Financial Shocks, Bank Intermediation, and Monetary Policy in a DSGE Model

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Abstract

We build upon the standard Bernanke, Gertler, and Gilchrist (1999) model of the financial accelerator by introducing a monopolistically competitive banking sector with interbank lending. Banks play an important role, most importantly through the cost of intermediation via loan production. Demand and supply side shocks originating from the financial sector have large macroeconomic effects, and monetary policy that responds to credit spreads linked to the intensive investment margin can mitigate these effects. Our model incorporates a realistic yet parsimonious banking sector, and highlights the importance of understanding the costs and benefits of bank intermediation for a smoothly functioning economy.

Keywords: Credit spreads; financial shocks; monetary policy; intermediation costs

JEL Classification Codes: E44, E52, E58, G18, G21

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The recent financial crisis featured a decline in the ability of financial institutions to intermediate borrowing and lending between households and firms, and even borrowing and lending between different financial institutions. However, as the two benchmark models developed to analyze the post-war business cycle fluctuations in developed economies, the Real Business Cycle (RBC) model and the Dynamic New Keynesian (DNK) model, feature zero (or extremely limited) borrowing and lending between agents in equilibrium, and therefore no role for financial intermediation to facilitate borrowing and lending, much of the macroeconomic literature was silent on the relationship between financial intermediation and macroeconomic volatility. Over the last two decades, beginning with Bernanke and Gertler (1989), economists began to introduce credit frictions into models that allowed for borrowing and lending in equilibrium. Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999) then showed that these credit frictions could amplify the macroeconomic fluctuations induced by certain shocks, hence why the credit frictions are often referred to as the "financial accelerator." Yet even in these financial accelerator models, the role of financial intermediaries themselves were relegated to a bit part, and there was certainly no characterization of shocks that could impact the ability of the financial intermediary to facilitate the borrowing and lending process.

In the aftermath of the financial crisis, there has been an explosion of theoretical papers attempting to link financial factors – including, but not limited to, financial intermediation – to macroeconomic fluctuations and the optimal response of monetary policy to financial shocks.¹ Gertler and Kiyotaki (2009), Cúrdia and Woodford (2010), Del Negro et. al. (2010), and Dib (2010) are few among the many of such papers. Our primary contribution to this literature is adding a relatively simple, realistic, and well-defined financial intermediation sector into a large-scale dynamic stochastic general equilibrium (DSGE) model.

¹There has also been an increase in empirical work aimed at quantifying the relationship between credit spreads and the macroeconomy. An abbreviated sample of this includes Gilchrist et. al. (2009), Nolan and Thoenissen (2009), Faust et. al. (2011), and Gilchrist and Zakrajšek (2011).
We then analyze the relationship between this financial intermediation sector and macroeconomic volatility by examining both the indirect effect of the sector on the propagation of non-financial shocks and the direct effect of financial shocks that inhibit financial intermediation. Applying the model, we then ask if traditional monetary policy rules adapted to respond to financial shocks can reduce macroeconomic volatility.

Our model is an extension of the seminal work of Bernanke, Gertler, and Gilchrist (1999) (henceforth, BGG). The BGG model contains an implicit financial intermediary that is conceptually equivalent to a perfectly competitive banking sector with no marginal costs for originating bank loans or accepting deposits that can borrow at the risk-free interest rate set by the central bank. We begin to extend the BGG model by introducing bank production functions that imply a positive marginal cost for banks to originate loans and accept deposits. Among other things, positive marginal intermediation costs add endogenous interest rate spreads into the financial accelerator model. We then introduce monopolistic competition in the banking sector. Monopolistic competition implies that banks set interest rates to maximize profits. Additionally, monopolistic competition allows for heterogeneity in the banking sector, and we introduce heterogeneity in the marginal cost of loans and deposits across banks. Finally, our model allows for banks to secure funding via interbank loans at a rate equal to the risk-free rate set by the central bank plus an exogenous premium. Through bank heterogeneity, the model allows for positive interbank lending in equilibrium.

