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# **An Exploration of Technology Diffusion**

**Diego A. Comin  
Bart Hobijn**

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# An Exploration of Technology Diffusion

Diego Comin

Bart Hobijn

Harvard Business School and NBER

Federal Reserve Bank of New York

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## Abstract

We develop a model that, at the aggregate level, is similar to the one sector neoclassical growth model, while, at the disaggregate level, has implications for the path of observable measures of technology adoption. We estimate our model using data on the diffusion of 15 technologies in 166 countries over the last two centuries. We evaluate the implications of our estimates for aggregate TFP and per capita income. Our results reveal that, on average, countries have adopted technologies 47 years after their invention. There is substantial variation across technologies and countries. Over the past two centuries, newer technologies have been adopted faster than old ones. The cross-country variation in the adoption of technologies accounts for at least a quarter of per capita income differences.

**keywords:** economic growth, technology adoption, cross-country studies.

**JEL-code:** E13, O14, O33, O41.

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# 1 Introduction

Most cross-country differences in per capita output are due to differences in total factor productivity (TFP), rather than to differences in the levels of factor inputs.<sup>1</sup> These cross-country TFP disparities can be divided into two parts: those due to differences in the range of technologies used and those due to non-technological factors that affect the efficiency with which all technologies and production factors are operated. In this paper, we explore the importance of the range of technologies used to explain cross-country differences in TFP.

Existing studies of technology adoption are not well suited to answer this question. On the one hand, macroeconomic models of technology adoption (e.g. Parente and Prescott, 1994, and Basu and Weil, 1998) use an abstract concept of technology that is hard to match with data. On the other hand, the applied microeconomic technology diffusion literature (Griliches, 1957, Mansfield, 1961, Gort and Klepper, 1982, among others) involves the estimation of diffusion curves for a relatively small number of technologies and countries. These diffusion curves, however, are purely statistical descriptions which are not embedded in an aggregate model. Hence, it is difficult to use them to explore the aggregate implications of the empirical findings.<sup>2</sup>

In this paper we bridge the gap between these two literatures by developing a new model of technology diffusion. Our model has two main properties. First, at the aggregate level it is similar to the one sector neoclassical growth model. Second, at the disaggregate level it has implications for the path of observable measures of technology adoption. These properties allow us to estimate our model using data on specific technologies and then use it to evaluate the implications of our estimates for aggregate TFP and per capita income.

A technology, in our model, is a group of production methods that is used to produce an intermediate good or service. Each production method is embodied in a differentiated capital good. A potential producer of a capital good decides whether to incur a fixed cost of adopting the new production method. If he does, he will be the monopolist supplying the capital good that embodies the specific production method. This decision determines whether or not a production method is used, which is the extensive margin of adoption.

The size of the adoption costs affects the length of time between the invention and the eventual adoption of a production method, i.e. its adoption lag. Once the production method has been introduced, its productivity determines how many units of the associated capital good are demanded, which reflects the intensive margin

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<sup>1</sup>Klenow and Rodríguez-Clare (1997), Hall and Jones (1999), and Jerzmanowski (2004).

<sup>2</sup>Another strand of the literature has also used more aggregate measures of diffusion to explore the determinants of adoption lags (Saxonhouse and Wright, 2000, and Caselli and Coleman, 2001) or the diffusion curve (Manuelli and Seshadri, 2003) for one technology. Our paper differs from these three studies in that (i) we specifically develop an aggregate model to assess the implications of technology adoption differentials for per capita GDP disparities, and (ii) our analysis covers a wide range of technologies and countries.

of adoption. Our model is thus very similar in spirit to the barriers to riches model of Parente and Prescott (1994), which yields endogenous TFP differentials across countries due to different adoption lags.

The endogenous adoption decisions determine the growth rate of productivity embodied in the technology through two channels. First, because new production methods embody a higher level of productivity their adoption raises the average productivity level of the production methods in use. This is what we call the *embodiment effect*. Second, an increase in the range of production methods used also results in a gain from variety that boosts productivity. This is the *variety effect*.

When the number of available production methods is very small, an increase in the number of methods has a relatively large effect on embodied productivity. As this number increases, the productivity gains from such an increase decline. Thus, the variety effect leads to a non-linear trend in the embodied productivity level. Since adoption lags affect the range of production methods used and thus the variety effect, adoption lags affect the curvature in the path of embodied productivity.

Our model maps this curvature in embodied productivity into similar non-linearities in the evolution of observable measures of technology adoption, such as the number of units of capital that embody a given technology or the output produced with this technology. These measures capture both the extensive as well as the intensive margin of adoption of these technologies. We use our theory to derive reduced form equations that describe how these two margins depend on adoption lags as well as on economy-wide conditions that determine aggregate demand.

Our model is broadly consistent with the empirical diffusion literature in that it predicts an S-shape diffusion pattern for conventional adoption measures that only capture the extensive adoption margin. However, the actual diffusion curves implied by the reduced form equations are not S-shaped. This is because our measures incorporate both the extensive and intensive adoption margins. S-shape curves provide a poor approximation to the evolution of technology measures that incorporate the latter.<sup>3</sup>

We use data from Comin, Hobijn, and Rovito (2006) to explore the adoption lags for 15 technologies for 166 countries. Our data cover major technologies related to transportation, telecommunication, IT, health care, steel production, and electricity. We obtain precise and plausible estimates of the adoption lags for two thirds of the 1278 technology-country pairs for which we have sufficient data. There are three main findings that are especially worth taking away from this exploration of technology diffusion.

First, adoption lags are large. The average adoption lag is 47 years. There is, however, substantial variation in these lags, both across countries and across technologies. The standard deviation in adoption lags is 39 years. An analysis of variance yields that 54% of the variance in adoption lags is explained by variation across technologies, 18% by cross-country variation, and 11% percent by the covariance between the two. The remaining 17% is unexplained. We also find that newer technologies have been adopted faster than

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<sup>3</sup>See Comin et al. (2008) for a detailed explanation of this argument.

older ones. This acceleration in technology adoption has taken place during the whole two centuries that are covered by our data. Thus, it started long before the digital revolution or the post-war globalization process that have often been cited as the driving forces behind rapid diffusion of technologies in recent decades.

Second, the remarkable development records of Japan in the second half of the Nineteenth Century and the first half of the Twentieth Century and of the, so-called, East Asian Tigers in the second half of the Twentieth Century all coincided with a catch-up in the range of technologies used with respect to industrialized countries. All these development ‘miracles’ involved a substantial reduction of the technology adoption lags in these countries relative to those in (other) OECD countries.

Third, our model can be used to quantify the aggregate implications of the estimated adoption lags for cross-country per capita income differentials. Doing so yields that cross-country differences in the timing of adoption of new technologies seems to account for at least a quarter of per capita income disparities.

The rest of the paper is organized as follows. Because the focus of our analysis is on technology, we devote the second section of our paper on a detailed explanation of the assumptions we make about the technology structure in our model economy. We then proceed in two directions.

First, in Section 3 we show how our assumptions nest a version of the one-sector neoclassical growth model with adoption lags, we introduce a set of simplifying assumptions about the technology, as well as add preferences, endogenize the technology adoption decision, and show how these assumptions yield the neoclassical growth model.

Second, in Section 4, we discuss how we identify and estimate the adoption lags for the country-technology pairs in our data. In Section 5, we present our estimates, use them for country case-studies, and quantify their implications for cross-country TFP differentials. In Section 6, we conclude by presenting directions for future research. For the sake of brevity, most of the mathematical derivations are relegated to Appendix B.

## 2 Technology and adoption lags

### 2.1 Technology

*Final output production:*

We present our theoretical analysis in the context of a one sector model. The output of the unique final good,  $Y$ , is produced competitively by combining a continuum of intermediate goods,  $Y_\tau$ , as follows:

$$Y = \left[ \int Y_\tau^{\frac{1}{\theta}} d\tau \right]^\theta, \text{ with } \theta > 1. \quad (1)$$

*Description of technology:*

A technology, indexed by  $\tau$ , is a group of production methods that are jointly used to produce a particular intermediate good,  $Y_\tau$ . Each production method associated with a given technology,  $\tau$ , corresponds to a

different capital vintage. For example, the technology of sail ships is used to provide the output of sail merchant shipping services which is used in the production of aggregate output. A new version of a sail ship constitutes a new capital vintage used for the production of sail merchant shipping services.

Productivity growth is embodied in new capital vintages. The embodied productivity of new vintages grows at a rate  $\gamma_\tau$  across vintages. This rate is technology-specific, as indicated by the subindex  $\tau$ . The productivity of a given vintage is constant over time.

Some innovations represent a significant breakthrough with respect to existing technologies. For example, the first version of the steam ship. We interpret that these innovations as the beginning of a new technology used to produce a new intermediate good, in this case steam shipping services.

The classification of capital vintages into technologies is important for two reasons. First, because the productivity growth rate,  $\gamma_\tau$ , varies across technologies. Second, because the elasticity of substitution between production methods may be different within a technology as opposed to between technologies.

Each instant, a new production method appears exogenously for each of the existing technologies. New technologies appear at pre-specified times. This characterizes the evolution of the world technology frontier.

We denote the set of production methods in the world for technology  $\tau$  as  $\bar{V}_\tau$ . Let  $v_\tau$  be the first capital vintage introduced for the production of intermediate  $\tau$ , then  $\bar{V}_\tau = [v_\tau, t]$  and the collection  $\{\bar{V}_\tau\}_\tau$  is the world technology frontier. A country does not necessarily use all the capital vintages that are available in the world for the production of intermediate  $\tau$ . We denote the set of vintages actually used by  $V_\tau = [v_\tau, t - D_{\tau,t}]$ . Here  $D_{\tau,t} \geq 0$  denotes the age of the best technology vintage that is adopted. It reflects the amount of time between when the best technology in use became available and when it was adopted; the *adoption lag* of technology  $\tau$ .<sup>4</sup>

*Intermediate goods production:*

The amount of intermediate good  $\tau$  produced is a CES aggregate of the output produced with each of the capital vintages used, i.e. each of the vintages in  $V_\tau$ . In particular:

$$Y_\tau = \left( \int_{v \in V_\tau} Y_{v\tau}^{\frac{1}{\mu}} dv \right)^\mu, \quad (2)$$

where  $\mu > 1$ .

$Y_{v\tau}$  is the output produced with vintage  $v$ . This output is produced competitively by combining the capital used for production method  $v$  and labor as follows:

$$Y_{v\tau} = Z_{v\tau} L_{v\tau}^{1-\alpha} K_{v\tau}^\alpha, \quad (3)$$

where  $Z_{v\tau}$  is the productivity level embodied in the capital good of vintage  $v$ . We assume that  $Z_{v\tau}$  is constant

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<sup>4</sup>In what follows, to simplify notation, we only include the time subscript  $t$  when necessary for the exposition.

for a given production method and varies across vintages in the following way

$$Z_{v\tau} = Z_{\underline{v}\tau} e^{\gamma_\tau(v-\underline{v}\tau)}, \quad (4)$$

where  $\gamma_\tau$  represents the growth rate of productivity embodied in new vintages used in the production of intermediate  $\tau$ .

*Capital goods production and technology adoption:*

Capital goods are produced by monopolistic competitors. Each of them holds the patent of the capital good used for a particular production method. It takes  $Q_\tau$  units of final output to produce one unit of capital of any vintage of technology  $\tau$ . This production process is assumed to be fully reversible.  $Q_\tau$  declines at a constant rate  $q_\tau$ .

By introducing investment-specific technological progress in this way, we allow for a trend in the relative price of capital goods when measured in the particular units used in our data set. For example, when we measure the number of trucks,  $q_\tau$  reflects the decline in the price per truck relative to the final good price.

The capital goods suppliers rent out their capital goods at the rental rate  $R_{v\tau}$  and capital goods depreciate at the technology-specific rate  $\delta_\tau$ .

## 2.2 Factor demands

*Final good demand:*

Given the intermediate goods prices,  $\{P_\tau\}$ , final good producers' demand for intermediate  $\tau$  is given by

$$Y_\tau = Y \left( \frac{P}{P_\tau} \right)^{\frac{\theta}{\theta-1}}, \text{ where } P = \left( \int P_{\tau'}^{-\frac{1}{\theta-1}} d\tau' \right)^{-(\theta-1)}. \quad (5)$$

We use the final good as the numeraire good throughout our analysis and, accordingly, normalize  $P = 1$ .

*Intermediate goods demand:*

The demand for the output produced with vintage  $v$  of technology  $\tau$ ,  $Y_{v\tau}$ , is

$$Y_{v\tau} = Y_\tau \left( \frac{P_\tau}{P_{v\tau}} \right)^{\frac{\mu}{\mu-1}}, \text{ where } P_\tau = \left( \int_{v \in V_\tau} P_{v\tau}^{-\frac{1}{\mu-1}} dv \right)^{-(\mu-1)}. \quad (6)$$

Labor is homogenous, competitively supplied at the real rate  $W$  and perfectly mobile across sectors. Recall that the rental rate of the capital good that embodies vintage  $v$  of technology  $\tau$  is  $R_{v\tau}$ . Since  $Y_{v\tau}$  is produced competitively, its price equals its marginal cost of production:

$$P_{v\tau} = \frac{1}{Z_{v\tau}} \left( \frac{W}{(1-\alpha)} \right)^{1-\alpha} \left( \frac{R_{v\tau}}{\alpha} \right)^\alpha. \quad (7)$$

*Capital goods suppliers:*

The supplier of each capital good recognizes that the rental price he charges for the capital good,  $R_{v\tau}$ , affects the price of the output associated with the capital good and, therefore, its demand,  $Y_{v\tau}$ . The resulting demand curve faced by the capital good supplier is

$$K_{v\tau} = Y_{\tau} P_{\tau}^{\frac{\mu}{\mu-1}} Z_{v\tau}^{\frac{1}{\mu-1}} \left( \frac{1-\alpha}{W} \right)^{\frac{1-\alpha}{\mu-1}} \left( \frac{\alpha}{R_{v\tau}} \right)^{\epsilon}, \text{ where } \epsilon \equiv 1 + \frac{\alpha}{\mu-1}. \quad (8)$$

Here,  $\epsilon$  is the constant price elasticity of demand that the capital goods supplier faces. As a result, the profit maximizing rental price equals a constant markup times the marginal production cost of a unit of capital. Because of the durability of capital and the reversibility of its production process, the per-period marginal production cost of capital is  $Q_{\tau}$  times the technology-specific user-cost of capital. Thus, the rental price that maximizes the profits accrued by the capital good producer is

$$R_{v\tau} = R_{\tau} = \frac{\epsilon}{\epsilon-1} Q_{\tau} (r + \delta_{\tau} + q_{\tau}) = \frac{\epsilon}{\epsilon-1} Q_{\tau} UC_{\tau}, \quad (9)$$

where  $\frac{\epsilon}{\epsilon-1}$  is the constant gross markup factor.

### 2.3 Aggregates at the intermediate good level

The lack of data at the capital vintage level makes it impossible to conduct empirical analyses at this level of aggregation. Therefore, we derive the technology-specific aggregates for which data are available.

