditional on the platform's exclusion policy (L_M).

As in the one-sided case, $\lambda_M(L_M)$ is decreasing and $\overline{q_M}(L_M)$ is increasing in L_M.

The platform's profits are:

$$\Pi^P = P_M N_M + P_W N_W$$

which it maximizes over (P_M, P_W, L_M).

Using (6) and (7) to switch price and demand variables, we can optimize the platform's profits over (N_M, N_W, L_M): Omitting quality-related arguments for concision, we have:

$$\max_{N_M, N_W, L_M} \left\{ \begin{array}{c} N_W \left(V_W \left(\overline{q_M} \left(L_M \right) \right) + \alpha_W \left(\overline{q_M} \left(L_M \right) \right) N_M - N_W \right) \\ + N_M \left(\alpha_M N_W - \frac{N_M}{\lambda_M (L_M)} \right) \end{array} \right\}$$

Taking the first-order conditions in N_W and N_M respectively, solving simultaneously and plugging back into the profit expression, we obtain the expression of platform profits as a function of the level of exclusion L_M:

$$\Pi^P \left(L_M \right) = \frac{V_W^2 \left(\overline{q_M} \left(L_M \right) \right)}{4 \left[1 - \lambda_M \left(L_M \right) \times \overline{\alpha}^2 \left(\overline{q_M} \left(L_M \right) \right) \right]}$$

where:

$$\overline{\alpha} \left(\overline{q_M} \left(L_M \right) \right) = \frac{1}{2} \left[\alpha_M + \alpha_W \left(\overline{q_M} \left(L_M \right) \right) \right]$$

is the average magnitude of indirect network effects.

Comparing this expression with (2) above, it is apparent that the implicit quantity vs. quality tradeoff and resulting analysis of exclusion are very similar.

3 Exclusion with competing platforms

I now turn to the case of platform competition. In order to keep things as simple as possible, I assume the two platforms are differentiated a la Hotelling, such that the utility derived by a user $x \in [0, 1]$ of quality q adopting platform $i \in \{1, 2\}$ is:

$$U \left(x, q, i \right) = V \left(\overline{q}_i \right) + \alpha N_i - P_i - t \left[\left(2 - i \right) x + \left(i - 1 \right) \left(1 - x \right) \right]$$

where $\overline{q}_i$ is the average quality of users on platform i, t are the unitary transportation costs and P_i is the access price charged by platform i.

There are two important differences relative to the monopoly case. First, the quality of an individual user no longer affects her utility (she does however care about the average quality of other users joining the same platform). This assumption is less restrictive than it might seem. Given the Hotelling setting, one could introduce a utility term depending on q: as long as that term does not depend on the platform the user joins, everything is *exactly* the same as in the formulation chosen above. Second, average user quality is assumed to only affect the standalone utility term $V(\overline{q}_i)$ but not the network effect α, which is constant throughout this section. This is obviously done for simplification purposes but it is straightforward (though tedious) to show that all of the following analysis would go through virtually unchanged if we also allowed α to depend on $\overline{q}_i$.

I will also assume right away that q has a binary distribution: a fraction λ of users are of quality $q = 1$ (high) and a fraction $(1 - \lambda)$ are of quality $q = 0$ (low). The distribution of q is independent of the distribution of x across users.

For platform $i \in \{1, 2\}$ there are thus two possibilities only:

- if it chooses to exclude low quality users, then $L_i = 1$ and $\overline{q}_i = 1$ so that:

$$V(\overline{q}_i) = V(1) \equiv V_H$$

- if it does not exclude, then $L_i = 0$ and $\overline{q}_i = \lambda$ so that:

$$V(\overline{q}_i) = V(\lambda) \equiv V_L$$

Finally, I also assume that standalone utilities are sufficiently large so that the market is always covered and that even the furthest user from a platform derives positive utility from it in all cases.

3.1 Platform demands

A subtle and unique issue arises when competing platforms resort to exclusion: what exactly happens to users excluded by one platform? If both platforms choose the same exclusion levels ($L_1 = L_2$)

then this is not an issue. On the other hand, suppose platform 2 excludes low quality users whereas platform 1 does not. In this case, we have to take into account the fact that the users excluded by platform 2 will join platform 1. Accordingly we will treat this case separately.

