Do Bonuses Enhance Sales Productivity? A Dynamic Structural Analysis of Bonus-Based Compensation Plans

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Abstract

We estimate a dynamic structural model of sales force response to a bonus based compensation plan. The paper has two main methodological innovations: First, we implement empirically the method proposed by Arcidiacono and Miller (2010) to accommodate unobserved latent class heterogeneity with a computationally light two-step estimator. Second, the bonus setting helps estimate discount factors in a dynamic structural model using field data. This is because, quarterly and annual bonuses help generate the instruments necessary to identify both discount factors in a hyperbolic discounting model.

Substantively, the paper sheds insights on how different elements of the compensation plan enhance productivity. We find clear evidence that: (1) bonuses enhance productivity; (2) overachievement commissions help sustain the high productivity of the best performers even after attaining quotas; and (3) sales people exhibit present bias consistent with hyperbolic discounting. Given such present bias, frequent quarterly bonuses tied to high demand end-of-quarter months, serve as pacers to keep the sales force on track to achieve their annual sales quotas.
1. Introduction

Personal selling is one of the most important elements of the marketing mix, especially in the context of B2B firms. An estimated 20 million people work as salespersons in the United States (Albers et al. 2008). Sales force costs average about 10% of sales revenues and as much as 40% of sales revenues for certain industries (Albers et al. forthcoming). In the aggregate, U.S. firms spent over $800 billion on sales forces in 2006, a number that is three times larger than advertising spending (Zoltners, Sinha and Lorimer 2008).

Marketing researchers routinely create response models for marketing mix instruments such as price, sales promotion and advertising. Meta-analysis of various research studies estimate that the sales force expenditure elasticity is about 0.35 (Albers et al., 2008), relative to about 0.22 for advertising (Assmus, Farley and Lehmann 1983) and about -2.62 for price (Bijmolt et al. 2005). While relative sales force expenditure elasticity is useful in determining the relative effectiveness of different instruments in the marketing mix, they give us little insight on how to design a sales force compensation plan, which is widely understood to be the primary tool by which firms can induce the sales force to exert the optimal levels of effort and thus to optimize the use of sales force expenditures.

A compensation plan can consist of many components: salary, commissions, and bonuses on achieving a certain threshold of performance called quotas. Figure 1 shows a variety of compensation plans that include combinations of these components. According to Joseph and Kalwani (1998), only about 25% of firms use a pure commission-based plan; the rest used some form of quotas. As per the Incentive Practices Research Study (2008) by ZS Associates, 73%, 85% and 89% in the pharma/biotech, medical devices and high tech industries respectively uses quota based compensation.

This paper has two substantive goals: First, to gain insight on how a firm should design its compensation plan. Specifically, should a firm offer quotas and bonuses in addition to commissions? Despite the ubiquity of quota-based compensation, there is considerable controversy in the theoretical (e.g., Holmstrom and Milgrom 1987; Lal and Srinivasan 1993) and empirical literature (Oyer 1998; Steenburgh 2008) about the effectiveness of quotas and bonuses relative to straight linear commission plans. Our paper sheds light on this controversy by estimating a dynamic structural model of how the sales
force responds to alternative compensation instruments and specific levels of commission rates, quotas and bonus levels.

Second, what should be the frequency of bonuses? Should one use a monthly, quarterly or annual bonus? Should one use a quarterly bonus in addition to an annual bonus? In the education literature, researchers have argued that frequent testing leads to better performance outcomes (Bangert-Drowns et al. 1991). Can quarterly quotas serve a similar role to improve outcomes? Like in the education literature, where frequent exams keep students prepared for the comprehensive final exam; frequent quota-bonus plans may serve as a mechanism to keep the sales force motivated to perform in the short-run well enough to be in striking distance of the overall annual performance quota.

Methodologically, the paper offers two key innovations. First, we empirically implement unobserved heterogeneity in a latent class framework within a computationally light two step conditional choice probability (CCP) framework to estimate the dynamic structural model. Though the use of two step estimation approaches have recently gained popularity (Hotz and Miller 1993; Bajari, Benkard and Levin 2007), due to ease of computation relative to traditional nested fixed point estimation approaches (e.g., Rust 1987), their use in empirical applications have been limited by their inability to accommodate unobserved heterogeneity. Arcidiacono and Miller (2010) propose an approach that allows accommodation of latent class heterogeneity within the two-step estimation framework. However, there are few empirical applications of this approach. To the best of our knowledge, ours is among the first empirical papers applying the Arcidiacono and Miller approach to account for unobserved heterogeneity in the two-step dynamic structural estimation framework.2

Second, and of great importance to the dynamic structural modeling literature, we estimate rather than assume discount factors. It is well-known in the literature on dynamic structural models that discount factors cannot be identified in standard applications because there are no instruments that provide exclusion restrictions across current and future period payoffs (Rust 1994). Hence the standard approach is to assume discount factors. In contrast to this, in our application, we have natural instruments in the form of bonuses: in

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2 Two concurrent working papers that have implemented this approach in economics are Finger (2008) and Beauchamp (2010).
non-bonus periods, bonuses should have no impact on current period payoffs, but only on future payoffs. This enables us to estimate discount factors from the data. Further, the psychology literature has shown strong evidence of hyperbolic discounting or present bias, where in contrast to the constant exponential discounting (Samuelson 1937), researchers have shown evidence of ‘hyperbolic discounting’ (Thaler 1981). The key idea of hyperbolic discounting is that individuals discount the immediate future from the present more than they do for the same time interval starting at a future date and hence a declining discount rate. The most frequently given example is the preference reversal shown between two delayed rewards. An individual may prefer 100$ today to 120$ in a year but may also prefer 120$ in two years to 100$ in a year.

Hyperbolic discounting is typically mathematically represented using the following quasi-hyperbolic discount function at time $t$: $D(t)=\beta^t\delta^t$ (Phelps and Pollak (1968), Elster (1979), Laibson (1997, 1998). Hence we need to estimate two discount parameters: a short-run present bias factor ($\beta<1$) and a long-term discount factor ($\delta$). When $\beta=1$, the model reduces to the single parameter exponential discounting model. Fortunately, the presence of bonuses at different frequencies (quarterly and annual) provides us instruments necessary to identify both the discount parameters.

There are three specific modeling and estimation challenges in the structural estimation of response to compensation plans, especially those with quotas and bonuses. First, in a typical structural model, one observes the agent's action in response to the firm's action. For example in a consumer response model, one observes consumers’ choices in response to the firm's choice of marketing mix such as price, advertising or sales promotion. In contrast, for a sales force response model, one does not observe the actions of the sales force, i.e., the exerted effort. One only observes the outcome of the agent's effort, i.e., sales, which is correlated (but not a one-to-one mapping) with effort. Hence one has to make an inference about the agent's action (effort) that leads to sales from the observed realized sales. This requires some modeling assumptions on the link between sales and effort.3

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3 The issue has parallels in empirical channel response models. For example, Sudhir (2001) makes an inference about manufacturer actions (wholesale prices) from the observed retail price and sales to infer competition between manufacturers.
A second challenge is that unlike marketing mix variables that change over time, the compensation plan remains stationary over at least a year. How can one identify how the sales force will respond to a compensation plan, when there is no variability in the plan? Here we draw on an empirical insight from Steenburgh (2008) that can help identify the sales force response, when the compensation plan involves payments for reaching quotas. The insight is that in any given period, a sales agent's optimal effort depends on her state: how close the person is to achieving her quota. A sales agent may find it optimal to reduce effort when she is close or very far from achieving quota, but may stretch herself to reach the quota, when she has a moderate chance of achieving this quota. This implies that the optimal level of effort (and therefore sales) would vary from period to period as a function of the agent's state (distance to quota).

A third issue follows from the discussion of the second. While quotas enable identification of sales force response, it also induces inter-temporal dynamics in optimal sales force response behavior. An agent has to be concerned not just with the current payoff by expending effort, but the future payoff that she can obtain by being in a more favorable state that can facilitate in obtaining a bonus. This implies that the estimated structural model needs to account for forward-looking behavior on the part of sales agents. This requires a dynamic structural model.

We estimate the dynamic structural model of sales force response to various features of the compensation plan using sales force output and compensation data from a Fortune 500 firm that sells office durable goods. This firm used Plan F in figure 1b. In addition, the quotas and bonuses are provided at two different frequencies: quarterly and annual. As the compensation structure of the focal firm features almost all dimensions in typically used compensation plans, we observe how the sales force responds to these different dimensions of the plan. This rich plan provides us two key benefits: First, the presence of bonuses allows us to identify and estimate discount factors, which would otherwise be impossible. Further, since the bonuses are at different frequencies, we are able to estimate discount factors (present bias factor and long-term discount factor) in a hyperbolic discounting model. Second, even though theoretically one can perform counterfactuals of any type of compensation plan if we can estimate structural parameters (other than discount factors) for a sales person with a less rich compensation plan, an analyst or manager should have greater
faith in the counterfactuals, based on parameters that were estimated from observed responses to different elements of the compensation plan.

We conclude the introduction with a numerical example that clarifies (1) how bonuses can efficiently stretch sales people to exert more effort for a given level of compensation; (2) how a person’s distance to quota can induce heterogeneity in effort. Let the utility function of the salesperson that trades off effort (e) and income from sales (s), who has sold S units at the beginning of the new period be:

$$U(s, e, S) = -de^2 + rs + B \cdot 1_{s \geq Q}$$

where –d is the disutility parameter and r is the commission rate (d>0, r>0) and B is the bonus obtained upon reaching quota (Q). For simplicity, assume a one-to-one-mapping between sales and effort, i.e., s=e.

We begin with the case where all agents that are identical in their distance to quota in the current period, specifically where S=0, i.e., agents have sold nothing thus far. Consider the pure commission case with no bonus where d=1, r=10 and B=0. In this case, the optimal effort is e*=5. For the bonus case, assume Q=10, and B=30. In this case, the optimal effort is higher at e*=10. The compensation cost to the firm is $130. To achieve the same level of effort from a pure commission plan, the commission rate r has to be increased to 20 and would cost the firm $200. Figure 2a illustrates these results graphically. Thus the quota-bonus plan is more efficient in obtaining the same level of sales.