*Technology level output:*

The factor demands for the capital vintage specific output  $Y_{v\tau}$  allow us to write intermediate output production in the following Cobb-Douglas form

$$Y_{\tau} = Z_{\tau} K_{\tau}^{\alpha} L_{\tau}^{1-\alpha}, \quad (10)$$

where

$$K_{\tau} \equiv \int_{v \in V_{\tau}} K_{v\tau} dv, \text{ and } L_{\tau} \equiv \int_{v \in V_{\tau}} L_{v\tau} dv, \quad (11)$$

and the TFP composite associated with intermediate  $\tau$  is

$$Z_{\tau} = \left( \int_{v \in V_{\tau}} Z_{v\tau}^{\frac{1}{\mu-1}} dv \right)^{\mu-1}. \quad (12)$$

Just like for the underlying capital vintage specific outputs, the total wage bill paid to labor used to produce intermediate  $\tau$  exhausts a constant fraction  $(1-\alpha)$  of the revenue generated by the sale of intermediate good  $\tau$  and the rental costs of capital exhaust the rest. Moreover, the price of the intermediate equals the marginal cost of production

$$P_{\tau} = \frac{1}{Z_{\tau}} \left( \frac{Y}{L} \right)^{1-\alpha} \left( \frac{R_{\tau}}{\alpha} \right)^{\alpha}. \quad (13)$$



*Technology specific TFP and adoption lags:*

The endogenous level of TFP in the production of intermediate  $\tau$  at time  $t$  (12) can be expressed as

$$Z_{\tau t} = \left( \frac{\mu - 1}{\gamma_\tau} \right)^{\mu-1} Z_{v_\tau} \underbrace{e^{\gamma_\tau(t-D_{\tau,t}-v_\tau)}}_{\text{embodiment effect}} \underbrace{\left[ 1 - e^{-\frac{\gamma_\tau}{\mu-1}(t-D_{\tau,t}-v_\tau)} \right]^{\mu-1}}_{\text{variety effect}} \quad (14)$$

From this equation, it can be seen that our model introduces two mechanisms by which the adoption lags,  $D_{\tau,t}$ , affect the level of TFP in the production of intermediate  $\tau$ : (i) the *embodiment effect*; and (ii) the *variety effect*.

First, as newer vintages with higher embodied productivity are adopted in the economy, the level of embodied productivity increases. This mechanism is captured by the ‘embodiment effect’ term of (14) which reflects the productivity embodied in the best vintage adopted in the economy.

The range of vintages available for production also affects the level of embodied productivity of technology  $\tau$ . In particular, an increase in the measure of vintages adopted leads to higher productivity through the gains from variety. This is captured by the ‘variety effect’ term in expression (14).

### 3 One-sector growth model

In order to develop a one-sector aggregate growth model using the notion of technology introduced above, we abstract from most of the cross-technology heterogeneity. In particular, we assume that: (i) the growth rate of embodied technological change is the same across technologies, such that  $\gamma_\tau = \gamma$  for all  $\tau$ ; (ii) the rate of investment specific technological change is zero for all technologies, such that  $q_\tau = q = 0$  for all  $\tau$  and we normalize  $Q = 1$ ; (iii) all types of capital goods are subject to the same physical depreciation rate, i.e.  $\delta_\tau = \delta$  for all  $\tau$ ; and, finally, the within technology elasticity of substitution across vintages is the same as the between technologies elasticity of substitution,  $\theta = \mu$ .

At each instant a new technology  $\tau$  is introduced, such that  $\tau$  indexes both the technology as well as its time of introduction. A new vintage of each existing technology is introduced at every instant. New vintages of a given technologies are more productive than older vintages. Furthermore, for a given date of introduction, vintages of more modern technologies are more productive than those of older technologies.

The following expression for the productivity embodied in vintage  $v$  of technology  $\tau$  captures these assumptions:

$$Z_{v,\tau} = Z_0 e^{\chi\tau} e^{\gamma v}, \text{ where } \chi, \gamma > 0. \quad (15)$$

Note that  $\gamma$  can be interpreted as *within* technology embodied technological change while  $\chi$  can be interpreted as *between* technology embodied technological change.

### 3.1 Preferences, technology adoption decision, and market structure

*Preferences:*

A measure one of households populate the economy. They inelastically supply one unit of labor every instant, at the real wage rate  $W$ , and derive the following utility from their consumption flow

$$U = \int_0^{\infty} e^{-\rho t} \ln(C_t) dt, \quad (16)$$

where  $C_t$  denotes per capita consumption and  $\rho$  is the discount rate. The representative household has an initial wealth level of  $S_0$  and cannot run Ponzi schemes.

*Capital goods production and technology adoption:*

In order to become the sole supplier of a particular capital vintage, the capital good producer must undertake an investment, in the form of an up-front fixed cost. We interpret this investment as the adoption cost of the production method associated with the capital vintage.

The cost of adopting vintage  $v$  for technology  $\tau$  at instant  $t$  is assumed to be

$$\Gamma_{vt} = \bar{V} (1 + b) \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{\vartheta}{\vartheta-1}} P_{v,\tau} Y_{v,\tau}, \text{ where } \vartheta > 0, \quad (17)$$

and  $\bar{V}$  is the steady state stock market capitalization to GDP ratio.<sup>5</sup>

We include  $\bar{V}$  in the cost function for normalization purposes. The parameter  $\vartheta$  reflects how much faster adoption costs are rising for the available vintages than the benefits of adoption and the parameter  $b$  reflects barriers to adoption in the sense of Parente and Prescott (1994).

### 3.2 Optimality conditions and aggregation

*Consumers:*

The representative consumer's path of consumption is characterized by the following Euler equation

$$\frac{\dot{C}}{C} = r - \rho \quad (18)$$

and lifetime budget constraint:

$$\int_0^{\infty} (C_s - W_s) e^{-\int_0^s r_{s'} ds'} ds = S_0. \quad (19)$$

*Aggregate technology:*

In a similar way to the intermediate goods production function, we obtain an aggregate production function. That is, final output can be represented by a Cobb-Douglas production function of the form

$$Y = AK^\alpha L^{1-\alpha}, \quad (20)$$

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<sup>5</sup>In particular,  $\bar{V} = \frac{\alpha}{\epsilon} \frac{1}{\left\{ \rho + \frac{1}{\vartheta-1} (x+\gamma) \right\}}$ .

where the aggregate factor inputs satisfy

$$K = \int_{-\infty}^t K_\tau d\tau \text{ and } L = \int_{-\infty}^t L_\tau d\tau = 1, \quad (21)$$

and aggregate TFP is equal to

$$A = \left( \int_{-\infty}^t \int_{V_\tau} Z_{v,\tau}^{\frac{1}{\theta-1}} dv d\tau \right)^{\theta-1} \quad (22)$$

*Technology adoption and adoption lags:*

The flow profits that the capital goods producer of vintage  $v$  makes are equal to

$$\pi_{v,\tau} = \frac{\alpha}{\epsilon} P_{v,\tau} Y_{v,\tau} = \frac{\alpha}{\epsilon} \left( \frac{Z_{v,\tau}}{A} \right)^{\frac{1}{\theta-1}} Y \quad (23)$$

The market value of each capital goods supplier equals the present discounted value of the flow profits. That is,

$$M_{v,\tau,t} = \int_t^\infty e^{-\int_t^s r_{s'} ds'} \pi_{v,s} ds = \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{1}{\theta-1}} \left( \frac{Z_{t,\tau}}{A_t} \right)^{\frac{1}{\theta-1}} V_t Y_t. \quad (24)$$

where

$$V_t = \frac{\alpha}{\epsilon} \int_t^\infty e^{-\int_t^s r_{s'} ds'} \left( \frac{A_t}{A_s} \right)^{\frac{1}{\theta-1}} \left( \frac{Y_s}{Y_t} \right) ds \quad (25)$$

is the stockmarket capitalization to GDP ratio.<sup>6</sup>

Optimal adoption implies that, every instant, all the vintages for which the value of the firm that produces the capital good is at least as large as the adoption cost will be adopted. That is, for all vintages,  $v$ , that are adopted at time  $t$

$$\Gamma_{v\tau} \leq M_{v\tau} \quad (26)$$

This holds with equality for the best vintage adopted if there is a positive adoption lag.<sup>7</sup>

The adoption lag that results from this condition equals

$$D_\tau = \max \left\{ \frac{\theta-1}{\gamma\vartheta} \{ \ln(1+b) - \ln V + \ln \bar{V} \}, 0 \right\} = D \quad (27)$$

and is constant across technologies,  $\tau$ .

The resulting aggregate TFP level equals

$$A_t = A_0 e^{(\chi+\gamma)(t-D_t)}, \quad (28)$$

<sup>6</sup>This can be interpreted as the stockmarket capitalization if all monopolistic competitors are publicly traded companies.

<sup>7</sup>If the frontier vintage,  $t$ , is adopted, and there is no adoption lag, then  $\Gamma_{t\tau} \leq M_{t\tau}$ . For simplicity, we ignore the possibility that, for the best vintages, already adopted  $\Gamma_{v\tau} > M_{v\tau}$ . In that case, no new vintages are adopted. This possibility is included in the mathematical derivations in Appendix B.

where  $A_0 > 0$  is a constant that depends on the model parameters.<sup>8</sup> Hence, aggregate TFP in this model is endogenously determined by the adoption lags induced by the barriers to entry.

Moreover, the total adoption costs across all vintages adopted at instant  $t$  equal

$$\Gamma = \bar{V} (1 + b) \left( \frac{\chi + \gamma}{\theta - 1} \right) e^{-\frac{\theta}{\theta-1} \gamma D} Y \left( 1 - \dot{D} \right), \quad (29)$$

where  $\dot{D}$  denotes the time derivative of the adoption lags.

### 3.3 Equilibrium

The equilibrium path of the aggregate resource allocation in this economy can be defined in terms of the following eight equilibrium variables  $\{C, K, I, \Gamma, Y, A, D, V\}$ . Just like in the standard neoclassical growth model, the capital stock,  $K$ , is the only state variable. The eight equations that determine the equilibrium dynamics of this economy are given by

(i) The consumption Euler equation, (18).

(ii) The aggregate resource constraint<sup>9</sup>

$$Y = C + I + \Gamma. \quad (30)$$

(iii) The capital accumulation equation

$$\dot{K} = -\delta K + I. \quad (31)$$

(iv) The production function, (20), taking into account that in equilibrium  $L = 1$ .

(v) The adoption cost function, (29).

(vi) The technology adoption equation, (27), that determines the adoption lag.

(vii) The stockmarket to GDP ratio, (25).<sup>10</sup>

(viii) The aggregate TFP level, (28).

In addition to these equations that pin down the equilibrium dynamics, the lifetime budget constraint, (19), pins down the initial level of consumption. We derive the balanced growth path and approximate transitional dynamics of this economy in Appendix B. The growth rate of this economy on the balanced growth path is  $(\chi + \gamma) / (1 - \alpha)$ .

*Below the surface:*

Underlying these aggregate dynamics, there is a continuum of diffusion curves for the expanding set of

<sup>8</sup>In particular  $A_0 = Z_0 \left( \frac{\theta-1}{\gamma} \right)^{\theta-1} \left( \left( \frac{\theta-1}{\chi} \right) - \left( \frac{\theta-1}{\chi+\gamma} \right) \right)^{\theta-1}$ .

<sup>9</sup>We assume that adoption costs are measured as part of final demand, such that  $Y$  can be interpret as GDP.

<sup>10</sup>The dynamics of  $V_t$  are what are considered in the system of equilibrium equations. The law of motion of the stockmarket to GDP ratio is given by  $\frac{\dot{V}}{V} = \left\{ \alpha \frac{\epsilon-1}{\epsilon} \frac{Y}{K} - \delta + \frac{1}{\theta-1} \frac{\dot{A}}{A} - \frac{\dot{Y}}{Y} \right\} - \frac{\alpha}{\epsilon} \frac{1}{V}$ .

vintages and technologies. Where aggregate TFP grows at the constant rate  $\chi + \gamma$ , the technology-specific TFP level is given by

$$Z_{\tau t} = \left( \frac{\theta - 1}{\gamma} \right)^{\theta - 1} Z_0 e^{(\chi + \gamma)\tau} e^{\gamma(t - D_t - \tau)} \left[ 1 - e^{-\frac{\gamma}{\theta - 1}(t - D_t - \tau)} \right]^{\theta - 1}. \quad (32)$$

Just like (14), (32) has a variety effect which introduces a non-linearity in  $Z_{\tau}$ . This non-linearity is critical for our empirical application.

As we show below, the evolution of technology-specific TFP governs the speed of diffusion as well as the shape of the diffusion curve of a technology. The variety effect thus drives the non-linearity in the diffusion curve. Since the measure of varieties adopted depends on the diffusion lag, the curvature of the diffusion curve allows us to identify the adoption lags in the data.

## 4 Empirical application

The simplifying assumptions that we made for the one-sector growth model are useful because they yield a tractable aggregate production function representation. They do, however, ignore cross-technology variation that is likely to be important in the data. For our empirical investigation, we reintroduce the cross-technology variation by allowing  $\delta_{\tau}$ ,  $\gamma_{\tau}$ ,  $q_{\tau}$  to be different across technologies. Our aim is to estimate the adoption lags for different technology-country pairs. To make this estimation practically feasible, we assume that adoption lags differ across countries and technologies but are constant over time. In this section, we describe our measures of technology diffusion, derive their reduced form equations and describe the method we use to estimate these equations. Before doing so, however, we relate our measures of diffusion to more traditional measures introduced by Griliches (1957) and Mansfield (1961).

### 4.1 Measures of diffusion

The empirical literature on technology diffusion has mainly focused on the analysis of the share of potential adopters that have adopted a technology. Such shares capture the extensive margin of adoption. Computing these measures requires micro level data that are not available for many technologies and countries. As a result, over the last 50 years, the diffusion of only relatively few technologies in a very limited number of countries has been documented.

Depending on the level of aggregation we are interested in, our model delivers two counterparts to this traditional diffusion measure: (i) the share of employees involved in the production of good  $\tau$  that use a technology vintage  $v'$  or more advanced, and (ii) the share of workers in the economy using (any vintage of) technology  $\tau$ .

The most remarkable finding of the traditional diffusion literature is that, for a majority of the technologies for which it has been possible to construct the diffusion measures, the diffusion curves are S-shaped. Our model is roughly consistent with this finding. In particular, before a given vintage has been adopted in the country, the diffusion measure is zero. At the moment in which the technology is adopted, the diffusion measure starts to increase at a decreasing rate until it reaches a plateau where it has fully diffused. As a result, the evolution of our model counterparts to the traditional diffusion measures is approximately S-shaped.

However, the implications of our model for adoption shares are not our main focus. The richness of our model allows us to explore its predictions for alternative measures of technology diffusion for which data is more widely available. In particular, we focus on (i)  $Y_\tau$ , the level of output of the intermediate good produced with technology  $\tau$ ; (ii)  $K_\tau$ , the capital inputs used in the production of this output.

These variables have two advantages over the traditional measures. First, they are available for a broad set of technologies and countries. Second, they capture (directly or indirectly) the number of units of the new technology that each of the adopters has adopted. This intensive margin is important to understand cross-country differences in adoption patterns. For spindles, for example, Clark (1987) argues that this margin is key to explaining the difference in adoption and labor productivity between India and Massachusetts in the Nineteenth century.

In order to see how our measures relate to the traditional diffusion measures consider the following decomposition of  $K_\tau$  and  $Y_\tau$

$$K_\tau = \left(\frac{L_\tau}{L}\right) \left(\frac{K_\tau/L_\tau}{K/L}\right) \left(\frac{K}{Y}\right) Y \quad (33)$$

$$Y_\tau = \left(\frac{L_\tau}{L}\right) \left(\frac{Y_\tau/L_\tau}{Y/L}\right) Y \quad (34)$$

The first component of these expressions measures the share of the labor inputs devoted to technology  $\tau$ . This captures the extensive margin of adoption, and is similar to the measures most commonly used in empirical microeconomic studies. The second component measures the intensive margin of adoption of the technology  $\tau$  relative to the economy. In expression (33) this corresponds to the technology-specific capital-labor ratio relative to the economy wide ratio. In expression (34) it is measured by the labor productivity in the production of intermediate  $\tau$  relative to aggregate labor productivity. This term reflects what Clark (1987) perceived was the difference between Massachusetts and India. Namely, distortions in the adoption of new technologies that caused an inefficiently low intensity of adoption.<sup>11</sup> The third component of (33) is the capital output ratio and reflects the fact that more capital intensive economies tend to have more capital embodying all the technologies, including the new ones. Finally, the last component in both expressions

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<sup>11</sup>Interestingly, it can easily be shown, that including these distortions in our model is isomorphic to increasing the cost of adopting the new technology. Hence, our estimates of the adoption lags will include the effects of such distortions.

reflects the size of the economy.

These different components are not separately distinguishable in the data. To make further progress in our exploration of technology diffusion, we use our model to derive estimable reduced form equations for  $K_\tau$  and  $Y_\tau$ . These reduced form equations relate the paths of the first two components of (33) and (34) to the adoption lags,  $D_\tau$ . This allows us to relate the technology adoption measures to observable variables and to estimate the adoption lags.