3.1.1 Both platforms choose the same exclusion level

When both platforms choose the same exclusion level ($L_1 = L_2$) we have $V(\overline{q}_1) = V(\overline{q}_2)$ so that:

$$\begin{aligned} N_1 &= \widetilde{\lambda}\left[\frac{1}{2} + \frac{\alpha(N_1 - N_2) - \Delta P_1}{2t}\right] \\ N_2 &= \widetilde{\lambda}\left[\frac{1}{2} + \frac{\alpha(N_2 - N_1) + \Delta P_1}{2t}\right] \end{aligned}$$

where

$$\widetilde{\lambda} \equiv \begin{cases} 1 & \text{if} \quad L_1 = L_2 = 0 \\ \lambda & \text{if} \quad L_1 = L_2 = 1 \end{cases}$$

is the share of users *willing* to join platform i who are actually allowed to join for $i \in \{1, 2\}$ and:

$$\Delta P_1 \equiv P_1 - P_2$$

is the price differential in favor of platform 2.

Solving for (N_1, N_2) simultaneously, we obtain:

$$N_1 = \frac{t - \alpha\widetilde{\lambda} - \Delta P_1}{2\left(\frac{t}{\widetilde{\lambda}} - \alpha\right)} \quad \text{and} \quad N_2 = \frac{t - \alpha\widetilde{\lambda} + \Delta P_1}{2\left(\frac{t}{\widetilde{\lambda}} - \alpha\right)} \tag{8}$$

3.1.2 Platform 2 excludes; platform 1 does not

When platform 1 chooses $L_1 = 0$ and platform 2 chooses $L_2 = 1$ we have:

$$N_1 = \left[\frac{1}{2} + \frac{\alpha(N_1 - N_2) - \Delta V - \Delta P_1}{2t}\right] + \underbrace{(1-\lambda)\left[\frac{1}{2} - \frac{\alpha(N_1 - N_2) - \Delta V - \Delta P_1}{2t}\right]}_{\text{users excluded by platform 2}}$$

$$N_2 = \lambda\left[\frac{1}{2} - \frac{\alpha(N_1 - N_2) - \Delta V - \Delta P_1}{2t}\right]$$

where:

$$\Delta V \equiv V_H - V_L$$

Solving for (N_1, N_2) simultaneously, we obtain:

$$N_1 = \frac{t\left(\frac{2}{\lambda} - 1\right) - \alpha - \Delta V - \Delta P_1}{2\left(\frac{t}{\lambda} - \alpha\right)} \quad \text{and} \quad N_2 = \frac{t - \alpha + \Delta V + \Delta P_1}{2\left(\frac{t}{\lambda} - \alpha\right)} \tag{9}$$

3.2 Equilibria

I can now turn to the derivation of the competitive equilibria. The two platforms simultaneously choose their respective prices P_i and their respective exclusion levels L_i. Assuming 0 marginal costs, platform i's profits are:

$$\Pi_i = P_i N_i$$

Clearly, there are three candidate equilibria: two symmetric (one with $L_1 = L_2 = 1$ and one with $L_1 = L_2 = 0$) and one asymmetric ($L_1 = 0$ and $L_2 = 1$).

3.2.1 Symmetric equilibrium with no exclusion

To derive this equilibrium I start by assuming $L_1 = L_2 = 0$ and use expressions (8) with $\widetilde{\lambda} = 1$. Optimizing each platform's profits over P_i, the candidate equilibrium prices are given by:

$$P_1 = \frac{1}{2}(t - \alpha + P_2) \quad \text{and} \quad P_2 = \frac{1}{2}(t - \alpha + P_1)$$

leading to $P_1 = P_2 = t - \alpha$ and candidate equilibrium profits $\Pi_1 = \Pi_2 = \frac{1}{2}(t - \alpha)$.

I now have to impose that neither platform has an incentive to unilaterally deviate by excluding low quality users and adjusting its price accordingly. This deviation is treated in the appendix, leading to the following conclusion:

Proposition 2 *In a symmetric competitive equilibrium with no exclusion, both platforms make profits:*

$$\Pi^*_{NE} = \frac{1}{2}(t - \alpha)$$

This equilibrium exists if and only if:

$$\Delta V \leq 2\left[\sqrt{(t-\alpha)\left(\frac{t}{\lambda}-\alpha\right)}-(t-\alpha)\right] \tag{10}$$

■

3.2.2 Symmetric equilibrium with exclusion

To derive this equilibrium I start by assuming $L_1 = L_2 = 1$ and use expressions (8) with $\widetilde{\lambda} = \lambda$. Optimizing each platform's profits over P_i, the candidate equilibrium prices are $P_1 = P_2 = t - \alpha\lambda$ and candidate equilibrium profits $\Pi_1 = \Pi_2 = \frac{\lambda}{2}(t - \alpha\lambda)$.