Next let us consider the scenario where d=2, r=10, Q=10 and B=30, where agents are different in their distance to quotas such as S=0 (far away from quota), S=5 (moderately close to quota) and S=7 (close to quota). Figure 2b shows the results graphically, with maximum effort of e* when S=5, relative to when S=3 or S=7.

For an inter-temporal model, where bonuses occur every few periods, the decision to stretch to obtain a bonus would depend on how close to bonus the sales person would be in the bonus period. But this also means that the sales person needs to look forward dynamically when exerting effort in earlier periods in order to be in a "good" state to reach the quota and receive a bonus in the bonus periods.
The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 summarizes the institutional details of the firm and data used for the empirical analysis. We present the model and the estimation methodology in sections 4 and 5. Section 6 discusses the estimation results and the counterfactual analysis. Section 7 concludes.

2. Related Literature

The literature review is in two parts: We begin with the discussion of the literature relating to the substantive issue of the role of nonlinearities such as quotas and bonuses in compensation plans. Following this, we discuss the empirical literature on structural estimation of worker productivity.

In the theoretical literature, Basu et al. (1985) apply the principal agent framework of Holmstrom (1979) and demonstrate that a combination of salary plus commission (usually nonlinear with respect to sales) will be optimal. Rao (1990) also shows a similar result on the optimality of nonlinear compensation plans. However, Holmstrom and Milgrom (1987) and Lal and Srinivasan (1993) question the need for nonlinear compensation schemes. Using the specific assumptions of linear exponential utility and normal errors (LEN) they show that a linear commission incentive scheme can achieve the best possible outcomes for the firm.

Yet, why do most firms have quota based compensation plans? Why are compensation plans nonlinear? Raju and Srinivasan (1996) suggest that even though a commission over quota plan may not be theoretically optimal, they provide the best compromise between efficiency and ease of implementation. Others argue that quota based plans offer high powered incentives that can motivate salespeople to work harder (e.g., Darmon 1997). Park (1995) and Kim (1997) demonstrate that a quota-bonus plan may lead to the first-best outcome, but in their framework, quota-bonus plan is just one of many possible plans that lead to first best outcomes. Oyer (2000) shows that when participation constraints are not binding, a quota-bonus plan with linear commissions beyond quotas can be uniquely optimal because it can concentrate the compensation in the region of effort where the marginal revenue from effort minus the cost of compensation is maximized.
There is limited empirical work addressing this issue. Based on an analysis of aggregate sales across different industries in different quarters, Oyer (1998) concludes that the negative effects of quota based plans encouraging sales people to maneuver the timing of orders are greater than the benefits obtained from more effort. Steenburgh (2008) questions whether aggregate data can be used to reach this conclusion. Using individual sales performance data from the same firm used in this study (utilizes compensation plan F in figure 1b), he finds that the net improvement in revenues from effort dominates the inefficiencies induced by inter-temporal dynamic considerations.

Our work is related to several static structural models of worker behavior such as Ferrall and Shearer (1999) and Paarsch and Shearer (2000), who endogenize the optimal contract choice of the firm, given linear contracts. In contrast to these papers, we seek to understand the response to nonlinear incentives, which require us to model the dynamic response of sales agents. However, we do not model the contract choice, because we do not have data on selection across contracts.

Copeland and Monnet (2008) estimate a dynamic structural model of worker productivity in a check-sorting environment with nonlinear incentives; unlike sales force productivity, there is limited unobserved uncertainty in check sorting productivity. Much of the variation in productivity here can be explained by observed characteristics such as machine breakdowns etc.

A contemporaneous paper by Misra and Nair (2009) on sales force compensation is closest to this paper in methods and substantive context. In terms of methodology, both papers use the two-step estimation technique; however our paper innovates on two key dimensions. First, we accommodate latent class heterogeneity within the two-step estimation framework—an issue that has been an econometric challenge for the literature for close to two decades. Misra and Nair sidestep the unobserved heterogeneity issue by estimating each sales person’s utility function separately.\(^4\) Second, unlike Misra and Nair, who assume discount factors, we contribute to the broader dynamic structural modeling

\(^4\) This approach to accommodate heterogeneity is similar to the estimation of individual level utility functions in conjoint analysis or scanner panel data, by using a large number of observations related to a particular individual. Further, the approach requires that sales people will exert effort equally across all customers---an assumption they show is valid in their data, but unlikely to hold in general. Our method of using latent classes works in the more common situation where there are limited observations per individual.
literature by estimating discount factors (specifically hyperbolic discount factors) using field data. Two characteristics of the data facilitate identification of discount factors. First, bonus payoffs are in the future and are excluded from current payoffs of the non-bonus months. This helps us estimate discount factors. Second, bonuses are at two different temporal distances, quarterly and annual, and as a result this helps us estimate both the present bias factor and the long-term discount factor in a hyperbolic discounting model.

The key substantive difference is that we focus on the value of quotas with bonuses (plan F in figure 1b), while Misra and Nair consider the role of quotas with floors and ceilings on commissions (plan D in figure 1b). Misra and Nair conclude that quotas reduce performance. This is because of two characteristics of their quotas: First, the quota ceiling (beyond which sales people receive zero additional compensation) limits the effort of the most productive sales people who would normally have exceeded the ceiling. Second, the company followed an explicit policy of ratcheting quotas based on past productivity, which reduced incentives of sales people to work hard in any given period, because it makes future rewards more difficult to attain due to ratcheted higher quotas. In contrast, we find that quotas coupled with bonuses enhance performance. In the plan we consider, the company offers extra overachievement commissions for exceeding quotas and use a group quota updating procedure that minimizes ratcheting effects. Thus the two papers offer complementary perspectives that enhance our understanding about how quotas impact performance.

3. Institutional Details and Model-Free Evidence of Dynamics

We begin by describing the details of the compensation plan. Then we provide model-free evidence to highlight the salient characteristics of the data to be modeled.

3.1. The Compensation Plan

The focal firm under study is a highly regarded multinational Fortune 500 company that sells durable office products. Products range from simple machines targeted for local small businesses with a price tag of less than a thousand dollars to highly sophisticated systems of machinery for multinational companies and government agencies costing several hundreds of thousands of dollars. In addition to its line of products, the firm also provides services such as equipment maintenance and system consulting. The sales force is directly
employed by the focal firm. While evaluating performance, only one sales agent receives credit for a unit of sales and the firm traditionally does not encourage group work or team cooperation among the sales force.

Our analysis focuses on data from 348 sales people during the three year period 1999-2001, for whom we have complete data. The firm’s compensation structure follows the pattern in Plan F of Figure 1b and the details of the compensation schedule for the period of analysis are described in Table 1. Every month, sales people receive a fixed monthly salary and commissions per volume of sales generated in that month. There are no caps on revenues for which an agent could obtain commissions or overachievement commissions. In the first three quarters, a quarterly lump-sum bonus is given when each of the quarterly quotas are met. And at the end of the year (i.e., end of the fourth quarter) an annual lump-sum bonus is paid if the annual quota is met and an overachievement commission is given for excess revenues beyond the annual quota. The single most important element, in terms of performance evaluation within the firm, is the annual quota; i.e., the firm views a salesperson as having a successful year if the annual quota is met. Overall, for a salesperson that meets all quotas, the salary component will be roughly 30% of total compensation.

3.2 Model Free Analysis

We consider three features of the data that can inform model development. First, we consider the nature of seasonality in the data. Second, we look at the evidence of forward looking behavior induced by bonuses and hence the need to develop a dynamic model. Finally, we test for the possibility of sales substitution across quarters by sales agents.

3.2.1 Seasonality

The descriptive statistics of the data is in Table 2. Figure 3a shows the average revenue generated for each calendar month by the sales force in the study. Sales is relatively high at the end of first three quarters (March, June, September) and much higher in the last quarter (December). At first glance, this suggests that sales agents are highly responsive to quotas and bonuses.

But could the higher sales at the end of quarters be due to higher demand during these periods, rather than just due to bonuses? Fortunately, an internal budgeting process, which is used to set quarterly and annual quotas, provides us with an instrument. This
instrument is the “monthly allocated quota” used internally by the firm to construct quarterly and annual quotas; \( \text{Quota}_{1\text{st quarter}} = M_1 + M_2 + M_3, \) \( \text{Quota}_{\text{annual}} = \sum_{t=1}^{12} M_t, \) where \( M_t \) is the monthly allocated quota for month \( t. \) Monthly allocated quotas are constructed by managers, with assistance from sales people, but also using other micro information such as past performance and future expectations for regions, and macro information such as economic forecasts from corporate headquarters. To avoid ratcheting problems, the managers are sensitive to not update quotas based purely on past performance of an individual sales person, but also account for performance across the region. Figure 3b shows the average monthly allocated quotas for the sales force. One can see that the shape of peaks and dips coincide with that of the monthly average revenue in figure 3a.

In addition to the direct sales force (the focus of our study), the focal firm also utilizes an indirect sales force who is compensated on a purely commission basis. Figure 3c shows the monthly revenue of the indirect sales force. The shape of the monthly revenue for the indirect sales force also mirrors that of the direct sales force and the monthly allocated quotas suggesting demand fluctuates from period to period independent of compensation. While the monthly allocated quotas has the advantage that they are more precisely related to each sales person’s region as well as firm expectations of seasonality, the indirect sales force numbers have the advantage of taking into account realized demand effects that were not known at the time of quota settings. We therefore use both these variables as controls for seasonality to isolate the effects of quotas and bonuses as opposed to pure demand seasonality effects. Without these controls, we may exaggerate the impact of quota-bonuses on sales.