## 4.2 Reduced form equations

Let's denote the technology measures for which we derive reduced form equations by  $m_\tau \in \{y_\tau, k_\tau\}$ . Small letters denote logarithms. By combining the log-linearized versions of the demand equation (5)

$$y_\tau = y - \frac{\theta}{\theta - 1} p_\tau, \quad (35)$$

and the intermediate goods price (13)

$$p_\tau = -\alpha \ln \alpha - z_\tau + (1 - \alpha)(y - l) + \alpha r_\tau, \quad (36)$$

we obtain the reduced form equation (37) for  $y_\tau$ <sup>12</sup>

$$y_\tau = y + \frac{\theta}{\theta - 1} [z_\tau - (1 - \alpha)(y - l) - \alpha r_\tau + \alpha \ln \alpha] \quad (37)$$

Similarly, we obtain the reduced form equation for  $k_\tau$  by combining the log-linear capital demand equation

$$k_\tau = \ln \alpha + p_\tau + y_\tau - r_\tau. \quad (38)$$

with (35) and (36).

These expressions depend on the adoption lag  $D_\tau$ , through the effect the lag has on  $z_\tau$ .

They also contain the technology-specific capital rental rate,  $r_\tau$ , for which we do not have data. However we can use the model to relate  $r_\tau$  to observable variables. In particular, equation (9) implies that the technology-specific capital rental rate is the product of the markup factor, the relative investment price, and the user cost. The user cost depends on the real interest rate,  $r$ . Unfortunately, we do not have historical cross-country data on real interest rates. However, log-linearization of the neoclassical growth model as well as of our aggregate model yields that the real interest rate is an approximate linear function of the growth

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<sup>12</sup>Using the fact that the final output is the numeraire, we can rewrite (37) as

$$y_\tau = y + \frac{\theta}{\theta - 1} ((z_\tau - z) + \alpha(r - r_\tau))$$

Intuitively,  $y_\tau$  depends on aggregate demand and on the technology-specific level of TFP relative to the overall level of TFP and on the relative rental for technology  $\tau$  capital relative to the aggregate rental rate.

rate of real GDP per capita. In the neoclassical growth model, this reflects that, along the transitional path, economies with a below steady state level of capital have a higher marginal product of capital. In our model, there is the additional effect of the adoption lags on the transitional path. We use this result to approximate

$$r_\tau \approx c_1 - q_\tau t + c_2 \Delta(y - l). \quad (39)$$

Here,  $\Delta$  denotes the first-difference operator in time, such that  $\Delta(y - l)$  is the growth rate of real GDP per capita. The trend part reflects investment specific technological change and  $c_1$  is a constant. Since the approximate log-linear relationship between the real interest rate and the growth rate of output depends both on technology and preference parameters, like the intertemporal elasticity of substitution, both  $c_1$  and  $c_2$  depend on preference parameters.

At this point we could estimate the reduced form equations. However, after careful investigation of the effect of the rate of embodied productivity,  $\gamma_\tau$ , on the dependent variables, we observe that to a first order approximation  $\gamma_\tau$  only affects  $y_\tau$  and  $k_\tau$  through the linear trend. More specifically, in Appendix B, we log-linearize (14) around  $\gamma_\tau = 0$  to obtain the approximation

$$z_\tau \approx z_{v_\tau} + (\mu - 1) \ln(t - T_\tau) - \frac{\gamma_\tau}{2} (t - T_\tau), \quad (40)$$

where  $T_\tau = v_\tau + D_\tau$  is the time that the technology is adopted.

In this approximation, the growth rate of embodied technological change,  $\gamma_\tau$ , only affects the linear trend in  $z_\tau$ . Intuitively, when there are very few vintages in  $V_\tau$  the growth rate of the number of vintages, i.e. the growth rate of  $t - T_\tau$ , is very large and it is this growth rate that drives growth in  $z_\tau$  through the variety effect. Only in the long-run, when the growth rate of the number of varieties tapers off, the growth rate of embodied productivity,  $\gamma_\tau$ , becomes the predominant driving force of the variety effect.

This result implies that, in first-order, both  $\gamma_\tau$  and  $q_\tau$  cause a linear trend in our technology measures. Thus,  $\gamma_\tau$ , is only separately identified from  $q_\tau$  through second-order effects, which are small for  $\gamma_\tau$  close to zero. Therefore, we present the estimates obtained using the log-linear approximation of  $z_\tau$  in the estimation, (40), and do not provide estimates of  $\gamma_\tau$ .<sup>13</sup>

Then, as we derive in Appendix B, the reduced form equation that we estimate is the same for both capital and output measures and is of the form

$$m_\tau = \beta_1 + y + \beta_2 t + \beta_3 ((\mu - 1) \ln(t - T_\tau) - (1 - \alpha)(y - l)) + \beta_4 \Delta(y - l) + \varepsilon_\tau, \quad (41)$$

where  $\varepsilon_\tau$  is the error term. The reduced form parameters are given by the  $\beta$ 's. We do not estimate  $\mu$  and  $\alpha$ . Instead, we calibrate  $\mu = 1.3$ , based on the estimates of the markup in manufacturing from Basu and

<sup>13</sup>We have also estimated the reduced form equations using the actual expression for  $z_\tau$ , (14). Because  $\gamma_\tau$  is locally non-identified at zero, this yields imprecise estimates of  $\gamma_\tau$ . However, it results in virtually identical estimates of the adoption lags.



Fernald (1997), and  $\alpha = 0.3$  consistent with the post-war U.S. labor share.<sup>14</sup>

### 4.3 Identification of adoption lags and estimation procedure

We use the reduced form equations to estimate country-technology-specific adoption lags. For this purpose, we make the following three assumptions: (i) Levels of aggregate TFP, relative investment prices, and units of measurement of the technology measures potentially differ across countries; (ii) technology-specific growth rates of investment specific technological change,  $q_\tau$ , embodied technological change,  $\gamma_\tau$ , as well as the growth rate of aggregate TFP, are the same across countries; (iii) preferences are potentially different across countries while technology parameters are the same except for the adoption lags.

In order to see how these assumptions translate into cross-country parameter restrictions, we consider which structural parameters affect each of the reduced form parameters. The fixed effect,  $\beta_1$ , captures four things (i) the units of the technology measure; (ii) the level of the relative price of investment goods,  $Q_\tau$ ; (iii) different TFP levels across countries, and (iv) differences in adoption lags. Because we assume that these things can vary across countries, we let  $\beta_1$  vary across countries as well. The trend-parameter,  $\beta_2$ , is assumed to be constant across countries because it only depends on the output elasticity of capital,  $\alpha$ ,<sup>15</sup> and on the trends in embodied and investment specific technological change,  $q_\tau$  and  $\gamma_\tau$ .  $\beta_3$  only depends on the technology parameter,  $\theta$ , and is therefore also assumed to be constant across countries. The growth rate of output per capita coefficient,  $\beta_4$ , is related to the relationship between the interest rate, user cost of capital, and output growth. Since this relationship depends on preference parameters, like the intertemporal elasticity of substitution, we let  $\beta_4$  vary across countries.

Given these cross-country parameter restrictions, the adoption lags are identified in the data through the non-linear trend component in equation (41), which is due to the variety effect. This is the only term affected by the adoption lag,  $D_\tau$ . It is also the only term which affects the curvature of  $m_\tau$  after controlling for the effect of observables such as income, per capita income and the growth in per capita income. Specifically, it causes the trend in  $m_\tau$  to monotonically decline with the time since adoption. This is the basis of our empirical identification strategy of  $D_\tau$ . Intuitively, our model predicts that, everything else equal, if at a given moment in time we observe that the trend in  $m_\tau$  is diminishing faster in one country than another, it must be because the former country has started adopting the technology more recently.

Because the adoption lag is a parameter that enters non-linearly in (41) for each country, estimating the system of equations for all countries together is practically not feasible. Instead, we take a two-step approach. We first estimate equation (41) using only data for the U.S. This provides us with estimates of

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<sup>14</sup>Our estimates of the adoption lags are robust to alternative calibration of  $\mu$  to a wide range of values both higher and lower than 1.3.

<sup>15</sup>The output elasticity of capital is one minus the labor share in our model. Gollin (2002) provides evidence that the labor share is approximately constant across countries.

the values of  $\beta_1$ ,  $\beta_4$ , and  $D_\tau$  for the U.S. as well as estimates of  $\beta_2$  and  $\beta_3$  that should hold for all countries. In the second step we separately estimate (41), and thus  $\beta_1$ ,  $\beta_4$ , and  $D_\tau$ , for all the countries in the sample besides the U.S. conditional on the estimates of  $\beta_2$  and  $\beta_3$  based on the U.S. data.

Besides practicalities, this two-step estimation method is preferable to a system estimation method for two other reasons. First, if we would apply a system estimation method, data problems for one country would affect the estimates for all countries. Since we judge the U.S. data to be most reliable, we use them for the inference on the parameters that are constant across countries. Second, our model is based on a set of stark neoclassical assumptions. These assumptions are more applicable to the low frictional U.S. economic environment than to that of countries in which capital and product markets are substantially distorted. Thus, we think that our reduced form equation is likely to be misspecified for some countries other than the U.S. Including them in the estimation of the joint parameters would affect the results for all countries.

We estimate all the equations using non-linear least squares. This means that the identifying assumption that we make is that the logarithm of GDP, of per capita GDP and the growth rate of per capita GDP are uncorrelated with the technology-specific error,  $\varepsilon_\tau$ . This identifying assumption essentially means that the causation goes from aggregate economic activity to the adoption of a particular technology and not the other way around. This is probably not an unreasonable assumption, since we focus on data for 15 out of many technologies that drive aggregate economic fluctuations. A piece of evidence that supports this assumption is that, while our individual measures of technology are highly non-linear, aggregate measures such as log TFP or log per capita GDP are almost linear.

Because we derive the reduced form equations from a structural model, the theory pins down the set of explanatory variables. However, even if one takes the theory as given, there are, of course, several potential sources of bias in our estimates. The most important is our assumption that  $D_\tau$  is constant over time. Because  $D_\tau$  is identified through the curvature in the data variations in  $D_\tau$  over time would be identified by changes in this curvature. There is simply too little variation in the data for this identification scheme. If there is time variation in  $D_\tau$  then our estimates would be skewed towards the adoption lag at the time of adoption. This is because the variation in the curvature of the non-linear trend being larger right after adoption than later on.

## 5 Results

We consider data for 166 countries and 15 technologies, that span the period from 1820 through 2003. The technologies can be classified into 6 categories; (i) transportation technologies, consisting of steam- and motorships, passenger and freight railways, cars, trucks, and passenger and freight aviation; (ii) telecommunication, consisting of telegraphs, telephones, and cellphones; (iii) IT, consisting of PCs and internet users;

(*iv*) medical, being MRI scanners; (*v*) steel, namely tonnage produced using blast oxygen furnaces; (*vi*) electricity.

The technology measures are taken from the CHAT dataset, introduced by Comin and Hobijn (2004) and expanded by Comin, Hobijn, and Rovito (2006). Real GDP and population data are from Maddison (2007). Appendix A contains a brief description of each of the 15 technology variables used.

Unfortunately, we do not have data for all 2490 country-technology combinations. For our estimation, we only consider country-technology combinations for which we have more than 10 annual observations. There are 1278 such pairs in our data. The third column of Table 1 lists, for each technology, the number of countries for which we have enough data.

For each of the 15 technologies, we perform the two-step estimation procedure outlined above. We divide the resulting estimates up into three main groups: (*i*) plausible and precise, (*ii*) plausible but imprecise, and (*iii*) implausible.

We consider an estimate *implausible* if our point estimate implies that the technology was adopted more than 10 years before it was invented. The 10 year cut off point is to allow for inference error. The sixth column of Table 1 lists the number of implausible estimates for each of the technologies. In total, we find implausible estimates in a bit less than one-third, i.e. 394 out of 1278, of our cases.

We have identified three main reasons why we obtain implausible estimates. First, as mentioned above, the adoption year  $T_\tau$  is identified by the curvature in the time-profile of the adoption measure. However, for some countries the data is too noisy to capture this curvature. In that case, the estimation procedure tends to fit the flatter part of the curve through the sample and infers that the adoption date is far in the past. Second, for some countries the data exhibit a convex technology adoption path rather than the concave one implied by our structural model. This happens in some African countries that have undergone dramatic events such as decolonization or civil wars. Third, for some countries we only have data long after the technology is adopted. In that case  $\ln(t - T_\tau)$  exhibits little variation and  $T_\tau$  is not very well-identified in the data. This can either lead to an implausible estimate of  $T_\tau$  or a plausible estimate with a high standard error.

Plausible estimates with high standard errors are considered *plausible but imprecise*. In particular, the cut off that we use is that the standard error<sup>16</sup> of the estimate of  $T_\tau$  is bigger than  $\sqrt{2003 - v_\tau}$ .<sup>17</sup> The number of plausible but imprecise estimates can be found in the fifth column of Table 1. These are 51 out of the 1278 cases that we consider.

The cases that are neither deemed implausible nor imprecise are considered *plausible and precise*. The fourth column of Table 1 reports the number of such cases for each technology. These represent 65 percent

<sup>16</sup>This standard error is conditional on the estimates of  $\beta_2$  and  $\beta_3$  that are based on U.S. data and, thus, do not take into account the inference error in these point estimates.

<sup>17</sup>This allows for longer confidence intervals for older technologies with potentially more imprecise data.

of all the technology-country pairs. Hence, our model, with the imposed U.S. parameters, yields plausible and precise estimates for the adoption lags for, a suprising, two-thirds of the technology-country pairs. In what follows, all our results are based on the sample of 833 plausible and precise estimates.<sup>18</sup>

Before we summarize the results for these 833 estimates, it is useful to start with an example. Figure 1 shows the actual and fitted paths of  $m_\tau$  for tonnage of steam and motor merchant ships for Argentina, Japan, Nigeria, and the U.S. The estimated adoption years,  $T_\tau$ , of electricity for these countries are 1870, 1959, 1901, and 1814, respectively. This means that, on average over the sample period, the pattern of U.S. steam and motor merchant ship adoption is consistent with a 1814 adoption date, according to our model. Given that the first steam boat patent in the U.S. was issued in 1788, we thus estimate that the U.S. adopted the innovations that enabled more efficient motorized merchant shipping services with an average lag of 26 years.

Given the estimates of  $\beta_2$  and  $\beta_3$  based on the U.S. data, the adoption years are identified through the curvature of the path of  $m_\tau$ . The U.S. path is already quite flat in the early part of the sample. This indicates an early adoption, i.e. a low  $T_\tau$ , and a short adoption lag. When we compare the U.S. and Argentina, we see that, in most years the path is more steep for Argentina than for the U.S. This is why we find a later adoption date and bigger adoption lag for Argentina than for the U.S. Since Japan's path is even steeper than that of Argentina, the lag for Japan is even larger. A similar analysis reveals why we find the 1959 adoption date for Nigeria.

The  $R^2$ 's associated with the estimated equations for electricity for Argentina, Japan, Nigeria, and the U.S. are 0.96, 0.20, 0.86, and 0.95, respectively. The  $R^2$  for Japan is very low because our model does not fit the, almost complete, destruction of the Japanese merchant fleet during WWII. The other  $R^2$ 's are not only high because the model captures the trend in the adoption patters but also because the model captures the curvature.

The last three columns of Table 1 summarize the properties of the  $R^2$ 's for the 833 plausible and precise estimates. Since we are imposing the US estimates for  $\beta_2$  and  $\beta_3$ , the  $R^2$  can be negative. The second to last column of Table 1 lists the number of cases for which we find a positive  $R^2$  for each technology.

In total, we find negative  $R^2$ 's for only 6.8 percent of the cases. Passenger railways and telegraphs are the two where negative  $R^2$  are more prevalent. This means that for those technologies the assumption that the U.S. estimates for  $\beta_2$  and  $\beta_3$  apply for all countries seems invalid. These are both technologies that have seen a decline in the latter part of the sample for the U.S. Such declines lead to estimates of the trend parameter,  $\beta_2$ , for the U.S. that do not fit the data for countries where these technologies have not seen such a decline (yet). Though present, such issues do not seem to be predominant in our results.