I now have to impose that neither platform has an incentive to unilaterally deviate by allowing low quality users and adjusting its price accordingly. The deviation is treated in the appendix, leading to the following conclusion:

Proposition 3 *In a symmetric competitive equilibrium with exclusion, both platforms make profits:*

$$\Pi_E^* = \frac{\lambda}{2}(t-\alpha\lambda)$$

This equilibrium exists if and only if:

$$\Delta V \geq (1-\lambda)\left(\frac{2t}{\lambda}-\alpha\right) \tag{11}$$

■

3.2.3 Asymmetric equilibrium

To derive this equilibrium I start by assuming $L_1 = 0$ and $L_2 = 1$. I now use expressions (9) and optimize each platform's profits over P_i to obtain the following candidate equilibrium prices:

$$\begin{aligned} P_1 &= \frac{1}{3}\left[t\left(\frac{4}{\lambda}-1\right)-3\alpha-\Delta V\right] \\ P_2 &= \frac{1}{3}\left[t\left(\frac{2}{\lambda}+1\right)-3\alpha+\Delta V\right] \end{aligned}$$

and candidate equilibrium profits:

$$\Pi_1 = \frac{1}{18}\frac{\left[t\left(\frac{4}{\lambda}-1\right)-3\alpha-\Delta V\right]^2}{\frac{t}{\lambda}-\alpha}$$
$$\Pi_2 = \frac{1}{18}\frac{\left[t\left(\frac{2}{\lambda}+1\right)-3\alpha+\Delta V\right]^2}{\frac{t}{\lambda}-\alpha}$$

I now have to impose that neither platform has an incentive to unilaterally deviate: there are two possible and distinct deviations here leading to two constraints (unlike the symmetric equilibria, where we only had one condition).

Proposition 4 *In the asymmetric competitive equilibrium with exclusion by platform 2 and no exclusion by platform 1, platforms make respective profits:*

$$\Pi_1^* = \frac{1}{18}\frac{\left[t\left(\frac{4}{\lambda}-1\right)-3\alpha-\Delta V\right]^2}{\frac{t}{\lambda}-\alpha}$$
$$\Pi_2^* = \frac{1}{18}\frac{\left[t\left(\frac{2}{\lambda}+1\right)-3\alpha+\Delta V\right]^2}{\frac{t}{\lambda}-\alpha}$$

This equilibrium exists if and only if:

$$\Delta V \leq (1-\lambda)\left(\frac{2t}{\lambda}-\alpha\right) \tag{12}$$

and:

$$\Delta V \geq 2\left[\sqrt{(t-\alpha)\left(\frac{t}{\lambda}-\alpha\right)}-(t-\alpha)\right]\times\frac{t\left(\frac{2}{\lambda}+1\right)-3\alpha}{2(t-\alpha)+\sqrt{(t-\alpha)\left(\frac{t}{\lambda}-\alpha\right)}} \tag{13}$$

■

3.3 Discussion, robustness and extensions

Examining the conditions for existence of each of the three possible equilibria (10-13), it is readily apparent that the symmetric equilibrium with exclusion can never co-exist with the asymmetric equilibrium (compare 11 with 12). It is also easily verified that the right hand side in (13) is always larger than the right hand side in (10),[5] which means that the asymmetric equilibrium can never

[5] This is because we must have $t > \alpha$.

co-exist with the symmetric equilibrium without exclusion. Finally, one can verify that the right hand side in (11) is always larger than the right hand side in (10), which means the two symmetric equilibria can never coexist either.

We can thus conclude that exclusion can arise as a symmetric competitive equilibrium whenever ΔV is sufficiently high, whereas no exclusion arises as a symmetric competitive equilibrium whenever ΔV is sufficiently low. The asymmetric equilibrium can (but does not necessarily) exist for intermediate value of ΔV. This conclusion is somewhat expected: the larger ΔV the larger the gain from excluding low-quality users.

What is much less expected is that, when α (preference for quantity) is larger, the symmetric equilibrium with exclusion is more likely to exist whereas the symmetric equilibrium with no exclusion is less likely to exist. Indeed, it is straightforward to show that both the right hand side of (11) and the right hand side of (10) are decreasing in α.