Why should we expect sales to be so seasonal and coincide by quarter? Given that the focal firm’s products are a B2B business selling discretionary good whose timing of purchase are also flexible, one possibility is that spending may expand during the month in which the fiscal year ends for its customers due to accounting procedures employed by clients. Figure 3d shows the distribution of fiscal year-ends across months for the year 2000

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5 The firm utilizes the indirect sales channel to perform sales activity in rural areas. The indirect sales channel is composed of approximately eight hundred small representative firms that resell the focal firm’s products. The focal firm does not directly compensate the sales force of these rep firms. The representative firms simply receive commissions from the focal firm, and they in turn give a percentage of that commission to the sales agents. This kind of compensation is the focus of Jiang and Palmatier (2010).
Indeed, we see that over 66% of the firms have a December fiscal year-end, which might explain a significant boost in sales during that month. End of quarter months also have peaks relative to other months, but the peaks are not as large as for December. Given that we have more direct proxies for seasonality in the form of monthly allocated quotas and indirect sales force revenues, we do not use the fiscal year-end information in our analysis.

3.2.2: Forward Looking Behavior induced by quarterly and annual bonuses

We begin by showing scatter plots and the best fitting nonparametric smoothed polynomial (and its 95% confidence interval) of normalized sales revenues (normalized by monthly allocated quotas) in the quarterly bonus months (March, June, September, December) against percentage of quota attained by the previous month in figure 4a. For March, June and September, the x axis is the percentage of quarterly quota completed (%QQ), while for December, the x axis is the percentage of annual quota completed (%AQ). The vertical line shows the %QQ and %AQ at which the salespeople on average achieve their monthly allocated quotas.

Two key elements stand out from the graphs. First, across the board one does not see a massive reduction in effort when salespeople get closer to achieving their quota. This must be partly because of the overachievement commission rate. Second, we can detect evidence of forward looking behavior. Early in the year (March and June), salespeople achieve their monthly targets, even at low levels of %QQ, while later in the year, they achieve their monthly targets only at fairly high levels of %QQ or %AQ. In March and June, even when there is little chance of achieving the quarterly quota, the sales person puts in effort to achieve quota. On its own, this might simply mean that salespeople are trying to obtain commissions. However, when seen in tandem with the fact that sales agents in similar states in September or December do not seek to accomplish their targets, one can infer that the sales person is forward looking. Early in the year, even if they are below targets, they still have hopes of receiving the large annual bonus by working hard. However, later in the year, such chances becomes less likely and sales people respond by reducing their effort. This is clear evidence of forward looking behavior. This also suggests an instrument for estimating discount factors. Annual bonuses should have little impact on current payoffs in
March, June or September, but only on future payoffs. Hence distance to annual quotas can serve as an instrument for estimating discount factors.

The next set of graphs presented in Figure 4b, shows the same relationship in the pre-bonus months (February, May, August and November). Here a new characteristic stands out. In the early months, February, May and even August, at all levels of %QQ, the salesperson on average sells above the monthly allocated quota. This is because hard work (and some luck in the form of positive shocks) may give a reasonable chance of attaining the smaller quarterly targets. However, in November, only at a very high level of %AQ, does the salesperson sell above the monthly allocated quota, because one has a very limited chance of making up the large gap in just two months.

This suggests again that in the pre-quarterly bonus months, the quarterly bonus in the future does have an impact on behavior, even though it does not have an immediate impact on the current payoff; again indicating forward looking behavior. However, the differential way in which the sales person responds to temporally different bonuses at any given point in time, provides an opportunity to identify the two discount factors in hyperbolic discounting: a long-term discount factor and a present bias factor.

This preliminary evidence also suggests a natural question for managers. Should the large annual bonus be split into a quarterly bonus (as in other months) and an annual bonus? This can prevent sales people from giving up in November, even if they do not have a chance of reaching the annual quota. At the same time, the cost of such a quarterly quota would be that early in the year, the incentive to stretch after reaching quarterly quotas would be reduced. How these two issues tradeoff is an empirical question, which we subsequently address in the counterfactual analysis.

3.2.3. Sales substitution across months

One possibility is that sales people giving up at the end of the quarter may be doing so to increase the odds of hitting quotas in subsequent quarters by simply not booking the sales in the current quarter. If this were true, then one should see a negative linkage between sales in month t and month t+1; and especially between the last month of a quarter and first month of the next quarter.
To test this, we report the results of a regression where sales in month $t$ is regressed against sales in month $t-1$ in table 3. Specifically to see if there might be any borrowing effects from the last month of a bonus period to the first period of the next bonus period we include a separate variable for the first month of each quarter. We also include individual level fixed effects. We do not find any evidence of substitution across months; in fact, we only find a positive relationship between the lagged sales, suggesting good months continue to be followed by good months. In particular, we do not find any significant coefficient for the first month of quarter (as in Model 2), suggesting little substitution across quarters. The first month of quarter also does not have any significant impact even if we separated the effect for people who are “way off target” in the last month and have therefore the greatest incentives to postpone purchases, as seen in Model 3. We defined “way off target” as those whose previous quarter sales were less than 50% of their quota. The results were robust and did not vary with alternative definitions of “way off target”.

4. Model

Based on the model-free evidence, we build a dynamic model of sales force response to the quota-based compensation scheme. The timing of the model is as follows:

1. At the beginning of each year, firm chooses the annual compensation plan.
2. Each month, agents observe their current state and exert effort in a dynamically optimal manner.
3. An idiosyncratic sales shock is realized; the shock plus agent's effort determines the agent's realized sales for the period. Agent receives compensation.
4. The realized sales of the current period affect the agent's state of the next period. Steps 2-3 are repeated each month until the end of the year and Steps 1-3 are repeated over the years.

A major challenge in building a model of sales force response to incentive compensation is that we do not observe effort, the agent’s choice variable. Given the unobservability of effort, we explicitly outline the key assumptions necessary to make

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6 We thank an anonymous reviewer for suggesting that we perform this test.

7 The positive correlation could be due to unobserved heterogeneity across salespeople. To test this, we estimate the same regression in Table 3, for each segment based on the segment classification from the final estimated model. Lagged effects then become insignificant, consistent with the notion that positive state dependence here is due to not accounting for unobserved heterogeneity.
progress. To appreciate the role of the assumptions, let us consider a hypothetical scenario, where one can observe effort. Then we would estimate the following two equation model - one for sales ($S$), and one for effort ($e$) as follows:

\[ S_t = \beta e_t + \gamma z^D_t + \nu_t + \epsilon_t \]

\[ e_t = h(z^E_t, z^D_t, \nu_t, \mu_t, \gamma) + \omega_t \]

Here $z^D_t$ are sales shifters, and $z^E_t$ are effort shifters. $\nu_t$ are unobserved shocks to the researchers that can be observed by the sales agents. $\epsilon_t$ and $\omega_t$ are the sales and productivity shocks respectively. $h$ is a function (that solves a static or dynamic model) that maps the observable shifters to effort.

When effort is unobserved, the only way we can make progress is to solve for the effort function based on observable characteristics. This would require us to assume away $\nu_t$ and $\omega_t$, i.e., we rule out potential endogeneity effects and productivity shocks. In addition, we require one more assumption that demand shifters cannot also impact effort, because it would be impossible to separate out the direct impact on sales and indirect impact of sales through effort. Thus to make progress, we require three key assumptions: (1) rule out potential endogeneity effects due to shocks that are unobservable to the researcher (but not to agent); (2) rule out productivity shocks; and (3) a variable may impact sales either directly or indirectly through effort, but not both.

We now describe the model in five parts: (i) the compensation plan (ii) the sales agent’s utility function (iii) the state transitions (iv) effort as a function of state variables and (v) the optimal effort choice by the sales agent.

### 4.1. Sales Response Model

We model the link between sales revenue ($S_{it}$) for salesperson $i$ at time $t$ and the effort ($e_{it}$) when faced with sales person $i$’s territory and own characteristics ($z_{it}, x_{it}$) as follows:

\[ S_{it} = s_{it}(e_{it}, z_{it}, x_{it}) + \epsilon_{it} \quad \cdots \quad (1) \]

where $\epsilon_{it}$ is an additive sales revenue shock that is not anticipated by the salesperson when choosing effort levels.
A special challenge in estimating a sales response model with respect to effort is that effort is not observed. This implies that typical endogeneity issues in choice of marketing mix such as price and advertising, where the marketing mix choice is correlated with certain market characteristics that are observable to the decision maker, but unobservable to the researcher, cannot be accounted for. We also require additional restrictions on the function $s_{it}$. First, we make the reasonable assumption that $s_{it}$ is a parametric monotonically increasing function of unobservable effort $e_{it}$. Second, we partition the territory characteristics variables into two groups: one set of variables $x_{it}$, can impact sales revenue only through their impact on effort; but no direct impact on sales revenue; a second set of variables $z_{it}$, can only have a direct impact on sales revenues, but no impact on effort. This is not an innocuous assumption, but necessary, because one cannot separately identify both the direct effect of a variable and its indirect effect through effort without observing effort—a characteristic of sales force compensation environments.

Examples of variables that affect sales person effort are “% Quota” (%AQ, %QQ) and sales person characteristics such as “tenure with the firm”. We believe it is reasonable to assume that these variables itself do not have a direct impact on sales revenues. Examples of variables that only have a direct effect are proxies for seasonal demand; i.e., periods with high seasonal demand can have higher levels of sales in a period irrespective of effort levels. For example, if sales revenue seasonality is due to a proportion of revenues arising from contract renewals etc., then one could have higher sales in such a period without more effort. More generally, it is possible that in periods of high demand, sales people can also exert higher levels of effort by responding differently to state variables. To accommodate the idea that seasonality can have both a direct and indirect impact (within a context where effort is not observed), we allow for the effort function in response to $x_{it}$ to vary across different months; hence the function is subscripted by $m$. This allows us to satisfy the restriction that variables can only have direct effect on sales or indirect effect through effort, but not both, yet accommodate different effort responses to seasonality.