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<sup>18</sup>Results that also include the imprecise estimates are both qualitatively and quantitatively very similar to the ones presented here.

The next to last and last columns of Table 1 list the sample mean and standard deviations of the distributions of positive  $R^2$ 's for each technology. Overall, the average  $R^2$ , conditional on being positive, is 0.82 and the standard deviation of these  $R^2$ 's is 0.10. Hence, even though we impose U.S. estimates for  $\beta_2$  and  $\beta_3$  across all countries, the simple reduced form equation, (41), derived from our model captures the majority of the variation in  $m_\tau$  over time for the bulk of the country-technology combinations in our sample.

We turn next to the estimates of the diffusion lags. The main summary statistics regarding these estimates are reported in Table 2. The average diffusion lag in our sample is 45 years with a median lag of 35. This means that the average adoption path of countries in our sample over all technologies is similar to that of a country that adopts the technology 45 years after its invention.

However, there is considerable variation both across technologies and countries. For steam- and motorships as well as railroads we find that it took about a century before they were adopted in half of the countries in our sample. This is in stark contrast with PCs and the internet, for which it took less than 15 years for half of the countries in our sample to adopt them.

Though we do not impose it, we find that the percentiles of the estimated adoption lags are similar for closely related technologies; passenger and freight rail transportation, cars and trucks, passenger and cargo aviation, and even for the upper percentiles of telegraphs and telephones.

Table 3 decomposes the variations in adoption lags into parts attributable to country effects and parts due to technology effects. Let  $i$  be the country index and let  $D_{i\tau}$  be the adoption lag estimated for country  $i$  and technology  $\tau$ . Table 3 contains the variance decomposition based on three regressions nested in the following specification

$$D_{i\tau} = D_i^* + D_\tau^* + u_{i\tau} \tag{42}$$

where  $D_i^*$  is a country fixed effect,  $D_\tau^*$  is a technology fixed effect, and  $u_{i\tau}$  is the residual. The first line of the table pertains to (42) with only country fixed effects. Country-specific effects explain about 30% of the variation in the estimated adoption lags. Technology-specific effects explain about twice as much, namely 66% of the variation. This can be seen from the second row of Table 3, which is computed from a version of regression (42) with only technology fixed effects. The last row of Table 3 shows that country and technology fixed effects jointly explain about 83% of the variation in the estimated adoption lags. Of this, 18% can be directly attributed to country effects, 54% can be directly attributed to technology effects, and the remaining 11% is due to the covariance between these effects that is the result of the unbalanced nature of the panel structure of our data.

Understanding the determinants of the cross-country or cross-technology adoption lags is beyond the goals of this paper. However, we do consider whether adoption lags tend to have gotten smaller over time. To this end, Figure 2 plots the invention date of each technology,  $\underline{v}_\tau$ , against the average adoption lag by technology as well as against the technology fixed effects,  $D_\tau^*$ , obtained from (42). The message from both

variables is the same. Newer technologies have diffused much faster than older technologies. In particular, technologies invented ten years later are on average adopted 4.3 years faster.

This finding is remarkably robust. As is clear from Figure 2, the average adoption lags of all 15 technologies covered in our dataset seem to adhere to this pattern. Moreover, the slope before and after 1950 is almost the same. Hence, the acceleration of the adoption of technologies seems to have started long before the digital revolution or the post-war globalization process.

Of course, this trend cannot go on forever. However, it has gone on at this pace for 200 years. If it persists, it will have major consequences for the cross-country differences in TFP due to the lag in technology adoption. In particular, the TFP gap between rich and poor countries due to the lag in technology adoption should be significantly reduced.

## 5.1 Case studies

Thus far, we have focused on computing a set of broad summary statistics that describe the properties of the estimated adoption lags. In addition to these broad patterns, these estimates also shed some light on a number of debates that focus on particular (groups of) countries and episodes. To see how, consider Table 4. For each technology, it contains the average adoption lag for different (groups of) countries relative to the average adoption lag for the technology.

### 5.1.1 U.S. and the U.K.

The U.S. and the U.K. have been the technological leaders over the last two centuries. Most of the major technologies invented over the last two centuries have been invented either in the U.S. or in the U.K. Table 4 shows that they also have adopted new technologies much faster than the rest of the world. The shorter adoption lags have surely contributed to their high levels of productivity and per capita income.

### 5.1.2 Japan

Until the Meiji restoration in 1867, Japan had an important technological gap with the western world. This is reflected in the Japanese adoption lag in steam and motor ships which is much longer than that in other OECD countries and is comparable to the lags in Latin America. Technological backwardness, surely, was a significant determinant of the development gap between Japan and other (now) industrialized countries; in 1870, Japan's real GDP per capita was 42 percent of the OECD average.

The industrialization process that was catalyzed by the Meiji restoration closed Japan's technological gap with the western world. This is reflected by Japan's adoption lags for the technologies invented in the 19th century, which are comparable to the lags in other OECD countries. The closing of the technology gap also diminished the development gap. By 1920, per capita GDP in Japan was 56 percent of the OECD

average. For those technologies invented in the Twentieth Century, Japan's adoption lag was significantly shorter than for the OECD average and it was comparable to the U.S. and also comparable to, if not shorter than, the U.K.'s. For blast oxygen steel, for example, the adoption lag that we estimate for Japan is 5 years shorter than for the U.S. and almost 6 years shorter than for the U.K. By 1980 Japan's per capita income was about the same as the U.K., 26 percent higher than the OECD average, and 33 percent lower than the U.S.

The estimated adoption lags for Japan thus seem to suggest that a large part of Japan's phenomenal rise in living standards between 1870 and 1980 involved closing the gap between the range technologies Japan used and those used by the world's industrialized leaders.

### 5.1.3 East Asian Tigers

Japan's phenomenal rise was outdone in the second half of the 20th century by the East Asian Tigers (EATs); Hong Kong, Korea, Taiwan and Singapore. These four countries experienced 'miraculous' growth in per capita GDP between 1960 and 1995 of around 6 percent per year.

There is disagreement about the sources of this growth. Young (1995) claims that factor accumulation is the main source of growth in the EATs, while Hsieh (2002) challenges this view and argues that the TFP growth experienced by the EATs is underestimated by Young (1995).<sup>19</sup>

Whether or not adoption lags show up as TFP or factor accumulation differentials depends on the extent to which capital stock data are quality adjusted. However, what we can say, based on our estimates, is that, just like for Japan, the growth spurt of the EATs has been associated with a substantial reduction in their technology adoption lags.

From Table 4, it is clear that the EATs had long adoption lags for early technologies. In particular, for technologies invented before 1950, the EATs' adoption lags were often longer than in Sub-Saharan Africa (SSA), and almost always longer than in Latin America. For newer technologies, however, the EAT's adoption lags are shorter than in Latin America and Sub-Saharan Africa. In fact, EATs adopted technologies invented since 1950 about as fast as OECD countries.

Young (1992) focuses on the sources of growth in Singapore and Hong Kong and argues that the lower TFP growth rate observed in Singapore reflects its faster rate of structural transformation towards the production of electronics and services, which did not allow agents to learn how to efficiently use older technologies. Some of the post-1950 technologies in our data set such as computers, cellphones, and the internet are surely significant for the production of both electronics and services. Hence, an implication of the Young hypothesis would be that the Singaporean adoption lags in these technologies are shorter than in Hong Kong. As can

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<sup>19</sup>More specifically, According to Hsieh, TFP growth was 2.2% in Singapore (vs. -0.7 for Young), 3.7% in Taiwan (vs. 2.1% for Young), 1.5% in Korea (vs. 1.7% for Young) and 2.3% in Hong Kong (vs. 2.7% for Young).

be seen from Table 5, this is not what we find. Singapore and Hong Kong are estimated to have the same adoption lags in PCs and the internet, 14 and 7 years respectively. Hong Kong is estimated to have adopted cellphones three years earlier than Singapore.

#### **5.1.4 Latin America**

Where the EATs are considered growth ‘miracles’, Latin American countries are often labeled as growth ‘failures’. Some of them, such as Chile and Argentina, were among the richest countries in the world during the late 1800s and the first half of the Twentieth Century (De Long, 1988). This designation is reflected in the fact that for the pre-1950 technologies Latin American countries adopted new technologies faster than the average country. Since World War II, however, they have failed to maintain their position in the development rankings and have been leapfrogged by numerous emerging economies, mostly in Asia. As can be seen from Table 4, this disappointing growth performance since 1950 coincides with longer lags in the adoption of new technologies in Latin American countries than in the average country.

#### **5.1.5 Sub-Saharan Africa**

Most Sub-Saharan countries have failed to grow at above average rates despite their low initial per capita income. This performance is consistent with the long lags in technology adoption reported in Table 4. For example, the adoption lags for passenger and freight aviation were, respectively, 22 and 33 years longer than for the average country. The extra lag was 24 years for steam and motor ships, 30 years for the telegraph, and 10 years for the telephone. The most recent technologies have also been adopted more slowly in Sub-Saharan countries than in the rest of the world. However, due to the overall decline in adoption lags, the difference between the lags of Sub-Saharan African countries and the average adoption lags for these technologies are much shorter, i.e. between 1 and 2 years.

## **5.2 Development accounting**

The brief case studies presented above suggest that variation in adoption lags may be associated with both cross-country and time series variation in per capita income. Next, we explore whether the anecdotes described above can be generalized. Specifically, we ask the following question: Are the adoption lags that we estimated a significant potential source of cross-country per capita income differences?

To answer this question, we have to approximate the aggregate effect of the estimated adoption lags for the 15 technologies on per capita GDP levels. We do so by using the equilibrium results of our one-sector growth model. If the only source of cross-country differentials in per capita GDP is adoption lags, then, in



steady state, the log difference of country  $i$ 's level of real GDP per capita with that of the U.S. is given by

$$(y_i - l) - (y_{USA} - l) = \frac{\chi + \gamma}{1 - \alpha} (D_{US} - D_i), \quad (43)$$

where  $(\chi + \gamma)$  is the growth rate of aggregate TFP, which is 1.4% for the U.S. private business sector during the postwar period. We observe the left hand side of (43) in our data and approximate the right hand side in the following way. We use  $\chi + \gamma = 0.014$  and  $\alpha = 0.3$ , consistent with postwar U.S. data. Moreover, we use the country fixed effects from (42) to approximate  $D_i \approx D_i^*$ . Hence, we assume that the country-specific adoption lags we have estimated for each country using our sample of technologies is representative of the average adoption lags across all the technologies used in production.

Figure 3 plots the data for both sides of (43) for 123 countries in our dataset. The correlation between both sides is 0.51. The solid line is the regression line while the dashed line is the 45°-line. The slope of the regression line is about 0.25, which can be interpreted as that our model and estimates explain about one fourth of the log per capita GDP differentials observed in the data.

The model seems to explain a much larger part of per-capita income differentials for high-income industrialized countries that make up the set of observations in the upper-right corner of the figure.<sup>20</sup> This may result from a downward bias in our estimates of  $D_i^*$  for the poor countries in our sample. Specifically, due to lack of data and/or plausible estimates for older technologies in poor countries, these technologies, which tend to be adopted more slowly, do not affect the estimate of  $D_i^*$  for poor countries. This may result in a downward bias of the average adoption lag for poor countries and in a lower cross-country dispersion in adoption lags and in TFP differentials due to differences in adoption.

In conclusion, our empirical exploration shows that adoption lags account for a substantial share of cross-country per capita income differences. The share they account for seems to be at least 25%, if not more.

## 6 Conclusion

In this paper we have built and estimated a model of technology diffusion and growth that has two main characteristics. First, at the aggregate level, it is similar to the one sector neoclassical growth model. Second, at the disaggregate level, it has implications for the path of observable measures of technology adoption, such as the number of units of capital that embody a given technology or the output produced with this technology.

The main focus of our analysis is on adoption lags. These lags are defined as the length of time between the invention and adoption of a technology. Our model provides a theoretical framework that links the adoption

<sup>20</sup>The slope for these countries is approximately 1.

lag of a technology to the level of productivity embodied in the capital associated with the technology. It also relates the path of the observable technology adoption measures over time to the path of embodied productivity and to economy-wide factors driving aggregate demand. The adoption lag determines the shape of a non-linear trend in embodied productivity as well as in the path of the technology measures. It is this non-linear trend term that allows us to identify adoption lags in the data.

We estimate adoption lags for 15 technologies and 166 countries over the period 1820-2003, using data from Comin, Rovito, and Hobijn (2006). Our model does a good job in fitting the diffusion curves. For two thirds of the technology-country pairs we obtain precise and plausible estimates of the adoption lags. In light of this result, we conclude that our model of diffusion provides an empirically relevant micro-foundation for a new set of measures of technology diffusion that are more comprehensive and easier to obtain than the measures used in the traditional empirical diffusion literature.

We obtain three key findings. The first is that adoption lags are large, 47 years on average, and vary a lot. The standard deviation is 39 years. Most of this variation is due to technology-specific variation, which contributes more than half of the variance of adoption lags in our sample. Over the two centuries for which we have data the average adoption lag across countries for new technologies has steadily declined.

The second finding is that the growth ‘miracles’ of Japan and the East Asian Tigers, though more than half a century apart, both coincided with a reduction of the technology adoption lags in these countries relative to those in their OECD counterparts.

Third, when we use our model to quantify the implications of the country-specific variation in adoption lags for cross-country per capita income differentials, we find that differences in technology adoption account for at least a quarter of per capita income disparities in our sample of countries.

Our exploration yields a set of precise estimates of the size of adoption lags across a broad range of technologies and countries. We plan on using these in subsequent work to investigate what are the key cross-country differences in endowments, institutions, and policies that impinge on technology diffusion.

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## A Data

The data that we use are taken from two sources. Real GDP and population data are taken from Maddison (2007). The data on the technology measure are from the Cross-Country Historical Adoption of Technology (CHAT) data set, first described in Comin, Hobijn, and Rovito (2006). The fifteen particular technology measures that we consider are:

1. **Steam and motor ships:**

*Definition:* Gross tonnage (above a minimum weight) of steam and motor ships in use at midyear.

*Invention year:* 1788; the year the first (U.S.) patent was issued for a steam boat design.

2. **Railways - Passengers:**

*Definition:* Passenger journeys by railway in passenger-KM.

*Invention year:* 1825; the year of the first regularly schedule railroad service to carry both goods and passengers.

3. **Railways - Freight:**

*Definition:* Metric tons of freight carried on railways (excluding livestock and passenger baggage).

*Invention year:* 1825; same as passenger railways.

4. **Cars:**

*Definition:* Number of passenger cars (excluding tractors and similar vehicles) in use.

*Invention year:* 1885; the year Gottlieb Daimler built the first vehicle powered by an internal combustion engine.

5. **Trucks:**

*Definition:* Number of commercial vehicles, typically including buses and taxis (excluding tractors and similar vehicles), in use.

*Invention year:* 1885; same as cars.

6. **Aviation - Passengers:**

*Definition:* Civil aviation passenger-KM traveled on scheduled services by companies registered in the country concerned.

*Invention year:* 1903; The year the Wright brothers managed the first succesful flight.

7. **Aviation - Freight:**

*Definition:* Civil aviation ton-KM of cargo carried on scheduled services by companies registered in the country concerned.

*Invention year:* 1903; same as aviation - passengers.

8. **Telegraph:**

*Definition:* Number of telegrams sent.

*Invention year:* 1835; year of invention of telegraph by Samuel Morse at New York University.

9. **Telephone:**

*Definition:* Number of telegrams sent.

*Invention year:* 1876; year of invention of telephone by Alexander Graham Bell.

10. **Cellphone:**

*Definition:* Number of users of portable cell phones.

*Invention year:* 1973; first call from a portable cellphone.

11. **Personal computers:**

*Definition:* Number of self-contained computers designed for use by one person.

*Invention year:* 1973; first computer based on a microprocessor.

12. **Internet users:**

*Definition:* Number of people with access to the worldwide network.

*Invention year:* 1983; introduction of TCP/IP protocol.

13. **MRIs:**

*Definition:* Number of magnetic resonance imaging (MRI) units in place.

*Invention year:* 1977; first MRI-scanner built.