We begin our analysis by investigating the role of the banking sector in propagating or mitigating business cycle fluctuations. First, we find that the addition of our banking sector to the BGG framework mitigates, but does not negate, the impact of the credit friction on macroeconomic fluctuations by affecting both the magnitude and persistence of non-financial shocks.\footnote{Qualitatively, our banking sector always mitigates the effect of the financial accelerator. The quantitative...} Second, we find that financial shocks, both on the demand and...
supply sides, can cause severe macroeconomic fluctuations. Holding the persistence and standard deviation fixed across the financial shocks, we find that a demand-side financial shock, as modeled by an increase in the riskiness of borrowers, has very small negative effects as the banking system increases the gross interest rate to offset the increase in default costs without changing key interest rate spreads. We find that supply-side financial shocks have much larger negative macroeconomic effects than the demand-side financial shock. Shocks to the marginal cost of producing loans have the largest macroeconomic effects due to the fact that the interest rate spread between borrowing and lending increases, which reduces both investment demand and the net wealth of entrepreneurs, which is exacerbated by the general equilibrium effects of the financial accelerator. Finally, we find that interbank lending shocks (comparable to increases in the LIBOR-OIS spread) have large immediate negative macroeconomic effects but with little to no persistence. The lack of a persistent macroeconomic effect of interbank lending shocks occurs because monetary policy endogenously responds to offset the interbank lending shock by reducing the central bank borrowing rate.

Given that we find that shocks to financial intermediation can cause large business cycle fluctuations, we employ our model in a monetary policy experiment. We begin the monetary policy experiment by asking whether or not the central bank can reduce macroeconomic and financial volatility by incorporating interest rate spreads in a traditional monetary policy rule. We find that a policy rule that targets interest rate spread stabilization, in addition to inflation and output gap stabilization, can reduce volatility relative to a policy rule that simply targets inflation and output gap stabilization. We believe this provides evidence for policymakers going forward of how an explicit (rather than implicit) objective of reacting to financial markets through interest rate spreads may help policymakers meet

importance of the banking sector on the effect of the financial accelerator depends on the calibration of the model. Additionally, in sensitivity analysis, we find that this result holds in general as long as there is a positive spread between the borrowing and lending interest rates.
their objective of mitigating business cycle fluctuations. Given the magnitude of financial shocks during the crisis, our estimates imply an economically relevant level of improvement that could have been made with such a policy (on the order of 1%-3% improvement in output fluctuations after a shock).

Our paper is related to a recent literature that incorporates the BGG financial accelerator credit friction into DSGE models. Christensen and Dib (2008) (henceforth, CD) estimate a model with and without the financial accelerator and find that the financial accelerator model matches the data better. Christiano, Motto, and Rostagno (2009) (henceforth, CMR) add the financial accelerator and a perfectly competitive banking sector to the Christiano, Eichenbaum, and Evans (2005) DSGE model. CMR’s main focus is to quantitatively match U.S. and E.U. data; our focus, on the other hand, is a more qualitative approach, carefully documenting the mechanisms and channels through which financial shocks originating in the banking sector cause volatility in the real economy, without an attempt to directly match quantitative moments.

In response to the financial crisis and the unprecedented monetary policy response, a subset of the growing literature related to financial shocks and macroeconomic volatility focuses primarily on studying the effects of monetary policy aimed at financial markets. Gertler and Kiyotaki (2009) use a variation of the BGG model in which there exists an agency problem between financial intermediaries and depositors, as well as the BGG agency problem between financial intermediaries and borrowers, to motivate spreads between the interest rates received by depositors and interest rates paid by borrowers. Our model, on the other hand, simply motivates banks (and interest rate spreads) by recognizing that banks use real resources such as labor when they accept deposits and originate loans. Additionally, Gertler and Kiyotaki (2009) focus on unconventional monetary policy, such as quantitative easing, rather than our focus on augmenting traditional monetary policy interest rate rules. Cúrdia and Woodford (2010) use a different framework in which the
financial sector is captured by an exogenous, reduced form interest rate spread. By using the BGG financial accelerator model as our benchmark instead, we introduce an endogenous interest rate spread that is a function of a parsimonious, yet realistic banking sector. Additionally, we solve the full financial accelerator model as opposed to a reduced form model, which allows the key elasticity of the financial accelerator model – the elasticity of the credit spread with respect to the leverage ratio – to vary as a function of the banking system. Overall, we feel that these differences enable our model to be used as a workhorse for analyzing monetary policy that incorporates financial market spreads.