Thus overall, we use the following model of sales response:

$$S_{it} = \alpha_i z_{it} + e_{im(t)}(x_{it}) + \varepsilon_{it} \quad \ldots \quad (2)$$

4.2 Compensation Plan
The compensation plan has three components. They are: (1) the monthly salary \( w_{it} \), (ii) end-of-quarter bonus, \( B_{iqt} \) for achieving the corresponding quarterly quota \( Q_{iqt} \), and end-of-year bonus \( B_{iyt} \) for achieving the corresponding annual quota \( Q_{iyt} \) (3) commission rate \( r_{it} \) per dollar worth of sales and an overachievement commission rate, \( r_{it}' \) given at the end of the year for sales over and above the annual quota for each individual \( i \) at time \( t \). We represent the compensation plan for a salesperson \( i \) by the vector \( \psi_{it} = \{w_{it}, Q_{iqt}, Q_{iyt}, B_{iqt}, B_{iyt}, r_{it}, r_{it}'\} \).

4.3 Sales person’s per-period utility

In each period \( t \), sales person \( i \) receives positive utility of wealth \( W_{it} \) earned based on realized sales and a disutility \( C(e_{it}; \theta_i) \) from exerting effort \( e_{it} \). Thus the utility function is defined as:

\[
U(e_{it}, S_{it}; \psi_{it}, \theta_i, \gamma_i) = \mathbb{E}[W(S_{it}; \psi_{it})] - \gamma_i \text{ var}[W(S_{it}; \psi_{it})] - C(e_{it}; \theta_i) \quad ^8
\]

where \( \gamma_i \) and \( \theta_i \) are each the risk aversion and disutility parameters respectively for salesperson \( i \).

Given the sales levels, and the compensation plan, the wealth for individual \( i \), \( W_{it} \) can be computed. \( W_{it} \) arises from four components, the per period salary component \( w_{it} \), the lump-sum bonus component \( B_{it} \), the commission component \( C_{it} \), and the overachievement commission component \( OC_{it} \). The detailed expressions of wealth is as follows,

\[
W_{it} = w_{it} + B_{it} + C_{it} + OC_{it}
\]

\[
B_{it} = 1_{qt} \cdot \frac{1}{1 + s_{it}(e_{it}(x_{it}), z_{it}; \alpha_i) + \varepsilon_{it}} \cdot B_{qt} + 1_{yt} \cdot \frac{1}{1 + s_{it}(e_{it}(x_{it}), z_{it}; \alpha_i) + \varepsilon_{it}} \cdot B_{yt}
\]

\[
C_{it} = r_{it} \cdot s_{it}(e_{it}(x_{it}), z_{it}; \alpha_i) + \varepsilon_{it}
\]

\[
OC_{it} = 1_{yt} \cdot \frac{1}{1 + s_{it}(e_{it}(x_{it}), z_{it}; \alpha_i) + \varepsilon_{it}} \cdot r_{it}' \cdot s_{it}(e_{it}(x_{it}), z_{it}; \alpha_i) + \varepsilon_{it} - Q_{iyt}
\]

\[
8 \quad \text{In the case of the CARA utility function (exponential utility function) with normal errors and a linear compensation plan, this functional form represents the certainty equivalent utility of the agent. Here we consider the utility function to be a second order approximation to a general concave utility function.}
\]
where \( x_{i1t} \) and \( x_{i2t} \) are the percentage of quarterly and annual quotas completed respectively by salesperson \( i \) until time \( t \). \( 1_{qt} \) and \( 1_{yt} \) are indicators for whether time \( t \) is a quarterly or annual bonus period.

In our empirical analysis, we use a quadratic functional form for the disutility function; specifically, \( C(e; \theta) = \theta e^2 \). Thus the set of structural parameters of the salesperson that needs to be estimated are \( \Omega_i = (\theta_i, \gamma_i) \).

4.4 State Variables

As discussed, the nonlinearity of the compensation scheme with quotas and bonuses introduces dynamics into the sales agent's behavior because there is an additional tradeoff between the disutility of effort today and a higher probability of lump-sum bonus and overachievement commissions tomorrow. To incorporate the dynamics of the model we consider the following stochastic state variables, the percentage of annual quota completed, the percentage of quarterly quota completed, and period type. These state variables evolve as follows:

1. Percentage of quarterly quota completed (%AQ)

\[
x_{i1t} = \begin{cases} 
0, & \text{if } t \text{ is start of quarterly quota period} \\
x_{i1(t-1)} + \frac{S_{i(t-1)}}{Q_{iqt}}, & \text{otherwise}
\end{cases}
\]

2. Percentage of annual quota completed (%QQ)

\[
x_{i2t} = \begin{cases} 
0, & \text{if } t \text{ is start of annual quota period} \\
x_{i2(t-1)} + \frac{S_{i(t-1)}}{Q_{iyt}}, & \text{otherwise}
\end{cases}
\]

Another state variable, the period type (month), would evolve deterministically as follows

\[
m_t = \begin{cases} 
1, & \text{if } t = 1 + 12k, \text{ where } k = 0, 1, 2, \ldots \\
m_{(t-1)} + 1, & \text{otherwise}
\end{cases}
\]

We also observe individual characteristic, specifically tenure with the focal firm \( (\tau) \), and therefore use it as an individual state variable that impacts effort. These state variables are collected in a state vector \( x_{it} = \{x_{i1t}, x_{i2t}, m_t, \tau_i\} \).
4.5 Optimal Choice of Effort

Given the parameters of the compensation scheme $\psi$, and the state variables and their transitions, each sales agent would choose an effort level conditional on her states to maximize the discounted stream of expected future utility flow. Alternatively, if this value function is below the reservation wage, the salesperson may choose to leave the firm.

The stream of utility flow, under the optimal effort policy function, conditional on staying at the firm, and the behavioral notion of quasi-hyperbolic discounting can be represented by a value function,

$$V(x; \psi, \Omega) = \max_e U(e, x; \psi, \Omega) + \beta \delta E \left[ \max_e U(e, x'; \psi, \Omega) + \delta E \left[ \max_e U(e, x''; \psi, \Omega) + \ldots \right] \right]$$

or differently put as,

$$V(x; \psi, \Omega) = \max_e U(e, x; \psi, \Omega) + \beta \delta E[V^\delta(x'; \psi, \Omega)]$$

where $\Omega$ is the primitives or the structural parameters of the underlying utility function, specifically the disutility parameter $\theta$ and the risk aversion parameter $\gamma$. $\beta$ and $\delta$ are the discount parameters. The long-term discount factor is $\delta$, the short-term discount factor is $\beta \delta$, where $\beta<1$ represents the present bias in hyperbolic discounting. If $\beta=1$, the model reduces to a single parameter exponential discount model. The expectation of the value function is taken with respect to both the present and future sales shocks.

5. Estimation

Traditionally, the nested fixed-point algorithm (NFXP) developed by Rust (1987) is used to estimate dynamic models. However, NFXP estimators are computationally burdensome as one has to solve the dynamic program numerically over each guess of the parameter space for every iteration. The two-step estimation first introduced by Hotz and Miller (1993) and extended by Bajari, Benkard, and Levin (2007) can serve to reduce the computation burden. In this approach, the model estimation proceeds in two steps. In the first step, the conditional choice probabilities of choosing a certain action as a function of state variables are estimated in a flexible non-parametric manner. Then, in the second step,
these conditional choice probabilities are used to estimate the structural parameters of the sales agent's utility function. For this approach to work, it is critical that the conditional choice probabilities are estimated accurately in the first step.

Until recently, it was believed that the accurate estimation of conditional choice probabilities for an agent is impractical when there is unobserved heterogeneity. Arcidiacono and Miller (2010) have proposed an EM–Algorithm based approach to accommodate unobserved heterogeneity in the first step of the two step estimation procedure. We provide one of the first applications of this approach – illustrating the empirical validity of the approach in practical applications. We now discuss the details of the two step estimation procedure.

5.1 Step 1

In this step, we need to estimate a flexible non-parametric mapping between observable states and actions of the sales person; this requires a non-parametric model of the monthly effort function $e_{im(t)}(x_it)$, that links effort and state in equation (2). We model the effort function non-parametrically as a combination of basis functions of the state variables. Thus the non-parametric effort function for each month is:

$$e_{im(t)} = \sum_{\ell=1}^{L} \rho_{\ell}(x_it) \cdot \lambda_{im(t)\ell} \quad ... \quad (3)$$

Where the $\ell^{th}$ basis function is $\rho_{\ell}(x_it)$. In this application, the $\ell^{th}$ basis function is the $\ell^{th}$ order Chebyshev polynomial.

From equations (2) – (3) we have the following sales response function to estimate.

$$S_{it} = \alpha_i z_{it} + \sum_{\ell=1}^{L} \rho_{\ell}(x_it) \cdot \lambda_{im(t)\ell} + \epsilon_{it}$$

For $z_{it}$, we use two variables: (1) an internal metric of monthly expected revenues for salesperson i, (2) the revenues of the indirect sales force.

The monthly expected revenues from the firm’s internal budgeting records take into account not only general territory characteristics for the time period, but also portions of sales that are likely to be generated with limited effort, because of contract renewals etc. The
revenues from the indirect sales force capture market shocks not anticipated when the monthly expected revenues were calculated, but may affect sales across the board, independent of the nonlinear nature of the compensation plan. We assume that the revenue shocks ($\epsilon_{it}$), come from a normal distribution that are i.i.d. within the same time period but whose variance differs across time periods; i.e., thus the variance in the revenue shocks will be higher in high demand periods than in low demand periods.