Furthemore, let us focus on the existence of the symmetric equilibrium with exclusion. Suppose that $V(\overline{q}) = v\overline{q}$. Then (11) is equivalent to:

$$v \geq \frac{2t}{\lambda} - \alpha$$

This equilibrium is more likely to exist for high λ and low t. In other words, a symmetric competitive equilibrium with exclusion is more likely to arise when the proportion of high-quality users is larger and competition between platforms is more intense.

[TO BE COMPLETED WITH PRECISE INTERPRETATION]

A model of competition between two-sided platforms with exclusion is provided in the precursor to this paper (Hagiu (2009)).

4 Conclusion

I have done two things in this paper. First, I have provided a simple model capturing the incentives that platforms (one-sided and multi-sided) have to exclude some participants who would be willing to pay the price of admission. The need for exclusion (or enforcing minimum "quality" standards -

cf. Leland (1979)) stems here from a fundamental tradeoff between the quality and the quantity of participating users. As soon as at least one side of the market values a quality attribute of at least one other side (which may or may not be correlated with willingness-to-pay for or cost of joining the platform), the platform may find it optimal to sacrifice quantity to a certain degree in order to increase the average quality of agents on the second side.

Second, I have shown that platforms' incentives to exclude are determined by several important considerations. Users' preferences for quality unambiguously increase the incentives to exclude. On the other hand, the effect of user preferences for quantity may depend on the competitive structure of the market. A monopoly platform will always be *less* likely to exclude when users care more about quantity. By contrast, competing platforms may be *more* likely to end up in an equilibrium with exclusion when users care more about quantity.

Meanwhile, the effects of the proportion of high-quality users in the overall population and of the relative cost advantage of high-quality users are ambiguous. Their sign is determined by the interplay of two forces. On the one hand, an increase in either the proportion of high-quality users in the overall population or in the relative cost advantage of high-quality users reduces platforms' benefits from exclusion - the potential gain in average quality is smaller. On the other hand, the same increase also reduces platforms' costs of exclusion - the loss of quantity is also smaller.

The current paper represents only an initial effort in exploring the various forces driving the choice of non-price exclusion by (multi-sided) platforms. Some immediate and promising extensions include a full analysis of competitive equilibria with multi-sided platforms and the introduction of imperfect signals perceived by platforms regarding user quality.

References

[1] Akerlof, G. (1970) "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, 84(3), 488-500.

[2] Armstrong, M. (2006) "Competition in Two-Sided Markets," *Rand Journal of Economics*, 37(3), 669-691.

[3] Boudreau, K. and A. Hagiu (2009) "Platform Rules: Multi-Sided Platforms As Regulators," in Annabelle Gawer (ed), *Platforms, Markets and Innovation*, Cheltenham, UK and Northampton, MA, US: Edward Elgar, forthcoming.

[4] Caillaud, B. and B. Jullien (2003) "Chicken and Egg: Competition Among Intermediation Service Providers," *Rand Journal of Economics*, 34(2), 309-328.

[5] Casadesus-Masanell, R. and H. Halaburda (2010) "When Does a Platform Create Value by Limiting Choice?" Harvard Business School working paper No. 11-030.

[6] Damiano E. and H. Li (2007) "Price Discrimination and Efficient Matching," *Economic Theory*, 30(2), 243-263.

[7] Damiano E. and H. Li (2008) "Competing Matchmaking," *Journal of the European Economic Association*, 6(4), 789-818.

[8] Evans, D. S., A. Hagiu and R. Schmalensee (2006) *Invisible Engines: How Software Platforms Drive Innovation and Transform Industries*, MIT Press.

[9] Hagiu, A. (2009) "Quantity vs. Quality and Exclusion By Two-Sided Platforms," Harvard Business School working paper No. 09-094.

[10] Hagiu, A. and H. Halaburda (2009) "Responding to the Wii?" Harvard Business School Case Study No. 709-448 and Teaching Note No. 709-481.

[11] Hagiu, A. and B. Jullien (2009) "Why Do Intermediaries Divert Search?", Harvard Business School working paper 08-010.

[12] Leland, H. E. "Quacks, Lemons and Licensing: A Theory of Minimum Quality Standards," *Journal of Political Economy*, Vol. 87, No. 6, pp.1328-1346, 1979.

[13] Parker, G. and M. W. Van Alstyne (2005) "Two-sided network effects: A theory of information product design," *Management Science*, 51(10), 1494-1504.