The outline of the rest of the paper is as follows. We begin in Section 1 by introducing the benchmark model with monopolistic competition in the banking sector. In Section 2, we discuss the calibration and solution of our model. In Section 3, we analyze the impact of the banking sector on the financial accelerator. Section 4 describes the effect of financial shocks on the key variables of our model. Section 5 addresses a monetary policy experiment. Lastly, section 6 concludes.

1 The Model

Our benchmark model is a DSGE model with the BGG financial accelerator: a stochastic growth model with money, monopolistic competition at the retail level, capital adjustment costs, nominal price rigidities, and a credit friction. Our main contribution is to add a monopolistically competitive banking sector with interbank lending. To focus on the effects of the banking sector, we maintain the small number of shocks and non-financial rigidities or frictions of the original BGG model. Because banks optimally set nominal interest rates to maximize profits in our model, we formulate all interest rates in nominal terms.

In the reduced form financial accelerator framework, the elasticity of the credit spread with respect to the leverage ratio is a fixed exogenous parameter. By solving the full financial accelerator model, which to our knowledge is a first, the steady state elasticity is a function of the size of the endogenous interest rate spread and the elasticity is allowed to move in response to shocks. See Appendix B for details.
As emphasized by CMR, nominal interest rates will introduce the Fischer debt-deflation nominal rigidity into the model.

There are six types of agents in the model: households, entrepreneurs, capital goods producers, retailers, banks, and a government. Households consume, provide labor, save (via deposits in banks) and hold money. Entrepreneurs produce wholesale goods using capital financed through net worth and external borrowing from banks. Asymmetric information between entrepreneurs and banks introduces the BGG credit friction, the financial accelerator, where the demand for external borrowing is a function of the entrepreneur’s leverage ratio. Capital goods producers build, and then sell, new capital to entrepreneurs while retailers purchase wholesale goods from entrepreneurs and resell them with a markup. Banks act as a financial intermediary between households and entrepreneurs and set interest rates to maximize their expected profits. The government buys the final good, creates money and conducts monetary policy by setting the benchmark interest rate each period.

The model description is organized as follows: section 1.1 describes the banking sector, section 1.2 describes entrepreneurs, and section 1.3 describes the government and monetary policy. Capital goods producers, households, retailers are standard and not central to development of the monopolistically competitive banking sector. Therefore, we defer the complete description of these agents to Appendix A.

1.1 The Banking Sector

The financial intermediation sector in our model is represented by a monopolistically competitive banking sector with an interbank lending market. Banks “produce” loans and deposits using real resources, introducing a real cost of financial intermediation into the model and endogenous interest rate spreads. Banks compete imperfectly with each other to provide loans to entrepreneurs and receive deposits from households. Additionally, banks have
access to funds outside their deposit base through interbank lending.

### 1.1.1 Monopolistic Competition in the Banking Sector

There is a mass of heterogeneous banks that compete in the lending and savings deposits markets. Banks compete on prices and set gross nominal interest rates on deposits, \( r_{t+1}^d(i) \), and entrepreneurial loans, \( r_{t+1}^b(i) \), to maximize their expected profits.\(^4\) We use a standard Dixit-Stiglitz aggregation function to describe the monopolistic competition. This has the simplifying feature that it implies that all banks essentially serve all entrepreneurs and therefore all banks face the same ex-ante and ex-post default rates.

Aggregate savings deposits and bank loans are given by

\[
D_t = \left[ \int D_t(i) \frac{\eta_d^{-1}}{\eta_d} \, di \right]^{\frac{\eta_d}{\eta_d-1}}, \tag{1}
\]

\[
B_t = \left[ \int B_t(i) \frac{\eta_b^{-1}}{\eta_b} \, di \right]^{\frac{\eta_b}{\eta_b-1}}, \tag{2}
\]

while the aggregate gross nominal interest rates are given by

\[
(r_{t+1}^d)^{-1} = \left[ \int (r_{t+1}^d(i)^{-1})^{1-\eta_d} \, di \right]^{\frac{1}{1-\eta_d}}, \tag{3}
\]

\[
r_{t+1}^b = \left[ \int (r_{t+1}^b(i)^{1-\eta_b} \, di \right]^{\frac{1}{1-\eta_b}}, \tag{4}
\]

where \( \eta_d \) and \( \eta_b \) are the elasticities of substitution in the deposit and loan markets, respectively.\(^5\) Following Dixit-Stiglitz, we can then write the standard monopolistic competition downward sloping demand curves for individual bank deposits and loans as a function of

\(^4\)Throughout this paper, nominal interest rates will be denoted in lower-case and real interest rates will be denoted in upper-case.