If one could estimate the sales response and effort response function at the level of each individual, we can simply obtain the individual level parameters of the effort and sales policy function by maximizing the log likelihood of the sample such as

$$
\hat{\Theta}_t = \text{argmax} \sum_{t=1}^{T} \log \left( L_{it} \left( S_{it} - \alpha_i z_{it} - \sum_{\ell=1}^{L} \rho_{\ell}(x_{it}) \cdot \lambda_{im(t)\ell} \right) \right)
$$

where the vector $\hat{\Theta}_t = \{\alpha_t, \lambda_t, \sigma_t\}$ contains the set of parameters of the sales response and effort policy functions and the distribution of sales shocks, where

$$
L_{it}(\epsilon) = \frac{1}{\sigma_{it}\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\epsilon}{\sigma_{it}})^2} \ldots (4)
$$

We accommodate unobserved heterogeneity by allowing for discrete segments. Assume that sales person $i$ belongs to one of $K$ segments, $k \in \{1, \ldots, K\}$ with segment probabilities $q_i=\{q_{i1}, \ldots, q_{iK}\}$. Let the population probability of being in segment $k$ be $\pi_k$. Let $L(S_{it}|x_{it}, z_{it}, k; \Theta_k)$ be the likelihood of individual $i$'s sales being $S_{it}$ at time $t$, conditional on the observables $x_{it}, z_{it}$, and the unobservable segment $k$, given segment parameters $\Theta_k$. Then the likelihood of observing sales history $S_i$ over the time period $t=1\ldots T$, given the observable history $x_i, z_i$, and the unobservable segment $k$ is given by:

$$
L_k(S_i|x_i, z_i; \Theta_k, \pi_k) = \prod_{t=1}^{T} q_{ik} L_{ikt} \ldots (5)
$$

where $L_{ikt} = L(S_{it}|x_{it}, z_{it}, k; \Theta_k)$. As noted earlier we assume the distribution of the revenue shocks to be normally distributed and hence use the normal likelihood for equation (5) as in equation (4). The parameter $\Theta_k = \{\alpha_{k1}, \lambda_{k1}, \ldots, \lambda_{k12}, \sigma_k\}$ is the vector of segment level parameters of the sales response and effort policy function for each month where each
\( \lambda_{km(t)} \) is the parameters that index the effort policy for segment \( k \) at month \( m \), and \( \sigma_k = \{ \sigma_{k1}, \ldots, \sigma_{k12} \} \) is the vector of parameters for the distribution of the revenue shocks for segment \( k \).

By summing over all of the unobserved states \( k \in \{1, \ldots, K\} \), we obtain the overall likelihood of individual \( i \):

\[
L(S_i \mid x_i, z_i; \Theta, \pi) = \sum_{k=1}^{K} q_{ik} L_k(S_i \mid x_i, z_i; \Theta_k, \pi_k)
\]

and hence the log-likelihood over the \( N \) sample of individuals becomes

\[
\sum_{i=1}^{N} \log (L(y_i \mid s_i, z_i; \Theta, \pi)) = \sum_{i=1}^{N} \log (\sum_{k=1}^{K} q_{ik} \prod_{t=1}^{T} L_{ikt}) \quad \ldots \quad (6)
\]

Directly maximizing the log-likelihood in (6) is computationally infeasible because the function is not additively separable so we take the approach of Arcidiacono and Jones (2003) and Arcidiacono and Miller (2010) to iteratively maximize the expected log-likelihood in equation (7)

\[
\sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} q_{ik} \log L(S_{it} \mid x_{it}, z_{it}; \Theta_k) \quad \ldots \quad (7)
\]

where \( q_{ik} \) is formally defined below as the probability that individual \( i \) is of segment type \( k \) given parameters values \( \Theta \), segment probabilities \( \pi \), and conditional on all of the observed data of individual \( i \).

\[
\Pr(k \mid s_i, x_i, z_i; \Theta, \pi) = q_{ik} = \frac{L_k(s_i \mid x_i, z_i; \Theta_k, \pi_k)}{L(S_i \mid x_i, z_i; \Theta, \pi)} \quad \ldots \quad (8)
\]

The iterative process is as follows.

We start with an initial guess of the parameters \( \Theta^0 \) and \( \pi^0 \). Natural candidate for such starting values would be to obtain the parameters from a model without unobserved heterogeneity and slightly perturbing those values.\(^9\) Given the parameters \( \{ \Theta^m, \pi^m \} \) from the \( m^{th} \) iteration, the update of the \( (m+1)^{th} \) iteration is as follows

---

\(^9\) We started the initial values from one tenth of the standard error from the parameter values obtain from a
(a) Compute \( q_{ik}^{(m+1)} \) using equation (8) with \( \Theta^m \) and \( \pi^m \)

(b) Obtain \( \Theta^m \) by maximizing (7) evaluated at \( q_{ik}^{(m+1)} \)

(c) Update \( \pi^{(m+1)} \) by taking the average over the sample such that

\[
\pi_k^{(m+1)} = \frac{1}{N} \sum_{i=1}^{N} q_{ik}^{(m+1)}
\]

We would iterate (a) – (c) till convergence.

For the basis functions in the effort policy, we use Chebyshev polynomials of state variables to approximate effort.\(^{10}\) As a result, we obtain the vector of parameters that index these basis functions \( \lambda^\prime \)’s, the vector of parameters for the sales policy \( \alpha^\prime \)’s, and the parameters of the revenue shocks \( \sigma^\prime \)’s for each segment \( k \). Also we obtain the population segment probabilities for each segment. More formally,

\[
\hat{\Theta} = \{\hat{\Theta}_1, ..., \hat{\Theta}_K\} = (\hat{\alpha}_k, \hat{\lambda}_{k1}, ..., \hat{\lambda}_{km}, \hat{\sigma}_k) \\
\hat{\pi} = \{\hat{\pi}_1, ..., \hat{\pi}_K\}
\]

Therefore, we obtain the sales revenue function \( \hat{S}(\cdot) \) and effort policy function \( \hat{e}(\cdot) \) for each segment.

**5.2 Step 2**

The key idea of the two-step estimation is that in the 1st stage we observe the agent’s optimal actions. Using these observed optimal actions we are able to construct estimates of the value function, which enables us to estimate the primitives of the model that rationalize these optimal actions.

Let the value function of a representative agent at state \( x \) that follows an action profile \( e \), conditional on the compensation plan \( \psi \), the sales profile \( S \) and the primitives of the utility function \( \Omega \) be represented as

\[
V(x; e; \psi, S, \Omega) = \mathbb{E} \left( \sum_{t=0}^{T} D(t)U(e(x_t), x_t, \epsilon_t; \Omega) \left| x_0 = x; \psi, S, \Omega \right\} \right)
\]

\[
\text{... (9)}
\]

single segment model. The initial values of the segment probabilities were set equally across segments.

where \( D(t) = \begin{cases} 1, & \text{if } t = 0 \\ \beta^t, & \text{otherwise} \end{cases} \) is the hyperbolic discount factor, and the expectation operator would be over the present and future sales shock \( \varepsilon_t \).

Using the estimated sales and effort policy function and the distribution of the sales shocks in the first stage, we are able to forward simulate the actions of sales agents to obtain the estimate of the value function. The detailed simulation procedure is as follows.

(a) From initial state of \( x_t \) calculate the optimal actions as \( e(x_t) \)

(b) Draw sales shock \( \varepsilon_t \) from \( f(\varepsilon) \)

(c) Update state \( x_{t+1} \) using the realized sales \( s(e(x_t)) + \varepsilon_t \)

(d) Repeat (a) – (c) until \( t=T \)

By averaging the sum of the discounted stream of utility flow over multiple simulated paths we can get the estimate of the value function \( \widehat{V}(x; e(x); \psi, S, \Omega) \).

Let \( e^*(x) \) be any deviation policy from a set of feasible policies that is not identical to the optimal policy and, by using the same simulation method proposed above, let the corresponding estimate of the value function be called the sub-optimal value function \( \widehat{V}(x; e^*(x); \psi, S, \Omega) \). By definition since \( e(x) \) is the effort policy and thus at an optimum, then any deviations from this policy rule would generate value functions of less or equal value to that of the optimal level.

Let us define the difference in the two value functions as,

\[
Q(v; \psi, S, \Omega) = V(x; e(x); \psi, S, \Omega) - V(x; e^*(x); \psi, S, \Omega)
\]

where \( v \in \mathcal{V} \) denotes a particular \( \{x, e^*(x)\} \) combination. Then if \( e(x) \) is the optimal policy, the function \( Q(v; \psi, \Omega) \) would always have value of greater or equal to zero. Thus our estimate of the underlying structural parameters \( \Omega \) would satisfy,

\[
\widehat{\Omega} = \arg\min \int (\min\{Q(v; \psi, S, \Omega), 0\})^2 dH(v)
\]

11 For each segment, we drew four hundred simulation draws over each period and computed the value functions.

12 As indicated in Bajari, Benkard, and Levin (2007), there are multiple ways to draw these suboptimal policy rules. Although the method of selecting a particular perturbation will have implications for efficiency the only requirement necessary for consistency is that the distribution of these perturbations has sufficient support to yield identification. We chose to draw a deviation policy from a normal distribution with mean zero and quarter of the variance from the revenue shock distribution, i.e. \( e^*(s) = e(s) + \eta \)
where $H(v)$ is the distribution over the set $\mathcal{V}$ of inequalities. Our empirical counterpart to $Q(v; \psi, S, \Omega)$ would be $\bar{Q}(v; \psi, \hat{S}, \Omega) = \bar{V}(x; \hat{e}(x); \psi, \hat{S}, \Omega) - \bar{V}(x; e^*(x); \psi, \hat{S}, \Omega)$ and as a result our estimates of the structural parameters are obtained from minimizing the objective function in equation (10).\(^{13}\)

$$\frac{1}{N_1} \sum_{j=1}^{N_1} \left( \min\{\bar{Q}(v_j; \psi, \hat{S}, \Omega), 0\} \right)^2 \quad \cdots \quad (10)$$

The above procedure is performed for each segment with the segment specific effort policies obtained in Step1. This allows us to estimate the structural parameters for each segment.\(^{14}\)

5.3. Identification

We provide a brief and informal discussion of identification. Realized sales is a function of effort and additive sales shocks. Given multiple observations of sales at different states, we can separately identify non-parametrically the density of sales shocks and a deterministic function of effort. We make a parametric assumption about a strictly monotonic relationship between sales and effort because it is not possible to identify this relationship non-parametrically.

Further, we assume a deterministic (but highly flexible) relationship between effort and observable states (percentage of quarterly and annual quotas achieved by previous month) at the level of each segment. Thus variation in sales (which is monotonically linked to effort) as a function of these state over time helps identify the effort disutility parameter. The risk parameter is identified by differences in response to variations in wealth levels over time.

The discount factor is typically not identified in dynamic choice models because instruments affect both the current utility and future utility (Rust 1994; Magnac and Thesmar 2002). In our model, the state variables (%QQ and %AQ) serve as identifying instruments. The choice of effort in non-bonus periods has two distinguishable wealth effects; (1) the

\(^{13}\) We drew two hundred deviation strategies to construct the objective function and hence $N_1=200$.