[14] Parker, G. and M. W. Van Alstyne (2008) "Innovation, Openness and Platform Control," working paper, Tulane University, Boston University and MIT.

[15] Piskorski, M. J., H. Halaburda and T. Smith (2008) "eHarmony," Harvard Business School Case Study No. 709-424.

[16] Rochet, J.-C. and J. Tirole, 2003 "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1(4), 990-1029.

[17] Rochet, J.-C., and J. Tirole (2006) "Two-Sided Markets: Where We Stand," *Rand Journal of Economics*, 37(3), 645-667.

5 Appendix

5.1 Existence conditions for competitive equilibria

Consider first the symmetric equilibrium with no exclusion and suppose that platform 2 deviates by excluding low-quality users and adjusting its price to maximize deviation profits. Using the expressions in (9) we obtain that platform 2's optimal deviation price is given by:

$$\begin{aligned} P_2' &= \frac{1}{2}\left(t - \alpha + \Delta V + P_1\right) \\ &= t - \alpha + \frac{\Delta V}{2} \end{aligned}$$

since $P_1 = t - \alpha$ in the symmetric equilibrium. This yields deviation profits of:

$$\Pi_2' = \frac{\left(t - \alpha + \frac{\Delta V}{2}\right)^2}{2\left(\frac{t}{\lambda} - \alpha\right)}$$

Imposing that these profits are lower than $\frac{1}{2}\left(t - \alpha\right)$ yields condition (10) in the text.

Second, consider the symmetric equilibrium with exclusion and suppose that platform 1 deviates by allowing low-quality users and adjusting its price to maximize deviation profits. Using the expressions in (9) we obtain that platform 1's optimal deviation price is given by:

$$\begin{aligned} P_1' &= \frac{1}{2}\left[t\left(\frac{2}{\lambda} - 1\right) - \alpha - \Delta V + P_2\right] \\ &= \frac{t}{\lambda} - \frac{\alpha\left(1 + \lambda\right)}{2} - \frac{\Delta V}{2} \end{aligned}$$

since $P_2 = t - \lambda\alpha$ in the symmetric equilibrium. This yields deviation profits of:

$$\Pi_1' = \frac{\left(\frac{t}{\lambda} - \frac{\alpha(1+\lambda)}{2} - \frac{\Delta V}{2}\right)^2}{2\left(\frac{t}{\lambda} - \alpha\right)}$$

Imposing that these profits are lower than $\frac{\lambda}{2}\left(t - \alpha\lambda\right)$ yields condition (11) in the text.

Third, consider the asymmetric equilibrium with platform 2 excluding and platform 1 not excluding low-quality users. Suppose that platform 1 deviates to exclusion. Using the expressions in (8) we obtain that platform 1's optimal deviation price is given by:

$$P_1' = \frac{1}{3}\left[t\left(\frac{1}{\lambda} + 2\right) - \frac{3\alpha\left(1+\lambda\right)}{2} - \frac{\Delta V}{2}\right]$$

yielding deviation profits of:

$$\Pi_1' = \frac{1}{18}\frac{\left[t\left(\frac{1}{\lambda} + 2\right) - \frac{3\alpha(1+\lambda)}{2} - \frac{\Delta V}{2}\right]^2}{\frac{t}{\lambda} - \alpha}$$

Imposing that these profits are lower than $\frac{1}{18}\frac{\left[t\left(\frac{4}{\lambda}-1\right)-3\alpha-\Delta V\right]^2}{\frac{t}{\lambda}-\alpha}$ yields condition (12) in the text.

Suppose that platform 2 deviates to non-exclusion. Using the expressions in (8) we obtain that platform 2's optimal deviation price is given by:

$$P_2' = \frac{1}{3}\left[t\left(\frac{2}{\lambda} + 1\right) - 3\alpha - \frac{\Delta V}{2}\right]$$

yielding deviation profits of:

$$\Pi_2' = \frac{1}{18}\frac{\left[t\left(\frac{2}{\lambda} + 1\right) - 3\alpha - \frac{\Delta V}{2}\right]^2}{t - \alpha}$$

Imposing that these profits are lower than $\frac{1}{18}\frac{\left[t\left(\frac{2}{\lambda}+1\right)-3\alpha+\Delta V\right]^2}{\frac{t}{\lambda}-\alpha}$ yields condition (12) in the text.

■