\(^5\)The price in the deposit function is inverted because the bank will see increased demand for deposits when it raises its interest rates. In other words, deposit interest is a cost to the bank that they would prefer to pay as little as possible; in equilibrium, this will lead to an optimal "markdown" instead of the standard markup.
aggregate demand and prices:

\[
D_{t+1}(i) = D_{t+1} \left( \frac{r_{t+1}^d}{r_{t+1}(i)} \right)^{-\eta_d}
\]

\[
B_{t+1}(i) = B_{t+1} \left( \frac{r_{t+1}^b}{r_{t+1}(i)} \right)^{\eta_b}.
\]

Banks use labor to originate entrepreneurial loans and to accept household savings deposits. For simplicity, we model the bank production functions as a linear technology; optimal deposit labor, \(H^d_t\), and loan labor, \(H^b_t\), are increasing functions of total deposits and loans:

\[
H^d_t(i) = \gamma^d(i)D_{t+1}(i)
\]

\[
H^b_t(i) = \gamma^b(i)B_{t+1}(i).
\]

We introduce heterogeneity into the banking sector by allowing the parameters \(\gamma^d\) and \(\gamma^b\) to vary across banks. Additionally, we introduce a financial supply shock by allowing bank-specific loan productivity to vary over time; the parameter \(\gamma^b(i)\) follows a first-order autoregressive process around the steady state value for each bank:

\[
\gamma^b_t(i) - \gamma^b(i) = \rho_{\gamma^b}(\gamma^b_{t-1}(i) - \gamma^b(i)) + e^b_t(i),
\]

where the shock persistence \(\rho_{\gamma^b}\) is fixed across banks and the shocks \(e^b_t(i)\) are i.i.d. white noise processes with standard deviation \(\sigma_{e^b}\). A positive shock to \(\gamma^b_t(i)\) implies bank \(i\) will be less productive and will need to use more labor to originate the same number of loans. It is this supply shock which captures shocks to the cost of bank intermediation.

\[\text{In our base model, the white noise shocks } e^b_t(i) \text{ are independent across banks. Given the persistence of the shocks, there is still likely to be contemporaneous correlation of } \gamma^b_t(i) \text{ across banks.}\]
Banks are allowed to borrow or loan from each other every period at a fixed nominal rate of interest \( r_{t+1}^L(i) \). The bank-specific interbank lending rate is a function of the benchmark nominal interest rate set by the central bank, \( r_{t+1}^{cb} \), and an exogenous spread shock, \( e_t^L(i) \), such that the spread between the interbank rate and the central bank rate follows a first-order autoregressive process with i.i.d shock \( e_t^L(i) \). We interpret the spread shock \( e_t^L(i) \) as the LIBOR-OIS spread focused on during the financial crisis as a measure of financial stress. We take no stand on the underlying mechanisms that can cause sharp movements in the LIBOR-OIS spread shock and simply model each \( e_t^L(i) \) as exogenous i.i.d. white noise processes with standard deviation \( \sigma_{e_t^L} \).\(^7\)

Banks take the interbank lending rate as fixed. Unlike the loan and deposit markets, banks do not use real resources to transfer money between themselves.\(^8\) Because banks have access to funds outside their stream of savings deposits, the marginal opportunity cost of money will be the interbank lending rate. Given the quantity of interbank borrowing, \( L_{t+1}(i) \), (or lending if \( L_{t+1}(i) < 0 \)), the banks’ balance sheet constraint implies they cannot lend more than they borrowed from households and other banks:

\[
B_{t+1}(i) \leq D_{t+1}(i) + L_{t+1}(i). \tag{10}
\]

In each period, banks receive the principal plus interest on non-defaulted loans originated in period \( t - 1 \) from entrepreneurs, period \( t \) deposits from households, and funds borrowed from other banks. Banks make payments of principal plus interest on period \( t - 1 \) deposits to households and loans from other banks, period \( t \) loans to entrepreneurs, and wage payments to households for labor. In addition, we assume there exists a collections agency that collects the assets of entrepreneurs that default and distributes them to households.