14. **Blast Oxygen Steel:**

*Definition:* Crude steel production (in metric tons) in blast oxygen furnances (a process that replaced bessemer and OHF processes).

*Invention year:* 1950; invention of Blast Oxygen Furnace.

15. **Electricity:**

*Definition:* Gross output of electric energy (inclusive of electricity consumed in power stations) in KwHr.

*Invention year:* 1882; first commercial powerstation on Pearl Street in New York City.

## B Mathematical details

### Derivation of equation (8):

The demand for capital of a particular vintage is given by the factor demand equation

$$R_{v\tau}K_{v\tau} = \alpha P_{v\tau} Y_{v\tau} \quad (44)$$

Since revenue generated from the output produced with the vintage is determined by the demand function (6), we can write

$$R_{v\tau}K_{v\tau} = \alpha Y_{\tau} P_{\tau}^{\frac{\mu}{\mu-1}} P_{v\tau}^{-\frac{1}{\mu-1}} \quad (45)$$

Moreover, the price of the output produced with this vintage is given by the equilibrium unit production cost (7), such that we can write

$$R_{v\tau}K_{v\tau} = \alpha Y_{\tau} P_{\tau}^{\frac{\mu}{\mu-1}} Z_{v\tau}^{\frac{1}{\mu-1}} \left( \frac{(1-\alpha)}{W} \right)^{\frac{1-\alpha}{\mu-1}} \left( \frac{\alpha}{R_{v\tau}} \right)^{\frac{\alpha}{\mu-1}} \quad (46)$$

such that

$$K_{v\tau} = Y_{\tau} P_{\tau}^{\frac{\mu}{\mu-1}} Z_{v\tau}^{\frac{1}{\mu-1}} \left( \frac{(1-\alpha)}{W} \right)^{\frac{1-\alpha}{\mu-1}} \left( \frac{\alpha}{R_{v\tau}} \right)^{\epsilon} \quad (47)$$

where

$$\epsilon = \frac{\mu}{\mu-1} - \frac{1-\alpha}{\mu-1} = 1 + \frac{\alpha}{\mu-1} \quad (48)$$

which is equation (8).

### Derivation of equation (9):

The Lagrangian associated with the dynamic profit maximization problem of the supplier of capital good  $v$  for intermediate  $\tau$  at time  $t$  equals

$$\mathcal{L}_{v\tau t} = \int_t^{\infty} e^{-\int_t^s r_{s'} ds'} H_{v\tau s} ds \quad (49)$$

where  $H_{v\tau s}$  is the current value Hamiltonian. We will drop the time subscript  $s$  in what follows. Here

$$\begin{aligned} H_{v\tau} &= (R_{v\tau}K_{v\tau} - QI_{v\tau}) + \\ &\lambda_{v\tau} \left( R_{v\tau}K_{v\tau} - \alpha Y_{\tau} P_{\tau}^{\frac{\mu}{\mu-1}} Z_{v\tau}^{\frac{1}{\mu-1}} \left( \frac{(1-\alpha)}{W} \right)^{\frac{1-\alpha}{\mu-1}} \left( \frac{\alpha}{R_{v\tau}} \right)^{\epsilon-1} \right) + \\ &\nu_{v\tau} (I_{v\tau} - \delta_{\tau}K_{v\tau}) \end{aligned} \quad (50)$$

Here  $\lambda_{v\tau}$  is the co-state variable associated with the demand function that the capital goods supplier faces and  $\nu_{v\tau}$  is the co-state variable associated with the capital accumulation equation.

The resulting optimality conditions read

$$\begin{aligned} \text{w.r.t. } R_{v\tau}: & \quad (1 + \lambda_{v\tau}) K_{v\tau} + (\epsilon - 1) \lambda_{v\tau} K_{v\tau} = 0 \\ \text{w.r.t. } I_{v\tau}: & \quad \nu_{v\tau} = Q \\ \text{w.r.t. } K_{v\tau}: & \quad (1 + \lambda_{v\tau}) R_{v\tau} - \delta \nu_{v\tau} = r \nu_{v\tau} - \dot{\nu}_{v\tau} \end{aligned} \quad (51)$$

The first optimality condition yields that

$$\lambda_{v\tau} = -\frac{1}{\epsilon} \quad (52)$$

while the second and third yield that

$$\begin{aligned} R_{v\tau} &= \frac{1}{(1 + \lambda_{v\tau})} Q (r + \delta_{\tau} + q_{\tau}) \\ &= \frac{\epsilon}{\epsilon - 1} Q (r + \delta_{\tau} + q_{\tau}) \end{aligned} \quad (53)$$

which is (9). Note that the resulting flow profits satisfy

$$\pi_{v\tau} = \frac{1}{\epsilon - 1} Q_\tau (r + \delta_\tau + q_\tau) K_{v\tau} = \frac{1}{\epsilon} R_{v\tau} K_{v\tau} \quad (54)$$

**Derivation of intermediate technology aggregation results:**

The factor demands for each of the vintage specific output types satisfy

$$WL_\tau = W \int_{v \in V_\tau} L_{v\tau} dv = (1 - \alpha) \int_{v \in V_\tau} P_{v\tau} Y_{v\tau} dv = (1 - \alpha) P_\tau Y_\tau \quad (55)$$

and

$$RK_\tau = R \int_{v \in V_\tau} K_{v\tau} dv = \alpha \int_{v \in V_\tau} P_{v\tau} Y_{v\tau} dv = \alpha P_\tau Y_\tau \quad (56)$$

Hence relative factor demands are the same as relative revenue levels

$$\frac{P_{v\tau} Y_{v\tau}}{P_\tau Y_\tau} = \left( \frac{Y_{v\tau}}{Y_\tau} \right)^{\frac{1}{\mu}} = \frac{L_{v\tau}}{L_\tau} = \frac{K_{v\tau}}{K_\tau} \quad (57)$$

which allows us to write

$$\begin{aligned} Y_{v\tau} &= Z_{v\tau} K_{v\tau}^\alpha L_{v\tau}^{1-\alpha} = Z_{v\tau} \left( \frac{Y_{v\tau}}{Y_\tau} \right)^{\frac{1}{\mu}} K_\tau^\alpha L_\tau^{1-\alpha} \\ &= (Z_{v\tau})^{\frac{\mu}{\mu-1}} \left( \frac{1}{Y_\tau} \right)^{\frac{1}{\mu}} (K_\tau^\alpha L_\tau^{1-\alpha})^{\frac{\mu}{\mu-1}} \end{aligned} \quad (58)$$

Such that we obtain that

$$\begin{aligned} Y_\tau &= \left( \int_{v \in V_\tau} Y_{v\tau}^{\frac{1}{\mu}} dv \right)^\mu = \left( \int_{v \in V_\tau} Z_{v\tau}^{\frac{1}{\mu-1}} dv \right)^\mu \left( \frac{1}{Y_\tau} \right)^{\frac{1}{\mu}} (K_\tau^\alpha L_\tau^{1-\alpha})^{\frac{\mu}{\mu-1}} \\ &= \left( \int_{v \in V_\tau} Z_{v\tau}^{\frac{\mu}{\mu-1}} dv \right)^{\mu-1} (K_\tau^\alpha L_\tau^{1-\alpha}) = Z_\tau K_\tau^\alpha L_\tau^{1-\alpha} \end{aligned} \quad (59)$$

The value of the unit production cost follows from the unit production cost of a Cobb-Douglas production function. The aggregation results at the highest level of aggregation can be derived in a similar way.

**Derivation of equation (14):**

This follows from

$$\begin{aligned} Z_\tau &= \left( \int_{v \in V_\tau} Z_{v\tau}^{\frac{1}{\mu-1}} dv \right)^{\mu-1} = \left( \int_{\underline{v}_\tau}^{t-D_\tau, t} (Z_{\underline{v}_\tau} e^{\gamma_\tau (v-\underline{v}_\tau)})^{\frac{1}{\mu-1}} dv \right)^{\mu-1} \\ &= Z_{\underline{v}_\tau} \left( \int_{\underline{v}_\tau}^{t-D_\tau, t} e^{\frac{\gamma_\tau}{\mu-1} (v-\underline{v}_\tau)} dv \right)^{\mu-1} = \left( \frac{\mu-1}{\gamma_\tau} \right)^{\mu-1} Z_{\underline{v}_\tau} \left[ e^{\frac{\gamma_\tau}{\mu-1} \gamma_\tau (t-D_\tau, t-\underline{v}_\tau)} - 1 \right]^{\mu-1} \\ &= \left( \frac{\mu-1}{\gamma_\tau} \right)^{\mu-1} Z_{\underline{v}_\tau} e^{\gamma_\tau (t-D_\tau, t-\underline{v}_\tau)} \left[ 1 - e^{-\frac{\gamma_\tau}{\mu-1} (t-D_\tau, t-\underline{v}_\tau)} \right]^{\mu-1} \end{aligned} \quad (60)$$

**Derivation of equation (24):**

Under the one-sector model assumptions, the price of intermediates produced with capital goods of vintage  $v$  and the aggregate price level equal

$$P_{v,\tau} = \frac{1}{Z_{v,\tau}} \left( \frac{W}{1-\alpha} \right)^{1-\alpha} \left( \frac{R}{\alpha} \right)^\alpha \quad \text{and} \quad P = \frac{1}{A} \left( \frac{W}{1-\alpha} \right)^{1-\alpha} \left( \frac{R}{\alpha} \right)^\alpha \quad (61)$$

As a consequence, the relative price of output produced with vintage  $v$  is given by the relative TFP level, i.e.

$$\frac{P_{v,\tau}}{P} = P_{v,\tau} = \frac{A}{Z_{v,\tau}} \quad (62)$$

From the demand function we obtain that the revenue from output produced with capital goods of vintage  $v$  is given by

$$P_{v,\tau} Y_{v,\tau} = \left( \frac{P}{P_{v,\tau}} \right)^{\frac{1}{\theta-1}} Y = \left( \frac{Z_{v,\tau}}{A} \right)^{\frac{1}{\theta-1}} Y \quad (63)$$

The flow profits that the capital goods producer of vintage  $v$  makes are equal to

$$\pi_{v,\tau} = \frac{\alpha}{\epsilon} P_{v,\tau} Y_{v,\tau} = \frac{\alpha}{\epsilon} \left( \frac{Z_{v,\tau}}{A} \right)^{\frac{1}{\theta-1}} Y \quad (64)$$



This means that the market value of each of the capital goods suppliers of vintage  $v$ , for each of the technologies, at time  $t$  equals the present discounted value of the above flow profits. That is,

$$M_{v,\tau,t} = \int_t^\infty e^{-\int_t^s r_{s'} ds'} \pi_{v,s} ds \quad (65)$$

$$= \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{1}{\theta-1}} \left( \frac{Z_{t,\tau}}{A_t} \right)^{\frac{1}{\theta-1}} \frac{\alpha}{\epsilon} \int_t^\infty e^{-\int_t^s r_{s'} ds'} \left( \frac{A_t}{A_s} \right)^{\frac{1}{\theta-1}} Y_s ds \quad (66)$$

$$= \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{1}{\theta-1}} \left( \frac{Z_{t,\tau}}{A_t} \right)^{\frac{1}{\theta-1}} \left[ \frac{\alpha}{\epsilon} \int_t^\infty e^{-\int_t^s r_{s'} ds'} \left( \frac{A_t}{A_s} \right)^{\frac{1}{\theta-1}} \frac{Y_s}{Y_t} ds \right] Y_t \quad (67)$$

$$= \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{1}{\theta-1}} \left( \frac{Z_{t,\tau}}{A_t} \right)^{\frac{1}{\theta-1}} V_t Y_t \quad (68)$$

**Derivation of equilibrium adoption lag, (27):**

The optimal adoption of technology vintages implies that the best vintage adopted at each instant satisfies

$$\Gamma_{v\tau} = M_{v\tau} \quad (69)$$

The adoption costs satisfy

$$\begin{aligned} \Gamma_{vt} &= \bar{V} (1+b) \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{\vartheta}{\theta-1}} P_{v,\tau} Y_{v,\tau} \\ &= \bar{V} (1+b) \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{1+\vartheta}{\theta-1}} \left( \frac{Z_{t,\tau}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \end{aligned} \quad (70)$$

Combining this with the market value of the capital goods supplier of capital good  $(v, \tau)$ , we obtain that the vintage that satisfies (69), solves

$$\left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{\vartheta}{\theta-1}} = \min \left\{ 1, \frac{1}{1+b} \left( \frac{V_t}{\bar{V}} \right) \right\} \quad (71)$$

such that

$$\ln Z_{v,\tau} - \ln Z_{t,\tau} = \min \left\{ 0, -\frac{\theta-1}{\vartheta} \{ \ln(1+b) - \ln V_t - \ln \bar{V} \} \right\} \quad (72)$$

which means that the adoption lag equals

$$D_{t,\tau} = \max \left\{ \frac{\theta-1}{\gamma\vartheta} \{ \ln(1+b) - \ln V_t - \ln \bar{V} \}, 0 \right\} = D_t \quad (73)$$

and constant across technologies,  $\tau$ .

**Best vintage adopted:**

In the main text, we present the equilibrium dynamics of the model for the particular case in which, at every instant, there are some vintages adopted. This does not have to be the case along all equilibrium paths of this economy. Here, in the appendix, we derive the general equilibrium dynamics of the model and subsequently explain how the one main text is a special case.

For these general dynamics, we define  $\bar{v}_t$  as the best vintage adopted until time  $t$ . This means that if  $\bar{v}_t > t - D_t$ , then, at instant  $t$ , there will be no additional vintages adopted. In the main text, we limited ourselves to the case in which, at any point in time,  $\bar{v}_t = t - D_t$ .

**Derivation of aggregate TFP, (28):**

This allows us to write aggregate total factor productivity as

$$\begin{aligned}
A_t &= \left( \int_{-\infty}^t \int_{\tau}^{\max\{\bar{v}_t, \tau\}} Z_{v, \tau}^{\frac{1}{\theta-1}} dv d\tau \right)^{\theta-1} \\
&= Z_0 \left( \int_{-\infty}^t e^{\frac{\chi}{\theta-1} \tau} \int_{\tau}^{\max\{\bar{v}_t, \tau\}} e^{\frac{\gamma}{\theta-1} v} dv d\tau \right)^{\theta-1} \\
&= Z_0 \left( \int_{-\infty}^{\bar{v}_t} e^{\frac{\chi}{\theta-1} \tau} \int_{\tau}^{\bar{v}_t} e^{\frac{\gamma}{\theta-1} v} dv d\tau \right)^{\theta-1} \\
&= Z_0 \left( \frac{\theta-1}{\gamma} \right)^{\theta-1} \left( \int_{-\infty}^{\bar{v}_t} e^{\frac{\chi}{\theta-1} \tau} \left[ e^{\frac{\gamma}{\theta-1} \bar{v}_t} - e^{\frac{\gamma}{\theta-1} \tau} \right] d\tau \right)^{\theta-1} \\
&= Z_0 \left( \frac{\theta-1}{\gamma} \right)^{\theta-1} \left( \int_{-\infty}^{\bar{v}_t} e^{\frac{\chi}{\theta-1} (\tau - \bar{v}_t)} - e^{\frac{\chi+\gamma}{\theta-1} (\tau - \bar{v}_t)} d\tau \right)^{\theta-1} e^{(\chi+\gamma)\bar{v}_t} \\
&= Z_0 \left( \frac{\theta-1}{\gamma} \right)^{\theta-1} \left( \left( \frac{\theta-1}{\chi} \right) - \left( \frac{\theta-1}{\chi+\gamma} \right) \right)^{\theta-1} e^{(\chi+\gamma)\bar{v}_t} \\
&= A_0 e^{(\chi+\gamma)\bar{v}_t}
\end{aligned} \tag{74}$$

which, under the assumption that  $\bar{v}_t = t - D_t$ , equals

$$A_t = A_0 e^{(\chi+\gamma)(t-D_t)} \tag{75}$$

#### Derivation of aggregate adoption costs, (29):

We derive the aggregate adoption costs at each instant of time by taking the limit of the adoption cost at a period of time of length  $dt$  starting at time  $t$  for  $dt$  going to zero. The total adoption costs between time  $t$  and  $t + dt$  in the economy are given by

$$\begin{aligned}
\Gamma_t dt &= \int_{-\infty}^{\bar{v}_t} \int_{\bar{v}_t}^{\bar{v}_t+dt} \bar{V} (1+b) \left( \frac{Z_{v, \tau}}{Z_{t, \tau}} \right)^{\frac{1+\vartheta}{\theta-1}} \left( \frac{Z_{t, \tau}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t dv d\tau \\
&\quad + \int_{\bar{v}_t}^{\bar{v}_t+dt} \int_{\tau}^{\bar{v}_t+dt} \bar{V} (1+b) \left( \frac{Z_{v, \tau}}{Z_{t, \tau}} \right)^{\frac{1+\vartheta}{\theta-1}} \left( \frac{Z_{t, \tau}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t dv d\tau
\end{aligned} \tag{76}$$