\(^{14}\) We also estimated a model with a second set of moment inequalities to reflect the participation constraint that employees continued to work at a firm because they at least obtained a reservation value (normalized to zero); i.e., $\min(V(x; e(x); \psi, S, \Omega), 0)$. The constraint was non-binding and did not impact our estimates.
increase in current utility through the commissions and (2) the increase in future utility through bonuses. Without these future bonuses, a sales person would just simply solve the myopic first order condition every period to decide on the optimal level of effort. However, with the future bonuses, a sales person can choose to increase effort to increase the likelihood of bonus, conditional on the state variables and depending on how much she values the future, which by definition is the discount factor.

Further, having two different future bonuses, one in the near future (quarterly bonuses) and the other in the distant future (annual bonuses and overachievement commissions) with different state variables affecting them, help us identify the hyperbolic discount parameters. Indeed, the model-free evidence in Figure 4 shows different responses to annual and quarterly bonuses.

6. Results

We first report the first stage estimates of the effort policy function; then we report estimates of structural parameters of sales agents' utility functions from the second stage. In doing so, we perform a grid search over the hyperbolic discount parameters (betas and deltas) to estimate the discount factors. We then perform several counterfactual simulations to address the substantive questions we seek to answer.

6.1 First Stage Estimates

We estimated segment level effort policy functions by estimating the non-parametric relationship between sales and state variables through Chebyshev polynomials of the state variables. Using the Bayesian Information Criterion (BIC) for model selection, we choose a three-segment model. We do not report the parameter estimates of the effort policy themselves, given that the coefficients associated with the Chebyshev polynomials have no intuitive meaning. However, for intuition, we show graphs of the effort policy function for the three segments as a function of percentage annual quota (%AQ) for select months in Figure 5. %AQ is normalized across sales agents, such that 1 implies at quota and 0.9 indicates 10% below quota and 1.1 indicates 10% above quota.

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15 We fitted up to fourth order polynomials for each month’s effort policy function. The best fitting Chebyshev models based on BIC were either quadratic or cubic polynomials depending on the month. We chose the best fitting polynomial in further analysis.
Table 4 shows the share of the three segments and their descriptive characteristics. Segment 2 is the largest with a share of 41%; Segments 1 and 3 have shares of 34% and 25% respectively. The average tenure with the firm is not very different across segments at approximately 12 years. Segment 3 has the highest annual quotas, followed by Segment 2 and Segment 1. Interestingly, Segments 2 and 3 with larger quotas achieve their quota targets more often than Segment 1 which has trouble meeting quota.

The parameter estimates for the sales policy function and the distribution of the revenue shocks for each segment are reported in tables 5a and 5b, respectively. The coefficient for monthly allocated quotas are positive for all segments as expected. The net effect for indirect sales is also positive for all segments after taking into account the interaction effect with allocated quotas. Table 5b reports the standard deviations of the revenue shock distribution for each segment by month. There are systematic differences across months; the size of the shocks is correlated with the magnitude of market demand in that month.

Figure 5a shows the Segment 3 exerts the most effort and is the most productive segment, and Segment 1 exerts the least effort and is the least productive segment. This is consistent with the allocated quotas and percentage of time quotas are achieved in Table 3. We also see a positive relationship between exerted effort and %AQ for all months shown. Segment 1 tends to increase effort along %AQ, but stabilize at a certain point. Segments 2 and 3 are similar, but with less concavity. As would be expected, early in the year (April, July), we even see a dip in effort for high %AQ. But, effort in December does not fall off even if the sales person has already reached or exceed quota (%AQ>1), likely due to the overachievement commissions in preventing sales people from lowering effort after achieving quota. Our results are consistent with Steenburgh (2008), who finds that sales people “give up” when far away from achieving quota, such as for all segments in our case, but do not slow down much once quota is reached.

Figure 5b shows the effect of tenure on effort for all segments. Sales people initially increase effort with experience. The peak effect is roughly around 18 years. This is probably due to the fact that in the early years of their careers, they want to work hard not only for monetary payments from increased wages but also other intangible incentives such as promotions or transfers to better job titles. However, after a certain amount of years,
these intangibles don’t matter as much as they begin to think of retirement so they start to slow down.

6.2 Discount Factor

We performed a grid search over the set of discount parameters in steps of 0.01 for delta and 0.1 for beta. Table 6 presents the mean absolute percentage errors (MAPE) associated with each set of hyperbolic parameters where a beta equals one represents exponential discounting. A beta of 0.8 and a delta of 0.95 has the lowest MAPE.\(^{16}\) Thus our estimates show a distinct present bias in that beta <1. Given that these are based on monthly data, the annual discount factor is effectively 0.46.

Frederick, Loewenstein and O’Donoghue (2002) have a comprehensive summary of the estimated discount factors from previous studies. The summary shows that the estimated discount factors vary extensively ranging from as low as a mere 0.02 to no discounting at all with a discount factor of 1. For purely monetary values, the estimated discount factor seems rather low. But as Frederick, Loewenstein and O’Donoghue (2002) point out, for behavioral aspects such as pain and thus in our case effort, the discount factors tend to be low and hence our estimates appear reasonable.

6.3 Second-Stage Structural Parameter Estimates

The first column of Table 7 reports the structural parameter estimates of the sales agent's utility function for a forward looking sales person, consistent with the model we developed earlier. Overall, the disutility parameters for all three segments are negative and significant. These estimates are consistent with the effort policy functions estimated in the first stage. Segment 3, which produces the greatest sales on average, has the lowest disutility for effort. Segment 1, which has the lowest sales, has the greatest disutility. The risk aversion coefficients for all segments are insignificant showing no direct evidence of risk aversion by the sales agents. This may be because in the range of incomes earned by the sales force, risk aversion is not a serious concern. The estimated model fits the observed sales revenue data reasonably well with a MAPE of 8.4%.

6.4 Assessing the value of a dynamic structural model

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\(^{16}\) We tested for finer granularity around beta for 0.8 and found that 0.8 is optimal.
How important is it to account for dynamics to model salesperson behavior? If salespeople are behaving in a dynamically optimal manner, not accounting for dynamics would bias the structural parameters. In a static model, any effort would be attributed to current payoff, not accounting for the large future bonuses. This will underestimate the salesperson disutility parameters and overestimate the effects of compensation on productivity.

The second column of Table 7 shows the estimates of the structural model without forward looking dynamics – discount factor set to zero. As predicted, the disutility parameters are smaller in magnitude relative to the forward looking model for all segments. For Segment 3 the bias is as large as 33% relative to the dynamic model estimate. The myopic model also has a poorer fit: a MAPE of 22.3% relative to the MAPE of 8.4% for the dynamic model.

We next compare the revenue and effort predictions between the dynamic and myopic models. To isolate the effects of forward looking behavior, we use the structural disutility parameter estimates from the dynamic model for both the forward looking and myopic models, but set the discount parameters to zero for the myopic model. Figure 6 compares the predicted revenues and effort of the myopic and dynamic models. The revenues are systematically lower for the myopic sales agent because the sales person does not take into account the effect of future bonuses and overachievement commission in current effort. The forward looking sales person anticipates that in an uncertain environment, there is a chance of bad shocks later, which may prevent getting to the quota, so they prepare for such a rainy day by working harder early on so that they are within striking target of quota even if a bad sales shock occurred.

The effort graph in Figure 6 enables us to isolate out the sales revenue cyclicality and focus on the differences in effort across dynamic and myopic agents. The myopic salesperson exerts much more effort in the bonus period, but the forward looking sales person smoothes effort over time, given the uncertainty in future demand shocks. The effort peak in the bonus periods are not as pronounced for the dynamic consumer. The observed effort smoothing is similar to consumption smoothing by forward looking consumers facing uncertain incomes in the development economics literature.
Next we assess the importance of accounting for hyperbolic discounting, by comparing the best fitting model with an exponential discounter. Figure 7 shows the effort level difference between hyperbolic discounters with a beta of 0.8 and delta of 0.95 and exponential discounters with beta of 1 and delta of 0.931 (the equivalent effective monthly rate based on hyperbolic discounting) for the two extreme segments: Segment 1 and 3. On average both segments exert less effort if they are hyperbolic discounters due to the present bias, but the difference is less pronounced in bonus months, where the reward is immediate and neither group discounts the immediate reward.

More interestingly, low productivity segment 1 decreases effort as the year progresses under both hyperbolic and exponential conditions because they recognize that the chance of meeting annual quota becomes lower as the year progresses, and she tends to give up. In contrast, the high productivity segment 3, which remains within striking distance of quota increases effort as the year progresses because the quota motivates them to stretch and perform better as they get closer to bonus. For both segments, the exponential discounter exerts greater overall effort and provides higher productivity than the hyperbolic discounters.

6.5 Counter–Factual Simulations

We now perform a series of counterfactual simulations that address the two sets of substantive questions we wish to answer. First, we address the issue of how valuable different components of the compensation plan are. The change in revenues under the alternative conditions is reported in Table 8. Second, we assess how quota-bonus frequency affects performance.

Value of Quotas and Bonuses

We compare changes in revenues and profits when the firm moves from the current compensation plan to a pure commission-only plan. We consider two cases: (1) where the commission rate is the same as the current commission rate; and (2) a higher commission rate is such that total compensation is exactly equal to the current compensation. We find that the revenues are about 20.5% greater with the current compensation plan compared to a pure commission plan; even when adjusting commission rates to be higher to make total compensation identical to current levels, we find that revenues are about 9% higher.

Value of Overachievement Compensation
We compare changes in revenues and profits when the firm eliminates the overachievement commission rate, which motivates sales people who are close to reaching their quota to continue exerting effort. Overall revenues drop by 9.5% and even accounting for the additional commission costs, profits are lower by 8.7% (assuming gross margin of 33%).