This is solved most easily in two parts. The integral

$$\int_{-\infty}^{\bar{v}_t} \int_{\bar{v}_t}^{\bar{v}_t+dt} \bar{V} (1+b) \left( \frac{Z_{v, \tau}}{Z_{t, \tau}} \right)^{\frac{1+\vartheta}{\theta-1}} \left( \frac{Z_{t, \tau}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t dv d\tau \tag{77}$$

$$= \bar{V} (1+b) \int_{-\infty}^{\bar{v}_t} \left[ \int_{\bar{v}_t}^{\bar{v}_t+dt} Z_{v, \tau}^{\frac{1+\vartheta}{\theta-1}} dv \right] \left( \frac{1}{Z_{t, \tau}} \right)^{\frac{1+\vartheta}{\theta-1}} \left( \frac{Z_{t, \tau}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t d\tau \tag{78}$$

$$= \bar{V} (1+b) \left( \frac{1}{Z_0} \right)^{\frac{\vartheta}{\theta-1}} e^{-\frac{\vartheta}{\theta-1} \gamma t} \left( \frac{1}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \int_{-\infty}^{\bar{v}_t} e^{-\frac{\vartheta}{\theta-1} \chi \tau} \left[ \int_{\bar{v}_t}^{\bar{v}_t+dt} Z_{v, \tau}^{\frac{1+\vartheta}{\theta-1}} dv \right] d\tau \tag{79}$$

Note that

$$\int_{\bar{v}_t}^{\bar{v}_t+dt} Z_{v, \tau}^{\frac{1+\vartheta}{\theta-1}} dv = Z_{\bar{v}_t, \tau}^{\frac{1+\vartheta}{\theta-1}} \frac{1}{\gamma} \frac{\theta-1}{1+\vartheta} \left[ e^{\frac{1+\vartheta}{\theta-1} \gamma \left( \frac{\bar{v}_t+dt - \bar{v}_t}{dt} \right)} dt - 1 \right] \tag{80}$$

such that

$$\lim_{dt \downarrow 0} \left[ \int_{\bar{v}_t}^{\bar{v}_t+dt} Z_{v, \tau}^{\frac{1+\vartheta}{\theta-1}} dv \right] / dt = Z_{\bar{v}_t, \tau}^{\frac{1+\vartheta}{\theta-1}} \frac{1+\vartheta}{\gamma} \tag{81}$$

This means that

$$\begin{aligned}
& \lim_{dt \downarrow 0} \left[ \bar{V}(1+b) \left( \frac{1}{Z_0} \right)^{\frac{\vartheta}{\theta-1}} e^{-\frac{\vartheta}{\theta-1}\gamma t} \left( \frac{1}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \int_{-\infty}^{\bar{v}_t} e^{-\frac{\vartheta}{\theta-1}\chi\tau} \left[ \int_{\bar{v}_t}^{\bar{v}_t+dt} Z_{v,\tau}^{\frac{1+\vartheta}{\theta-1}} dv \right] d\tau \right] / dt \quad (82) \\
&= \bar{V}(1+b) \left( \frac{1}{Z_0} \right)^{\frac{\vartheta}{\theta-1}} e^{-\frac{\vartheta}{\theta-1}\gamma t} \left( \frac{1}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \dot{\bar{v}}_t \int_{-\infty}^{\bar{v}_t} e^{-\frac{\vartheta}{\theta-1}\chi\tau} Z_{\bar{v}_t,\tau}^{\frac{1+\vartheta}{\theta-1}} d\tau \\
&= \bar{V}(1+b) e^{-\frac{\vartheta}{\theta-1}\gamma(t-\bar{v}_t)} \left( \frac{Z_0 e^{\gamma\bar{v}_t}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \dot{\bar{v}}_t \int_{-\infty}^{\bar{v}_t} e^{\frac{1}{\theta-1}\chi\tau} d\tau \\
&= \bar{V}(1+b) \frac{\theta-1}{\chi} e^{-\frac{\vartheta}{\theta-1}\gamma(t-\bar{v}_t)} \left( \frac{Z_0 e^{\gamma\bar{v}_t}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \dot{\bar{v}}_t e^{\frac{1}{\theta-1}\chi\bar{v}_t} \\
&= \bar{V}(1+b) \frac{\theta-1}{\chi} e^{-\frac{\vartheta}{\theta-1}\gamma(t-\bar{v}_t)} \left( \frac{Z_0 e^{(\gamma+\chi)\bar{v}_t}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \dot{\bar{v}}_t
\end{aligned}$$

Now let's look at the second term

$$\begin{aligned}
& \int_{\bar{v}_t}^{\bar{v}_t+dt} \int_{\tau}^{\bar{v}_t+dt} \bar{V}(1+b) \left( \frac{Z_{v,\tau}}{Z_{t,\tau}} \right)^{\frac{1+\vartheta}{\theta-1}} \left( \frac{Z_{t,\tau}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t dv d\tau \quad (83) \\
&= \bar{V}(1+b) Y_t \left( \frac{1}{A_t} \right)^{\frac{1}{\theta-1}} \int_{\bar{v}_t}^{\bar{v}_t+dt} \left[ \int_{\tau}^{\bar{v}_t+dt} Z_{v,\tau}^{\frac{1+\vartheta}{\theta-1}} dv \right] \left( \frac{1}{Z_{t,\tau}} \right)^{\frac{\vartheta}{\theta-1}} d\tau
\end{aligned}$$

Here

$$\left[ \int_{\tau}^{\bar{v}_t+dt} Z_{v,\tau}^{\frac{1+\vartheta}{\theta-1}} dv \right] = Z_0^{\frac{1+\vartheta}{\theta-1}} \frac{1}{\gamma} \frac{\theta-1}{1+\vartheta} \left[ e^{\frac{1+\vartheta}{\theta-1}\chi\tau} e^{\frac{1+\vartheta}{\theta-1}\gamma\bar{v}_t+dt} - e^{\frac{1+\vartheta}{\theta-1}(\gamma+\chi)\tau} \right] \quad (84)$$

which allows us to write

$$\begin{aligned}
& \bar{V}(1+b) Y_t \left( \frac{1}{A_t} \right)^{\frac{1}{\theta-1}} \int_{\bar{v}_t}^{\bar{v}_t+dt} \left[ \int_{\tau}^{\bar{v}_t+dt} Z_{v,\tau}^{\frac{1+\vartheta}{\theta-1}} dv \right] \left( \frac{1}{Z_{t,\tau}} \right)^{\frac{\vartheta}{\theta-1}} d\tau \quad (85) \\
&= \bar{V}(1+b) Y_t \left( \frac{1}{A_t} \right)^{\frac{1}{\theta-1}} Z_0^{\frac{1+\vartheta}{\theta-1}} \frac{1}{\gamma} \frac{\theta-1}{1+\vartheta} \int_{\bar{v}_t}^{\bar{v}_t+dt} \left[ e^{\frac{1+\vartheta}{\theta-1}\chi\tau} e^{\frac{1+\vartheta}{\theta-1}\gamma\bar{v}_t+dt} - e^{\frac{1+\vartheta}{\theta-1}(\gamma+\chi)\tau} \right] \left( \frac{1}{Z_{t,\tau}} \right)^{\frac{\vartheta}{\theta-1}} d\tau \\
&= \bar{V}(1+b) Y_t \left( \frac{1}{A_t} \right)^{\frac{1}{\theta-1}} \left( \frac{1}{e^{\frac{\vartheta}{\theta-1}\gamma t}} \right) Z_0^{\frac{1+\vartheta}{\theta-1}} \frac{1}{\gamma} \frac{\theta-1}{1+\vartheta} \int_{\bar{v}_t}^{\bar{v}_t+dt} \left[ e^{\frac{1}{\theta-1}\chi\tau} e^{\frac{1+\vartheta}{\theta-1}\gamma\bar{v}_t+dt} - e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\tau} \right] d\tau
\end{aligned}$$

Now, we continue by looking at

$$\begin{aligned}
& \int_{\bar{v}_t}^{\bar{v}_t+dt} \left[ e^{\frac{1}{\theta-1}\chi\tau} e^{\frac{1+\vartheta}{\theta-1}\gamma\bar{v}_t+dt} - e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\tau} \right] d\tau \quad (86) \\
&= \frac{\theta-1}{\chi} e^{\frac{1+\vartheta}{\theta-1}\gamma\bar{v}_t+dt} \left[ e^{\frac{1}{\theta-1}\chi\bar{v}_t+dt} - e^{\frac{1}{\theta-1}\chi\bar{v}_t} \right] \\
&\quad - \frac{1}{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)} \left[ e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\bar{v}_t+dt} - e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\bar{v}_t} \right] \\
&= e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\bar{v}_t} \left\{ \frac{\theta-1}{\chi} e^{\frac{1+\vartheta}{\theta-1}\gamma\left(\frac{\bar{v}_t+dt-\bar{v}_t}{dt}\right)} dt \left[ e^{\frac{1}{\theta-1}\chi\left(\frac{\bar{v}_t+dt-\bar{v}_t}{dt}\right)} dt - 1 \right] \right. \\
&\quad \left. - \frac{1}{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)} \left[ e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\left(\frac{\bar{v}_t+dt-\bar{v}_t}{dt}\right)} dt - 1 \right] \right\}
\end{aligned}$$

But this implies that

$$\begin{aligned}
& \lim_{dt \downarrow 0} \left[ \int_{\bar{v}_t}^{\bar{v}_t+dt} \left[ e^{\frac{1}{\theta-1}\chi\tau} e^{\frac{1+\vartheta}{\theta-1}\gamma\bar{v}_t+dt} - e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\tau} \right] d\tau \right] / dt \quad (87) \\
&= e^{\left(\frac{1+\vartheta}{\theta-1}\gamma + \frac{1}{\theta-1}\chi\right)\bar{v}_t} \left\{ \dot{\bar{v}}_t - \dot{\bar{v}}_t \right\} = 0
\end{aligned}$$

Hence, the second part of the integral is zero and the aggregate adoption cost at each instant in time are given by

$$\begin{aligned}\Gamma_t &= \bar{V}(1+b) \frac{\theta-1}{\chi} e^{-\frac{\vartheta}{\theta-1}\gamma(t-\bar{v}_t)} \left( \frac{Z_0 e^{(\gamma+\chi)\bar{v}_t}}{A_t} \right)^{\frac{1}{\theta-1}} Y_t \dot{\bar{v}}_t \\ &= \bar{V}(1+b) \left( \frac{\gamma}{\theta-1} \right)^{\frac{1}{\theta-1}} \left( \frac{1}{1-\frac{\chi}{\chi+\gamma}} \right) e^{-\frac{\vartheta}{\theta-1}\gamma(t-\bar{v}_t)} Y_t \dot{\bar{v}}_t\end{aligned}\quad (88)$$

**Equilibrium:**

Equilibrium in this case consists of the consumption Euler equation

$$\frac{\dot{C}_t}{C_t} = \left( \alpha \frac{\epsilon-1}{\epsilon} \frac{Y_t}{K_t} - \delta - \rho \right) \quad (89)$$

where we have used that the real interest rate is related to the marginal product of capital as follows

$$r_t = \alpha \frac{\epsilon-1}{\epsilon} \frac{Y_t}{K_t} - \delta \quad (90)$$

The resource constraint

$$Y_t = C_t + I_t + \Gamma_t \quad (91)$$

The capital accumulation equation

$$\dot{K}_t = -\delta K_t + I_t \quad (92)$$

The production function

$$Y_t = A_t K_t^\alpha \quad (93)$$

The aggregate TFP equation

$$A_t = A_0 e^{(\chi+\gamma)\bar{v}_t} \quad (94)$$

The adoption cost equation

$$\Gamma_t = \bar{V}(1+b) \left( \frac{\gamma}{\theta-1} \right)^{\frac{1}{\theta-1}} \left( \frac{1}{1-\frac{\chi}{\chi+\gamma}} \right) e^{-\frac{\vartheta}{\theta-1}\gamma(t-\bar{v}_t)} Y_t \dot{\bar{v}}_t \quad (95)$$

The adoption lag equation

$$D_t = \max \left\{ \frac{\theta-1}{\gamma\vartheta} \{ \ln(1+b) - \ln V_t - \ln \bar{V} \}, 0 \right\} \quad (96)$$

And the market value equation

$$V_t = \frac{\alpha}{\epsilon} \int_t^\infty e^{-\int_t^s r_{s'} ds'} \left( \frac{A_t}{A_s} \right)^{\frac{1}{\theta-1}} \frac{Y_s}{Y_t} ds \quad (97)$$

which is best written in changes over time

$$\frac{\dot{V}_t}{V_t} = \left\{ \alpha \frac{\epsilon-1}{\epsilon} \frac{Y_t}{K_t} - \delta + \frac{1}{\theta-1} \frac{\dot{A}_t}{A_t} - \frac{\dot{Y}_t}{Y_t} \right\} - \frac{\alpha}{\epsilon} \frac{1}{V_t} \quad (98)$$

and the technology adoption equation

$$\dot{\bar{v}}_t = \begin{cases} \max \left\{ 1 - \dot{D}_t, 0 \right\} & \text{if } \bar{v}_t = t - D_t \\ 0 & \text{if } \bar{v}_t > t - D_t \end{cases} \quad (99)$$

Because, in the main text we assumed that  $\bar{v}_t = t - D_t$  for all  $t$ , the dynamic equilibrium equations in the main text are based on the assumption that along the equilibrium paths considered  $\dot{\bar{v}}_t = 1 - \dot{D}_t$ , and thus that  $\dot{D}_t < 1$ .

**Balanced growth path:**

We will consider the balanced growth path in this economy in deviation from the trend

$$\bar{A}_t = A_0 e^{(\chi+\gamma)t} \quad (100)$$

The nine transformed/detrended variables on the balanced growth path are

$$C_t^* = \frac{C_t}{A_t^{\frac{1}{1-\alpha}}}, Y_t^* = \frac{Y_t}{A_t^{\frac{1}{1-\alpha}}}, I_t^* = \frac{I_t}{A_t^{\frac{1}{1-\alpha}}}, K_t^* = \frac{K_t}{A_t^{\frac{1}{1-\alpha}}}, \Gamma_t^* = \frac{\Gamma_t}{A_t^{\frac{1}{1-\alpha}}}, \text{ and } A_t^* = \frac{A_t}{A_t} \quad (101)$$

as well as

$$D_t, V_t, \text{ and } \bar{v}_t^* = \bar{v}_t - t \quad (102)$$

### Derivation of transformed dynamic system:

The resulting dynamic equations that define the transitional dynamics of the economy around the balanced growth path are the following Euler equation

$$\frac{\dot{C}_t^*}{C_t^*} = \left( \alpha \frac{\epsilon - 1}{\epsilon} \frac{Y_t^*}{K_t^*} - \delta - \rho \right) - \frac{1}{1 - \alpha} (\chi + \gamma) \quad (103)$$

The resource constraint

$$Y_t^* = C_t^* + I_t^* + \Gamma_t^* \quad (104)$$

The capital accumulation equation

$$\frac{\dot{K}_t^*}{K_t^*} = - \left[ \delta + \frac{1}{1 - \alpha} (\chi + \gamma) \right] + \frac{I_t^*}{K_t^*} \quad (105)$$

The production function

$$Y_t^* = A_t^* (K_t^*)^\alpha \quad (106)$$

The trend adjusted productivity level

$$A_t^* = e^{(\chi + \gamma)\bar{v}_t^*} \quad (107)$$

The aggregate adoption cost

$$\Gamma_t^* = \bar{V} (1 + b) \left( \frac{\gamma}{\theta - 1} \right)^{\frac{1}{\theta - 1}} \left( \frac{1}{1 - \frac{\chi}{\chi + \gamma}} \right) e^{\frac{\theta}{\theta - 1} \gamma \bar{v}_t^*} Y_t^* \left( \frac{\dot{\bar{v}}_t^*}{\bar{v}_t^*} + 1 \right) \quad (108)$$

The adoption lag

$$D_t = \max \left\{ \frac{\theta - 1}{\theta \gamma} \{ \ln(1 + b) - \ln V_t + \ln \bar{V} \}, 0 \right\} \quad (109)$$

and the market value transitional equation

$$\frac{\dot{V}_t}{V_t} = \left\{ \left[ \alpha \frac{\epsilon - 1}{\epsilon} \frac{Y_t^*}{K_t^*} - \delta \right] + \frac{1}{\theta - 1} \left\{ \frac{\dot{A}_t^*}{A_t^*} + (\chi + \gamma) \right\} - \left\{ \frac{\dot{Y}_t^*}{Y_t^*} + \frac{1}{1 - \alpha} (\chi + \gamma) \right\} \right\} - \frac{\alpha}{\epsilon} \frac{1}{V_t} \quad (110)$$

as well as the adoption law of motion

$$\dot{\bar{v}}_t^* = \begin{cases} \max \left\{ -\dot{D}_t, -1 \right\} & \text{if } \bar{v}_t^* = -D_t \\ -1 & \text{if } \bar{v}_t^* > -D_t \end{cases} \quad (111)$$

### Steady state equations:

The steady state is defined by the following equations

$$0 = \left( \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} - \delta - \rho \right) - \frac{1}{1 - \alpha} (\chi + \gamma) \quad (112)$$

The resource constraint

$$\bar{Y}^* = \bar{C}^* + \bar{I}^* + \bar{\Gamma}^* \quad (113)$$

The capital accumulation equation

$$0 = - \left[ \delta + \frac{1}{1 - \alpha} (\chi + \gamma) \right] + \frac{\bar{I}^*}{\bar{K}^*} \quad (114)$$

The production function

$$\bar{Y}^* = \bar{A}^* (\bar{K}^*)^\alpha \quad (115)$$

The trend adjusted productivity level

$$\bar{A}^* = e^{-(\chi+\gamma)\bar{D}} \quad (116)$$

The aggregate adoption cost

$$\bar{\Gamma}^* = \bar{V} (1+b) \left( \frac{\gamma}{\theta-1} \right)^{\frac{1}{\theta-1}} \left( \frac{1}{1-\frac{\chi}{\chi+\gamma}} \right) e^{\frac{\vartheta}{\theta-1} \gamma \bar{v}^*} \bar{Y}^* \quad (117)$$

The steady state adoption lag, assuming that  $b \geq 0$ , equals

$$\bar{D} = \frac{\theta-1}{\vartheta\gamma} \ln(1+b) \quad (118)$$

and the market value transitional equation

$$0 = \left\{ \left[ \alpha \frac{\epsilon-1}{\epsilon} \frac{Y_t^*}{K_t^*} - \delta \right] + \frac{1}{\theta-1} (\chi+\gamma) - \frac{1}{1-\alpha} (\chi+\gamma) \right\} - \frac{\alpha}{\epsilon} \frac{1}{\bar{V}} \quad (119)$$

as well as

$$\bar{v}^* = -\bar{D}$$

### Steady state solution:

Combining the Euler equation with the market cap to GDP equation, we obtain that

$$0 = \left\{ \rho + \frac{1}{\theta-1} (\chi+\gamma) \right\} - \frac{\alpha}{\epsilon} \frac{1}{\bar{V}} \quad (120)$$

Such that the steady state market cap to GDP ratio equals

$$\bar{V} = \frac{\alpha}{\epsilon} \frac{1}{\left\{ \rho + \frac{1}{\theta-1} (\chi+\gamma) \right\}} \quad (121)$$

The steady state trend adjusted level of productivity equals

$$\bar{A}^* = \left[ \left( \frac{1}{1+b} \right)^{\frac{1}{\vartheta}} \right]^{(\theta-1) \frac{(\chi+\gamma)}{\gamma}} \quad (122)$$

When we insert this into the Euler equation, we find that

$$0 = \left( \alpha \frac{\epsilon-1}{\epsilon} \left[ \left( \frac{1}{1+b} \right)^{\frac{1}{\vartheta}} \right]^{(\theta-1) \frac{(\chi+\gamma)}{\gamma}} \left( \frac{1}{\bar{K}^*} \right)^{1-\alpha} - \delta - \rho \right) - \frac{1}{1-\alpha} (\chi+\gamma) \quad (123)$$

which allows us to solve for the steady state capital stock

$$\bar{K}^* = \left[ \frac{\alpha \frac{\epsilon-1}{\epsilon} \bar{A}^*}{\delta + \rho + \frac{1}{1-\alpha} (\chi+\gamma)} \right]^{\frac{1}{1-\alpha}} = \left[ \frac{\alpha \frac{\epsilon-1}{\epsilon} \left[ \left( \frac{1}{1+b} \right)^{\frac{1}{\vartheta}} \right]^{(\theta-1) \frac{(\chi+\gamma)}{\gamma}}}{\delta + \rho + \frac{1}{1-\alpha} (\chi+\gamma)} \right]^{\frac{1}{1-\alpha}} \quad (124)$$

Such that

$$\begin{aligned} \bar{I}^* &= \left[ \delta + \frac{1}{1-\alpha} (\chi+\gamma) \right] \bar{K}^* \\ &= \left[ \delta + \frac{1}{1-\alpha} (\chi+\gamma) \right] \left[ \frac{\alpha \frac{\epsilon-1}{\epsilon} \left[ \left( \frac{1}{1+b} \right)^{\frac{1}{\vartheta}} \right]^{(\theta-1) \frac{(\chi+\gamma)}{\gamma}}}{\delta + \rho + \frac{1}{1-\alpha} (\chi+\gamma)} \right]^{\frac{1}{1-\alpha}} \end{aligned} \quad (125)$$

and output equals

$$\bar{Y}^* = \bar{A}^* (\bar{K}^*)^\alpha = \left\{ \left[ \left( \frac{1}{1+b} \right)^{\frac{1}{\vartheta}} \right]^{(\theta-1) \frac{(\chi+\gamma)}{\gamma}} \right\}^{\frac{1}{1-\alpha}} \left[ \frac{\alpha \frac{\epsilon-1}{\epsilon}}{\delta + \rho + \frac{1}{1-\alpha} (\chi+\gamma)} \right]^{\frac{\alpha}{1-\alpha}} \quad (126)$$

while the aggregate adoption cost is

$$\bar{\Gamma}^* = \bar{V} (1 + b) \left( \frac{\gamma}{\theta - 1} \right)^{\frac{1}{\theta - 1}} \left( \frac{1}{1 - \frac{\chi}{\chi + \gamma}} \right) e^{\frac{\vartheta}{\theta - 1} \gamma \bar{v}^*} \bar{Y}^* \quad (127)$$

and steady state consumption equals

$$\bar{C}^* = \bar{Y}^* - \bar{I}^* - \bar{\Gamma}^* \quad (128)$$

Note that, for steady state consumption to be positive, we need a restriction on the parameters, such that the total adoption costs do not fully exhaust productive capacity.

### Transitional dynamics:

The next thing is to linearize the transitional dynamics around the steady state. Note that this model has only one state variable, namely the capital stock  $K_t$ . The stock market capitalization to GDP ratio,  $V_t$ , is a jump variable and so are the adoption lag,  $D_t$ , the best vintage adopted,  $\bar{v}_t$ , and the trend adjusted productivity level,  $A_t^*$ .

The log-linearized equations are the Euler equation

$$\dot{\hat{C}}_t^* = \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \hat{Y}_t^* - \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \hat{K}_t^* \quad (129)$$

as well as the resource constraint

$$0 = \hat{Y}_t^* - \frac{\bar{C}^*}{\bar{Y}^*} \hat{C}_t^* - \frac{\bar{I}^*}{\bar{Y}^*} \hat{I}_t^* - \frac{\bar{\Gamma}^*}{\bar{Y}^*} \hat{Y}_t^* \quad (130)$$

the capital accumulation equation

$$\dot{\hat{K}}_t^* = \frac{\bar{I}^*}{\bar{K}^*} \hat{K}_t^* - \delta \frac{\bar{I}^*}{\bar{K}^*} \hat{K}_t^* \quad (131)$$

The production function

$$0 = \hat{Y}_t^* - \hat{A}_t^* - \alpha \hat{K}_t^* \quad (132)$$

The trend adjusted productivity level

$$0 = \hat{A}_t^* - (\chi + \gamma) (\bar{v}_t^* - \bar{v}^*) \quad (133)$$

The adoption lag equation

$$0 = (D_t - \bar{D}) + \frac{\theta - 1}{\vartheta \gamma} \hat{V}_t \quad (134)$$

The aggregate adoption cost

$$\dot{\bar{v}}_t^* = \hat{\Gamma}_t^* - \frac{\vartheta}{\theta - 1} \gamma (\bar{v}_t^* - \bar{v}^*) - \hat{Y}_t^* \quad (135)$$

and the market capitalization equation

$$\dot{\hat{V}}_t = \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \hat{Y}_t^* - \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \hat{K}_t^* + \frac{\chi + \gamma}{\theta - 1} \bar{v}_t^* - (\chi + \gamma) \bar{v}_t^* - \alpha \hat{K}_t^* + \frac{\alpha}{\epsilon} \frac{1}{\bar{V}} \hat{V}_t \quad (136)$$

which simplifies to

$$\dot{\hat{V}}_t + \left( 1 - \frac{1}{\theta - 1} \right) (\chi + \gamma) \bar{v}_t^* + \alpha \hat{K}_t^* = \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \hat{Y}_t^* - \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \hat{K}_t^* + \frac{\alpha}{\epsilon} \frac{1}{\bar{V}} \hat{V}_t \quad (137)$$

where we have assumed that, all along the equilibrium path  $\bar{v}_t^* = -D_t$ , such that

$$\dot{\bar{v}}_t^* = \begin{cases} \max \left\{ -\frac{\theta - 1}{\vartheta \gamma} \hat{V}_t, -1 \right\} & \text{if } \bar{v}_t^* = -D_t \\ -1 & \text{if } \bar{v}_t^* > -D_t \end{cases} \quad (138)$$

For our examples, we limit ourselves to the part of the transitional path for which  $\bar{v}_t^* = -D_t$  for all  $t$ . On that path, the transitional dynamics simplify, because then

$$\bar{v}_t^* - \bar{v}^* = -(D_t - \bar{D}) \quad (139)$$

and

$$\dot{\bar{v}}_t^* = -\dot{D}_t = \frac{\theta - 1}{\vartheta \gamma} \dot{\hat{V}}_t \quad (140)$$

which allows us to write

$$0 = \widehat{A}_t^* + (\chi + \gamma) (D_t - \bar{D}) \quad (141)$$

and

$$\frac{\theta - 1}{\vartheta\gamma} \dot{\widehat{V}}_t = \widehat{\Gamma}_t^* + \frac{\vartheta}{\theta - 1} \gamma (D_t - \bar{D}) - \widehat{Y}_t^* \quad (142)$$

as well as

$$\left[ 1 + (\theta - 2) \frac{\chi + \gamma}{\vartheta\gamma} \right] \dot{\widehat{V}}_t + \alpha \dot{\widehat{K}}_t^* = \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \widehat{Y}_t^* - \alpha \frac{\epsilon - 1}{\epsilon} \frac{\bar{Y}^*}{\bar{K}^*} \widehat{K}_t^* + \frac{\alpha}{\epsilon} \frac{1}{\bar{V}} \widehat{V}_t \quad (143)$$

### Derivation of equation (39)

Let

$$Q_{\tau,t} = q_{\tau,0} - q_{\tau}t \quad (144)$$

then, when we log-linearize (9) around the steady-state real interest rate  $\bar{r}$  and the implied steady-state user cost

$$\bar{u}c_{\tau} = (\bar{r} + \delta_{\tau} + q_{\tau}) \quad (145)$$

we obtain the approximation

$$\begin{aligned} r_{\tau} &\approx \left[ \ln \left( \frac{\epsilon}{\epsilon - 1} \right) + \bar{u}c_{\tau} - \frac{1}{\bar{u}c_{\tau}} \bar{r} + q_{\tau,0} \right] - q_{\tau}t + \frac{1}{\bar{u}c_{\tau}} r \\ &\approx c_r - q_{\tau}t + \frac{1}{\bar{u}c_{\tau}} r \end{aligned} \quad (146)$$

When we assume that  $r$  is approximately linear in the growth rate of per capita GDP, we obtain

$$r_{\tau} = c_1 - q_{\tau}t + c_2 r \quad (147)$$

which is (39).

### Derivation of equation (40)

Denote the adoption time by  $T_{\tau} = D_{\tau} + v_{\tau}$ . Consider the technology-specific TFP level

$$Z_{\tau t} = Z_{v_{\tau}} \left[ \left( \frac{\mu - 1}{\gamma_{\tau}} \right) \left( e^{\frac{\gamma_{\tau}}{\mu - 1}(t - T_{\tau})} - 1 \right) \right]^{\mu - 1} \quad (148)$$

We are interested in the behavior of this TFP for  $\gamma_{\tau} \downarrow 0$ . In that case, there is no embodied productivity growth and the increase in productivity after the introduction of the technology is all due to the introduction of an increasing number of varieties over time.