Figure 8 plots the effort level of sales agents who met and didn’t meet the annual quota, respectively. For those who met the annual quota, the effort level does not decline even when close to the quota because of the overachievement commission. In contrast, those who did not meet quota, decrease effort towards the end of the year as they are unlikely to meet quota and therefore overachievement commission has no impact on their earnings. Thus overachievement commission provides the incentives for the most productive sales people even if they have already met quota (or likely to meet quota). The increasing effort level of Segment 3 in Figure 7 also is consistent with this conjecture.

**Value of Cumulative Annual Quota**

Rather than have a cumulative annual quota, what would be the effect of replacing it with just a fourth quarter quota? To study this, we remove the overachievement commission (which is based on reaching the annual quota) and split the total bonus payments across all four quarters. Overall, revenues drop by 13.7%. This decrease is greater than the 9.5%, where we just dropped the overachievement commissions. Thus the cumulative annual quota induces sales agents to exert greater effort and raise revenues by 4.2%.

We also consider the case where we split the annual quota into a quarterly bonus and an annual bonus so that people do not “give up” in the last quarter when they are far away from quota. While this did increase the effort in the last quarter, it reduced revenues overall because sales people did not put in as much effort earlier in the year to be within striking distance of annual quota, because it is not as large. Total revenues drop by 1.8%

**Quota-Bonus Frequency**

We next investigate the value of quarterly bonuses relative to annual bonuses. The left panel of Figure 9 shows the comparison of effort between the current plan and when quarterly bonuses are eliminated and only an annual bonus is paid at the end of the year. Effort drops consistently across the year when there are no quarterly quotas. Overall revenues fall by 9.2%. Even in December, when there is the annual bonus on the table,
revenue falls by 3% and effort falls by 12%. Thus annual quotas and over-achievement commissions have less of an impact on year-end performance without quarterly bonuses. Why?

Not only does the quarterly quota induce sales agents to work harder in a given quarter but it also helps them achieve the annual quota by giving the incentive to stay on track of their annual goals. When the quarterly quota is removed, sales agents no longer have as much incentive to work hard early on. But this lack of incentive leads them to be farther away from the annual quota by December. Hence, annual quotas and over-achievement commission have little impact on effort as sales agents are more likely to give up meeting quota. Even when we sum-up all the bonuses (3 quarterly bonuses of $1500 + annual bonus of $4000) and give it to agents at the end of the year on achieving the annual quota, we see a 3% reduction in effort and 2% decline in revenues and profit.

To the best of our knowledge, there has been no analysis to-date on what is the appropriate frequency of quota and bonuses. There has been some descriptive work in the education literature on how frequent testing affects academic performance (for an extensive survey, see Bangert-Drowns et al. 1991) and some experimental work in the behavioral psychology (Heath, Larrick and Wu, 1999). The basic idea is that achieving short-term goals make achieving long-term goals more feasible. But this does not mean that one may increase the frequency of quota-bonuses indiscriminately. There is a tradeoff in terms of payouts and performance. In the education literature, more frequent testing takes time away from classroom learning, but also motivates students to learn earlier material better which helps them to do better on later material. In a similar way being on target with the sub-goal early on makes it more likely that one will be closer to achieving longer-term targets and therefore increases performance.

To further address the issue of frequency, we check how the sales force would react to bimonthly and monthly bonuses. For this analysis, we leave the annual quota and bonus as they are. We use the monthly allocated quotas to construct the bi-monthly and monthly quotas. For the monthly and bi-monthly bonuses, we make the bonus amount proportional

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17 Unlike the rest of the analysis where the value of bonuses appear less important due to the low hyperbolic discount factors, quarterly bonuses appear relatively more valuable because of their immediacy, relative to the annual bonus.
18 We thank an anonymous reviewer for encouraging us to test this counterfactual.
to the allocated quota, keeping total bonus amount equal to the earlier quarterly bonus amount. The estimated effort under the two scenarios is in figure 10. The bimonthly plan on the left-panel shows that effort now falls in the high demand end of quarter months, because bonuses are misaligned with peak demand periods. Also since the bonus amounts are now spread over more periods, the incentive and stretch effects of bonuses are lower. Overall, effort decreases by 2.2%.

The results of the monthly plan shown on the right panel shows a puzzling result for effort. Effort is lowest in the high demand end-of-quarter months. Since bonuses are now not as concentrated in the high demand months (even though it is higher than the low demand months), sales agents try to get their bonuses in the less demanding low demand months and give up effort in the high demand months. Overall effort is reduced by 4.3%. In summary, our results show that while frequent bonuses are important, they are more effective when these bonuses are aligned with peak periods of sales.

7. Conclusion

Even though personal selling is a primary marketing mix tool for most B2B firms to generate sales, there is little research on how the compensation plan used to motivate the sales-force affects performance. This paper developed and estimated a dynamic structural model of sales force response to a compensation plan with many components: salary, commissions, lump-sum bonus for achieving quotas, and different commission rates beyond achieving quotas. Our analysis helped us assess the impact of (1) different components of compensation and (2) frequency of quotas and bonuses on performance.

There has been a fair amount of controversy on the value of quotas and bonuses in the literature. Overall, we find that the quota-bonus scheme used by this firm increased performance of the sales force by serving as stretch goals and pushing employees to accomplish targets. Features such as overachievement compensation reduce the problems associated with sales agents slacking off when they get close to achieving their quota.

Further, quarterly bonuses serve as a continuous evaluation scheme to keep sales agents within striking targets of their annual quotas. In the absence of quarterly bonuses, a failure in the early periods to accomplish targets caused agents to fall behind more often than in the presence of quarterly bonuses. Thus, the quarterly bonus serves as a valuable sub-
goal which helps the sales force stay on track in achieving their overall goal. A key finding is that annual bonuses are not as effective for the sales force without quarterly bonuses. Increasing the frequency of bonuses to be monthly is counterproductive, because it does not induce the stretch motive linked to providing incentives in the highest demand periods (end of quarters).

We used recent innovations in the two-step dynamic structural model estimation to accommodate unobserved heterogeneity in sales force response. The approach is flexible, yet computationally feasible with minimal additional burden compared to traditional two-step methods. Features of our data allow us to separate seasonality in sales due to quotas as opposed to underlying consumer demand seasonality. This enables us to get better estimates of the response to compensation, because demand peaks that coincide with quota periods may be wrongly interpreted as a by-product of compensation.

We now discuss limitations of the paper, which provide promising avenues for future research. First, effort tends to be multi-dimensional and one possibility is that quotas and bonuses force people to focus on the effort that lead to final sales in bonus periods, while in other periods, they may focus on earlier stages of the selling process. It is not possible to identify such a multidimensional effort merely from the sales data as in this paper. Nevertheless new data from CRM databases which track customer stages through the selling process can help shed insight on this issue. This we believe is an exciting area for future research.

Second, compensation contracts can serve as a selection mechanism to draw the right type of sales people into the sales force. This paper does not address the selection issues. By looking at a longer panel of sales people's performance, one can use attrition information to address this issue. Looking at scenarios where contracts varied over time, can also shed light on this problem. Papers that have looked at varying contracts over time typically have focused on only contracts with linear commission rates (e.g., Paarsch and Shearer 1998). One needs more work on scenarios with richer contracts.

Finally, Chan, Li and Pierce (2009) investigate the effects of peer effects on sales performance in the presence of team based compensation in a reduced form analysis. It would be useful to extend structural analysis to settings involving team based compensation.
In summary, this paper provides important insights on how the sales force responds to a very rich compensation structure involving many components of compensation: salary, commissions, quota and bonuses at quarterly and annual frequencies. How employees respond to such rich compensation structures with bonuses, a reality at many firms, has not been investigated at all in the literature. This paper illustrates a rigorous framework to analyze this problem and obtains useful substantive insights. Nevertheless, the issues raised above provide an interesting agenda for future work.
References


Chan, Tat Y., Jia Li, and Lamar Pierce, “Competition and Peer Effects in Competing Sales Teams”, Working Paper, Washington University, St. Louis, 2009


Paarsch, Harry, J. and Bruce Shearer, "The Response of Worker Effort to Piece Rates: Evidence from the British Columbia Tree-Planting Industry," *Journal of Human Resources*, 34, pp. 643-667, 1999


<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Payment period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarterly Bonus</td>
<td>$1500 Awarded if quarterly revenue exceeds quarterly quota</td>
<td>Mar, Jun, Sep</td>
</tr>
<tr>
<td>Annual Bonus</td>
<td>$4000 Awarded if annual revenue exceeds annual quota</td>
<td>Dec</td>
</tr>
<tr>
<td>Base Commission</td>
<td>About 1.5%* paid in proportion to the revenue generated each month</td>
<td>Every month</td>
</tr>
<tr>
<td>Overachievement Commission</td>
<td>About 3%* paid in proportion to the total cumulative revenue surpassing the annual quota</td>
<td>Dec</td>
</tr>
</tbody>
</table>

*These numbers are approximate for confidentiality reasons.