For this reason, we consider

$$\lim_{\gamma_{\tau} \downarrow 0} Z_{v_{\tau}} \left[ \left( \frac{\mu - 1}{\gamma_{\tau}} \right) \left( e^{\frac{\gamma_{\tau}}{\mu - 1}(t - T_{\tau})} - 1 \right) \right]^{\mu - 1} \quad (149)$$

which, using de l'Hopital's rule, can be shown to equal

$$Z_{v_{\tau}} \left[ \lim_{\gamma_{\tau} \downarrow 0} \left( \frac{\mu - 1}{\gamma_{\tau}} \right) \left( e^{\frac{\gamma_{\tau}}{\mu - 1}(t - T_{\tau})} - 1 \right) \right]^{\mu - 1} = Z_{v_{\tau}} (t - T_{\tau})^{(\mu - 1)} \quad (150)$$



Taking the first order Taylor approximation around  $\gamma = 0$  yields that

$$\begin{aligned}
Z_{\tau t} &\approx Z_{v_{\tau}}(t - T_{\tau})^{(\mu-1)} + \\
&+ Z_{v_{\tau}} \left[ \lim_{\gamma_{\tau} \downarrow 0} \left( \left( \frac{\mu-1}{\gamma_{\tau}} \right) (t - T_{\tau}) e^{\frac{\gamma_{\tau}}{\mu-1}(t-T_{\tau})} - \left( \frac{\mu-1}{\gamma_{\tau}} \right)^2 \left( e^{\frac{\gamma_{\tau}}{\mu-1}(t-T_{\tau})} - 1 \right) \right) \right] (t - T_{\tau})^{(\mu-2)} \gamma_{\tau} \\
&= Z_{v_{\tau}}(t - T_{\tau})^{(\mu-1)} + \\
&+ Z_{v_{\tau}} \left[ \lim_{\gamma_{\tau} \downarrow 0} \left( \frac{\mu-1}{\gamma_{\tau}} \right)^2 \left( \left( \frac{\gamma_{\tau}}{\mu-1} (t - T_{\tau}) - 1 \right) e^{\frac{\gamma_{\tau}}{\mu-1}(t-T_{\tau})} + 1 \right) \right] (t - T_{\tau})^{(\mu-2)} \gamma_{\tau} \\
&= Z_{v_{\tau}}(t - T_{\tau})^{(\mu-1)} \\
&+ Z_{v_{\tau}} \left[ \lim_{\gamma_{\tau} \downarrow 0} (\mu-1)^2 \frac{\frac{\gamma_{\tau}}{(\mu-1)^2} (t - T_{\tau})^2 e^{-\frac{\gamma_{\tau}}{\mu-1}(t-T_{\tau})}}{2\gamma_{\tau}} \right] (t - T_{\tau})^{(\mu-2)} \gamma_{\tau} \\
&= Z_{v_{\tau}}(t - T_{\tau})^{(\mu-1)} + \frac{1}{2} Z_{v_{\tau}}(t - T_{\tau})^{\mu} \gamma_{\tau} \\
&= Z_{v_{\tau}}(t - T_{\tau})^{(\mu-1)} \left[ 1 + \frac{1}{2} (t - T_{\tau}) \gamma_{\tau} \right]
\end{aligned} \tag{151}$$

Hence, for  $\gamma_{\tau}$  close to zero,

$$z_{\tau t} \approx z_{v_{\tau}} + (\mu-1) \ln(t - T_{\tau}) + \frac{\gamma_{\tau}}{2} (t - T_{\tau}) \tag{152}$$

#### Derivation of equation (41)

Combining the five log-linearized equations, we obtain for  $m_{\tau} = y_{\tau}$  that

$$y_{\tau} = y - \frac{\theta}{\theta-1} p_{\tau} \tag{153}$$

$$\begin{aligned}
&= y - \frac{\theta}{\theta-1} \alpha \ln \alpha + \frac{\theta}{\theta-1} z_{\tau} - \frac{\theta}{\theta-1} (1-\alpha)(y-l) - \frac{\theta}{\theta-1} \alpha r_{\tau} \\
&= y + \left[ -\frac{\theta}{\theta-1} \alpha \ln \alpha + \frac{\theta}{\theta-1} z_{v_{\tau}} - \frac{\theta}{\theta-1} \alpha c_1 - \frac{\theta}{\theta-1} \frac{\gamma_{\tau}}{2} T_{\tau} \right] \\
&+ \left[ \frac{\theta}{\theta-1} \frac{\gamma_{\tau}}{2} - \frac{\theta}{\theta-1} \alpha q_{\tau} \right] t + \frac{\theta}{\theta-1} [(\mu-1) \ln(t - T_{\tau}) - (1-\alpha)(y-l)] \\
&- \frac{\theta}{\theta-1} \alpha c_2 \Delta(y-l)
\end{aligned} \tag{154}$$

$$= \beta_1 + y + \beta_2 t + \beta_3 ((\mu-1) \ln(t - T_{\tau}) - (1-\alpha)(y-l)) + \beta_4 \Delta(y-l) \tag{155}$$

Combining the five log-linearized equations, we obtain for  $m_{\tau} = k_{\tau}$  that

$$k_{\tau} = y + \ln \alpha - \frac{1}{\theta-1} p_{\tau} - r_{\tau} \tag{156}$$

$$= \left[ 1 + \frac{\alpha}{\theta-1} \right] [\ln \alpha - c_1] + \frac{1}{\theta-1} \left[ z_{v_{\tau}} + \frac{\gamma_{\tau}}{2} T_{\tau} \right] + \tag{157}$$

$$\left[ \left[ 1 + \frac{\alpha}{\theta-1} \right] q_{\tau} + \frac{1}{\theta-1} \frac{\gamma_{\tau}}{2} \right] t + \frac{1}{\theta-1} [(\mu-1) \ln(t - T_{\tau}) - (1-\alpha)(y-l)] \tag{158}$$

$$- \left[ 1 + \frac{\alpha}{\theta-1} \right] c_2 \Delta(y-l) \tag{159}$$

$$= \beta_1 + y + \beta_2 t + \beta_3 ((\mu-1) \ln(t - T_{\tau}) - (1-\alpha)(y-l)) + \beta_4 \Delta(y-l) \tag{160}$$

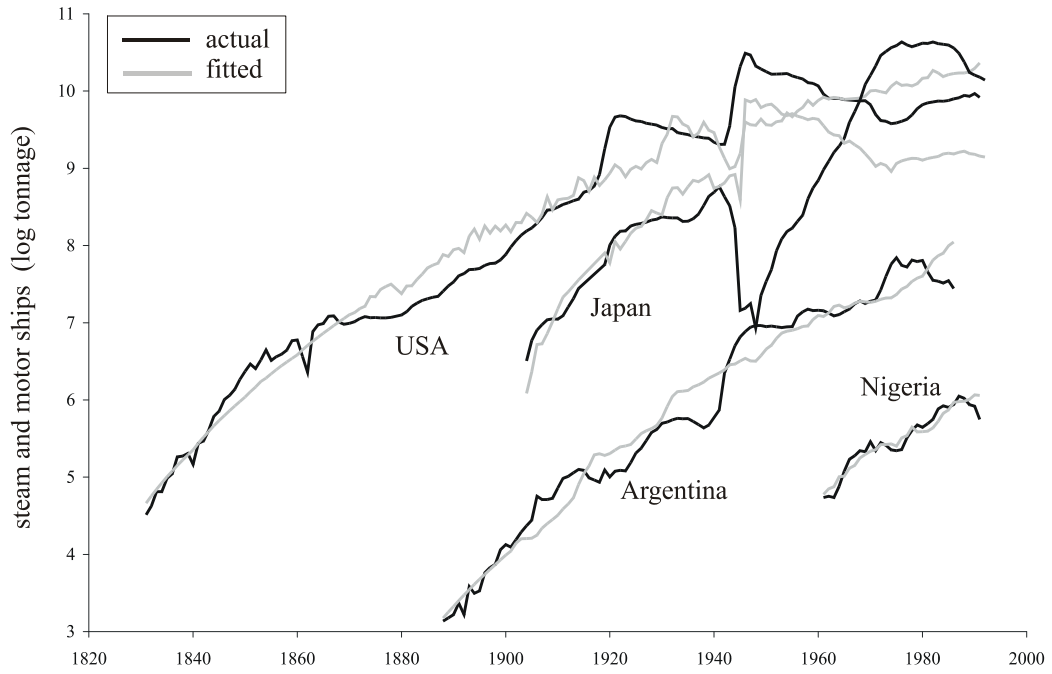


Figure 1: Actual and fitted tonnage of steam and motor ships for four countries

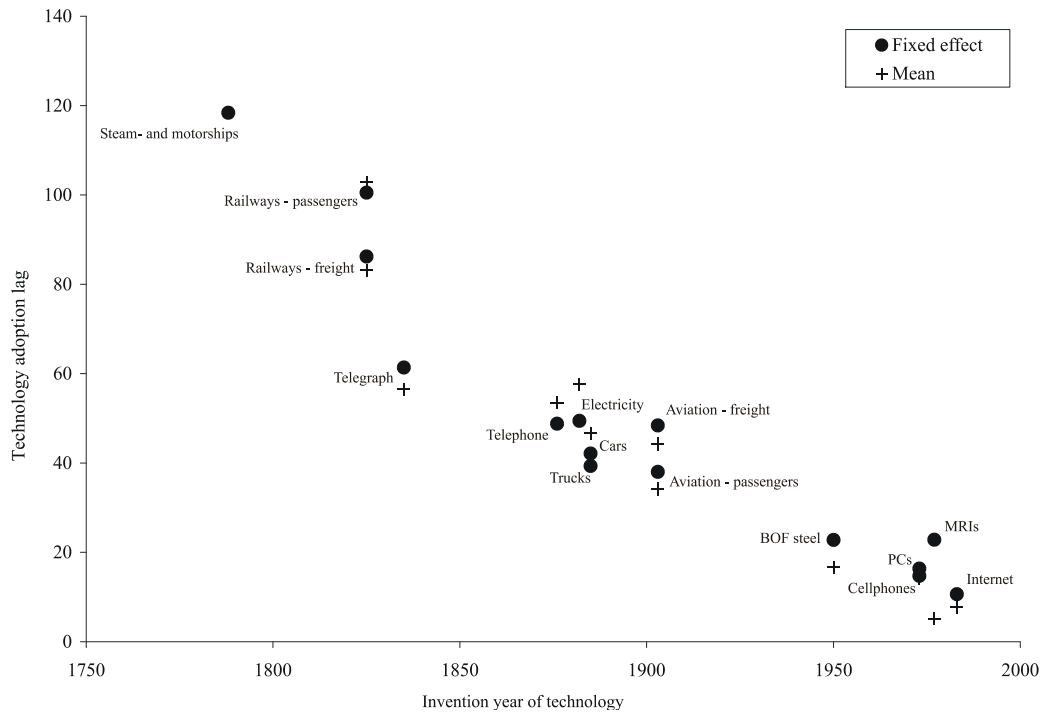


Figure 2: Technology adoption lags decrease for later inventions

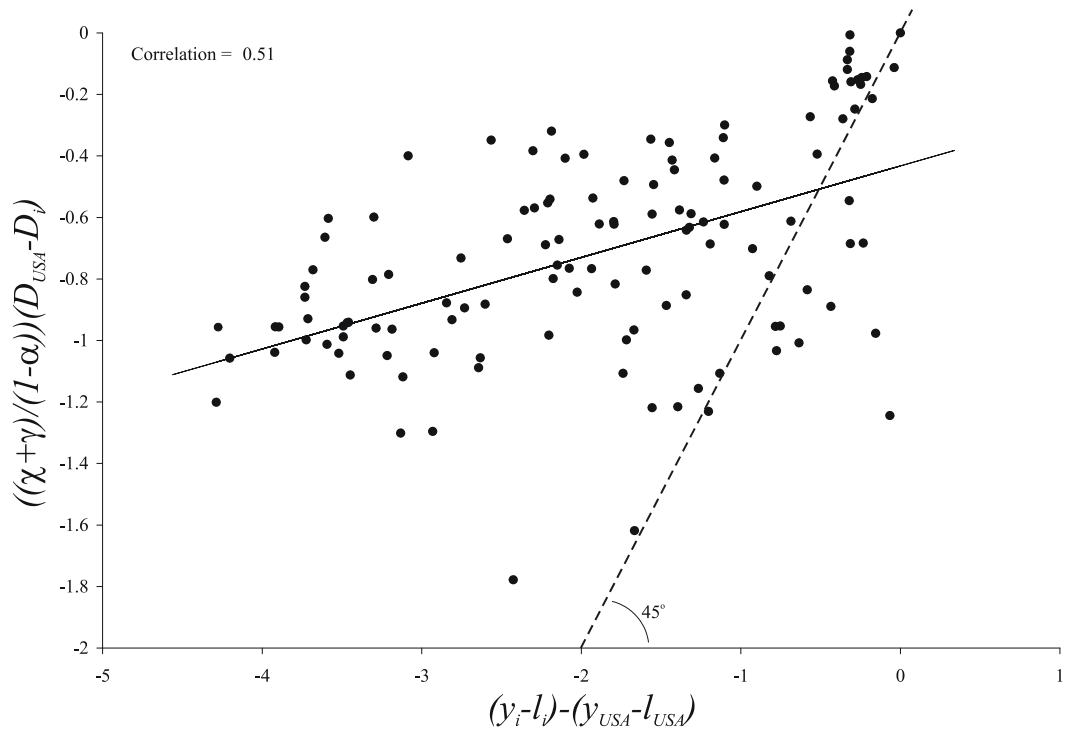


Figure 3: TFP part of technology adoption lags versus real GDP per capita.

Table 1: Quality of estimates

Technology	Invention year ( $\underline{y}_T$ )	Number of countries	Plausible		Implausible		$R^2$	
			Precise	Imprecise	Implausible	$R^2 > 0$	mean	stdev
Steam- and motorships	1788	64	54	0	10	54	0.81	0.14
Railways - Passengers	1825	80	58	3	19	40	0.54	0.19
Railways - Freight	1825	85	41	6	38	41	0.82	0.12
Cars	1885	123	76	6	41	66	0.67	0.22
Trucks	1885	109	58	6	45	52	0.68	0.23
Aviation - Passengers	1903	97	50	4	43	50	0.90	0.07
Aviation - Freight	1903	94	29	5	60	29	0.88	0.09
Telegraph	1835	67	45	4	18	32	0.59	0.22
Telephone	1876	142	69	9	64	66	0.86	0.12
Cellphones	1973	85	81	3	1	81	0.92	0.07
PCs	1973	71	68	1	2	68	0.94	0.06
Internet users	1983	60	59	0	1	59	0.95	0.05
MRIs	1977	12	12	0	0	12	0.93	0.03
Blast Oxygen Steel	1950	52	40	1	11	33	0.67	0.25
Electricity	1882	137	93	3	41	93	0.92	0.09
Total		1278	833	51	394	776	0.82	0.19

Table 2: Estimated adoption lags and goodness of fit

Technology	Invention year ( $x_r$ )	Number of countries	Adoption lags						
			mean	stdev	1%	10%	50%	90%	99%
Steam- and motorships	1788	54	118	50	26	56	115	179	180
Railways - Passengers	1825	58	103	24	26	68	106	126	137
Railways - Freight	1825	41	83	33	26	31	91	124	135
Cars	1885	76	47	21	10	20	46	67	101
Trucks	1885	58	40	20	5	15	36	65	89
Aviation - Passengers	1903	50	34	13	17	21	29	53	72
Aviation - Freight	1903	29	44	13	16	24	43	61	73
Telegraph	1835	45	56	34	11	17	53	104	114
Telephone	1876	69	54	33	-7	7	59	101	114
Cellphones	1973	81	15	4	1	10	16	19	20
PCs	1973	68	14	3	7	11	14	17	19
Internet users	1983	59	8	2	-2	4	8	11	11
MRIs	1977	12	5	2	2	3	5	7	9
Blast Oxygen Steel	1950	40	17	7	6	9	16	28	33
Electricity	1882	93	58	20	13	25	67	74	105
Total		833	47	39	3	9	36	102	178

Table 3: Analysis of variance

Total sum of squares = 1287350, N = 833					
	Model	Country	Technology	Residual	Total
	SS	effect	effect	SS	SS
Country effect alone	29%	29%		71%	100%
Technology effect	66%		66%	34%	100%
Joint effect	83%	18%	54%	17%	100%

Table 4: Adoption lags for country groups

Technology	Invention year ( $v_\tau$ )	USA	GBR	Japan	other OECD	Asian Tigers	Latin America	Sub-Saharan Africa	Other
Steam- and motorships	1788	-92	-76	-5	-50	32	-4	24	35
		1	1	1	14	4	10	3	20
Railways - Passengers	1825	-46	5		-26	10	-5	19	11
		1	1		14	2	10	9	24
Railways - Freight	1825	-36		-16	-22	5	7	24	17
		1		1	18	2	3	6	16
Telegraph	1835	-21	-42	-17	-22	21	-6	30	23
		1	1	1	17	3	8	2	16
Telephone	1876	-52	-33	-35	-28	25	-14	10	19
		1	1	1	17	4	13	11	30
Electricity	1882	-37		-32	-26	10	-4	11	7
		1		1	16	4	15	23	36
Cars	1885	-32	-32	-17	-14	17	-10	6	13
		1	1	1	19	4	14	15	27
Trucks	1885	-19		-8	-11	26	-6	-2	13
		1		1	18	4	13	10	17
Aviation - Passengers	1903	-7	-12	-8	-5	20	-4	22	2
		1	1	1	19	3	7	2	20
Aviation - Freight	1903	-16		-3	-3	18	-7	33	-1
		1		1	15	3	2	1	11
Blast Oxygen Steel	1950	-8	-8	-8	-3	8	2		3
		1	1	1	17	2	6		13
PCs	1973	-6	-4	-3	-1	0	2	1	1
		1	1	1	19	3	10	7	27
Cellphones	1973	-4	-4	-7	-3	-1	2	2	1
		1	1	1	19	4	15	9	34
MRIs	1977	-2			1				-4
		1			10				1
Internet	1983	-4	-2	-1	-2	0	1	2	2
		1	1	1	19	4	9	3	21

Table 5: Adoption dates of three recent technologies for EATs

	<b>Singapore</b>	<b>Hong Kong</b>	<b>Korea</b>	<b>Taiwan</b>
PCs	1987	1987	1987	
Cellphones	1987	1984	1986	1989
Internet	1991	1990	1990	1991