Table 2: Descriptive statistics – Sales force under study

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>% Achieving Quota</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>348</td>
<td>-</td>
</tr>
<tr>
<td>Average Salary (USD)</td>
<td>$3,585</td>
<td>-</td>
</tr>
<tr>
<td>Average tenure (years)</td>
<td>11.8</td>
<td>-</td>
</tr>
<tr>
<td>Average Q1 quota (000'USD)</td>
<td>232.4</td>
<td>51.1</td>
</tr>
<tr>
<td>Average Q2 quota (000'USD)</td>
<td>374.2</td>
<td>49.8</td>
</tr>
<tr>
<td>Average Q3 quota (000'USD)</td>
<td>397.1</td>
<td>42.8</td>
</tr>
<tr>
<td>Average full-year quota (000'USD)</td>
<td>1,639.3</td>
<td>49.9</td>
</tr>
</tbody>
</table>
Table 3: Testing for Sales Substitution Across Months

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Month Sales</td>
<td>0.302***</td>
<td>0.188***</td>
<td>0.188**</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.013)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Qtr 1st Month *Last Month Sales</td>
<td>-0.007</td>
<td>0.023*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
</tr>
<tr>
<td>Qtr 1st Month<em>Last Month Sales</em></td>
<td></td>
<td></td>
<td>0.029*</td>
</tr>
<tr>
<td>“Way off Target Last Qtr”</td>
<td></td>
<td></td>
<td>(0.015)</td>
</tr>
<tr>
<td>Qtr 1st Month<em>Last Month Sales</em></td>
<td></td>
<td></td>
<td>0.004</td>
</tr>
<tr>
<td>“Not Way off Target”</td>
<td></td>
<td></td>
<td>(0.024)</td>
</tr>
<tr>
<td>Monthly Allocated Quota</td>
<td></td>
<td>0.565***</td>
<td>0.566***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.021)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Indirect Sales</td>
<td>14.063***</td>
<td>14.15***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.130)</td>
<td>(2.229)</td>
<td></td>
</tr>
<tr>
<td>Sales person Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*: p<0.1  **: p<0.01  ***: p<0.001

Table 4: Descriptive Characteristics of Segments

<table>
<thead>
<tr>
<th></th>
<th>segment1</th>
<th>segment2</th>
<th>segment3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share</td>
<td>0.34</td>
<td>0.41</td>
<td>0.25</td>
</tr>
<tr>
<td>Tenure*</td>
<td>11.2</td>
<td>12.6</td>
<td>11.5</td>
</tr>
<tr>
<td>Percent achieving Q1 quota</td>
<td>35</td>
<td>62</td>
<td>61</td>
</tr>
<tr>
<td>Percent achieving Q2 quota</td>
<td>34</td>
<td>64</td>
<td>61</td>
</tr>
<tr>
<td>Percent achieving Q3 quota</td>
<td>29</td>
<td>58</td>
<td>59</td>
</tr>
<tr>
<td>Percent achieving annual quota</td>
<td>28</td>
<td>61</td>
<td>61</td>
</tr>
<tr>
<td>Average annual quota**</td>
<td>1,209.1</td>
<td>1,580.1</td>
<td>2,328.7</td>
</tr>
<tr>
<td>Average December revenue**</td>
<td>166.7</td>
<td>331.5</td>
<td>584.3</td>
</tr>
</tbody>
</table>

* Tenure is measured in years  ** Average quotas and revenues are indicated in USD(K)
Table 5a: Parameter Estimates – Sales Policy *

<table>
<thead>
<tr>
<th></th>
<th>seg1</th>
<th>seg2</th>
<th>seg3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly allocated quotas</td>
<td>0.40</td>
<td>0.79</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Indirect sales</td>
<td>-0.78</td>
<td>-2.15</td>
<td>-11.13</td>
</tr>
<tr>
<td></td>
<td>(1.36)</td>
<td>(0.77)</td>
<td>(1.56)</td>
</tr>
<tr>
<td>Indirect sales*Monthly allocated quota</td>
<td>0.05</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

*The estimated coefficients from the monthly effort policy functions estimated as coefficients of Chebyshev polynomials of the state variables are not reported. Selected Graphs are shown in Figure 5.

Table 5b: Revenue Shock Distribution – Standard Deviation

<table>
<thead>
<tr>
<th></th>
<th>seg1</th>
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<tbody>
<tr>
<td>Jan</td>
<td>17.64</td>
<td>25.39</td>
<td>47.72</td>
</tr>
<tr>
<td>Feb</td>
<td>23.71</td>
<td>33.14</td>
<td>58.33</td>
</tr>
<tr>
<td>Mar</td>
<td>45.14</td>
<td>57.19</td>
<td>103.08</td>
</tr>
<tr>
<td>Apr</td>
<td>30.34</td>
<td>42.71</td>
<td>80.51</td>
</tr>
<tr>
<td>May</td>
<td>34.10</td>
<td>41.03</td>
<td>85.23</td>
</tr>
<tr>
<td>Jun</td>
<td>59.42</td>
<td>80.33</td>
<td>183.98</td>
</tr>
<tr>
<td>Jul</td>
<td>33.41</td>
<td>47.10</td>
<td>92.10</td>
</tr>
<tr>
<td>Aug</td>
<td>38.47</td>
<td>50.88</td>
<td>91.19</td>
</tr>
<tr>
<td>Sep</td>
<td>56.55</td>
<td>78.87</td>
<td>134.56</td>
</tr>
<tr>
<td>Oct</td>
<td>47.80</td>
<td>66.52</td>
<td>115.25</td>
</tr>
<tr>
<td>Nov</td>
<td>46.51</td>
<td>64.24</td>
<td>115.91</td>
</tr>
<tr>
<td>Dec</td>
<td>92.85</td>
<td>135.98</td>
<td>283.47</td>
</tr>
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</table>
### Table 6: Optimal Discount Factor – Model Fit

**Mean Absolute Percentage Error by Discount Factors**

<table>
<thead>
<tr>
<th>δ</th>
<th>0.9</th>
<th>0.91</th>
<th>0.92</th>
<th>0.93</th>
<th>0.94</th>
<th>0.95</th>
<th>0.96</th>
<th>0.97</th>
<th>0.98</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0.4</td>
<td>0.920</td>
<td>0.922</td>
<td>0.938</td>
<td>0.910</td>
<td>0.909</td>
<td>0.923</td>
<td>0.918</td>
<td>0.938</td>
<td>0.935</td>
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<tr>
<td></td>
<td>0.5</td>
<td>0.945</td>
<td>0.926</td>
<td>0.920</td>
<td>0.932</td>
<td>0.900</td>
<td>0.902</td>
<td>0.882</td>
<td>0.892</td>
<td>0.899</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.920</td>
<td>0.0917</td>
<td>0.0899</td>
<td>0.0906</td>
<td>0.0901</td>
<td>0.0900</td>
<td>0.0896</td>
<td>0.0889</td>
<td>0.0886</td>
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<td></td>
<td>0.7</td>
<td>0.0901</td>
<td>0.0915</td>
<td>0.0908</td>
<td>0.0912</td>
<td>0.0908</td>
<td>0.0902</td>
<td>0.0894</td>
<td>0.0860</td>
<td>0.0889</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.0897</td>
<td>0.0891</td>
<td>0.0875</td>
<td>0.0875</td>
<td>0.0868</td>
<td>0.0841</td>
<td>0.0901</td>
<td>0.0883</td>
<td>0.0970</td>
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<tr>
<td></td>
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<td>0.0900</td>
<td>0.0889</td>
<td>0.0901</td>
<td>0.0911</td>
<td>0.0914</td>
<td>0.0917</td>
<td>0.0938</td>
<td>0.0981</td>
<td>0.1009</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.0927</td>
<td>0.0923</td>
<td>0.0912</td>
<td>0.0943</td>
<td>0.0934</td>
<td>0.0994</td>
<td>0.1025</td>
<td>0.1076</td>
<td>0.1116</td>
</tr>
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### Table 7: Parameter Estimates

<table>
<thead>
<tr>
<th>Segment</th>
<th>Disutility</th>
<th>Risk Aversion</th>
<th>With Forward Looking Behavior</th>
<th>Without Forward Looking Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Segment 1</strong></td>
<td>-0.254*** (0.035)</td>
<td>-0.0001 (0.002)</td>
<td><strong>Segment 2</strong></td>
<td>-0.232*** (0.038)</td>
</tr>
<tr>
<td></td>
<td><strong>Segment 3</strong></td>
<td><strong>Segment 3</strong></td>
<td></td>
<td>-0.132** (0.054)</td>
</tr>
<tr>
<td></td>
<td><strong>Segment 3</strong></td>
<td><strong>Segment 3</strong></td>
<td></td>
<td>-0.088** (0.043)</td>
</tr>
</tbody>
</table>

***p<0.01, **p<0.05
Table 8: Impact of Alternative Bonus Plans on Sales Revenues

<table>
<thead>
<tr>
<th>Counterfactual</th>
<th>Change in Revenues</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a. Only Pure Commissions</td>
<td>-20.5%</td>
</tr>
<tr>
<td>1b. Only Pure Commissions (adjusted to equal payout with bonus)</td>
<td>-9%</td>
</tr>
<tr>
<td>2a. No Bonus (Only Commissions + Overachievement Commission)</td>
<td>-15.2%</td>
</tr>
<tr>
<td>2b. No Bonus (Commissions adjusted to equal payout with bonus)</td>
<td>-4%</td>
</tr>
<tr>
<td>3. No overachievement commissions</td>
<td>-9.5%</td>
</tr>
<tr>
<td>4a. Cumulative Annual Quota replaced with quarterly quota</td>
<td>-4.2%</td>
</tr>
<tr>
<td>4b. Annual Bonus split into Quarterly and Annual Bonus</td>
<td>-1.8%</td>
</tr>
<tr>
<td>5a. Remove quarterly bonus</td>
<td>-9.2%</td>
</tr>
<tr>
<td>5b. Add all quarterly bonus into one lump sum annual bonus</td>
<td>-2.6%</td>
</tr>
<tr>
<td>6a. Replace quarterly bonus with bi-monthly bonus</td>
<td>-1.1%</td>
</tr>
<tr>
<td>6b. Replace quarterly bonus with monthly bonus</td>
<td>-2.1%</td>
</tr>
</tbody>
</table>

Figure 1: Plots of Incentive Compensation Schemes
Figure 2a: How Quotas and Bonus Serve as stretch goals

Figure 2b: Effort as a Function of Distance to Quotas

Figure 3: Sales / Monthly Allocated Quota / Indirect-Sales / Fiscal Year-Ends
Figure 4a: Sales and Percentage Quota Achieved – Bonus Months

Figure 4b: Sales and Percentage Quota Achieved – Pre-Bonus Months
Figure 5a: Effort Policy by Segment as a Function of % Quota

Figure 5b: The Effect of Tenure on Effort
Figure 6: Simulated Revenue & Effort—Static vs. Dynamic

Figure 7: Effort under Hyperbolic vs. Exponential Estimated Parameters

Hyperbolic discounters – beta: 0.8, delta: 0.95, Exponential discounters – beta: 1, delta: 0.931
Figure 8: Effect of Overachievement Commission

Figure 9: Effect of Quarterly Quotas

Figure 10: Effect of Changing Bonus Frequency