\(^7\)See Smith (2011) (and others cited therein) for explanations as to why the interbank interest rate spreads changed so dramatically.

\(^8\)Traditionally, interbank loans take little more than a phone call to execute.
banks in proportion to their loan market share. Bank $i$'s period $t$ nominal profit function is

$$\Pi_t^B(i) = (1 - F_{t-1}(\bar{\omega}_t))r_t^b(i)B_t(i) + \left( \frac{B_t(i)}{B_t} \right) (1 - \mu)\tilde{\phi}_t^y - r_t^d(i)D_t(i) - r_t^L(i)L_t(i) + D_{t+1}(i) + L_{t+1}(i) - B_{t+1}(i) - w_t \gamma_d^b(i)B_{t+1}(i) - w_t \gamma_d^b(i)D_{t+1}(i),$$

(11)

where $w_t$ is the nominal wage, $F_{t-1}(\bar{\omega}_t)$ is the proportion of entrepreneurs who are unable to repay their period $t - 1$ loan in full in period $t$, $\mu$ is the monitoring cost, and $\bar{\phi}_t$ is the nominal value of assets held by entrepreneurs who defaulted on their period $t - 1$ loans by failing to repay them in full in period $t$.\(^9\) Aggregate bank profits $\Pi_t^B$ are transferred lump-sum to households at the end of each period.

Banks take as given aggregate interest rates $r_{t+1}^d$ and $r_{t+1}^b$, the bank-specific interbank lending rate $r_{t+1}^L$, and aggregate deposits and loans. Banks choose interest rates $r_{t+1}^d(i)$ and $r_{t+1}^b(i)$ and borrowing on the interbank lending market $L_{t+1}(i)$ to maximize their expected real profits subject to the balance sheet constraint. The Lagrangian is given by

$$\mathcal{L}_t(i) = \mathbb{E}_t \sum_{k=0}^{\infty} \lambda_{t+k} \frac{\Pi_{t+k}^b(i)}{P_{t+k}} + \tilde{\mu}_{t+k}(i)(D_{t+1+k}(i) + L_{t+1+k}(i) - B_{t+1+k}(i)),$$

(12)

where $\lambda_{t+k} = \beta^k(C_t/C_{t+k})$ is the standard household intertemporal discount factor and $P_{t+k}$ is the price level in period $t + k$. Setting $\lambda_t = 1$, the first-order conditions are given by

$$\frac{1 - \gamma_d^d(i)w_t}{P_t} + \tilde{\mu}_t(i) = \left( \frac{\eta_d + 1}{\eta_d} \right) \mathbb{E}_t [\lambda_{t+1}r_{t+1}^d(i)P_{t+1}^{-1}]$$

(13)

$$\frac{1 + \gamma_b^b(i)w_t}{P_t} + \tilde{\mu}_t(i) = \mathbb{E}_t \left[ \lambda_{t+1} \left( 1 - F_t(\bar{\omega}_{t+1}) \right) \left( \frac{\eta_b - 1}{\eta_b} \right) r_{t+1}^b(i)P_{t+1}^{-1} + B_{t+1}(i) \left( 1 - \mu \right) \bar{\phi}_{t+1}^y P_{t+1}^{-1} \right]$$

(14)

\(^9\)The complete entrepreneur problem will be described in section 1.2.
\[ P_{t+1}^{-1} + \hat{\mu}_t(i) = \mathbb{E}_t[\lambda_{t+1}r_{t+1}^L(i)P_{t+1}^{-1}] . \] (15)

Let \( \bar{r}_{t+1}^e(i) \) be defined as the expected net nominal return to bank \( i \) for each dollar of entrepreneurial loans originated:

\[ \bar{r}_{t+1}^e(i) = \left( \frac{\eta_b - 1}{\eta_b} \right) (1 - F_i(\bar{\omega}_{t+1}))r_{t+1}^b(i) + B_{t+1}^{-1}(1 - \mu)\bar{\phi}_{t+1}^y . \] (16)

Substituting equations (15) and (16) into (13) and (14) yields

\[ \mathbb{E}_t[\lambda_{t+1}r_{t+1}^d(i)\pi_{t+1}^{-1}] = \frac{\eta_d}{\eta_d + 1} \left[ \mathbb{E}_t[\lambda_{t+1}r_{t+1}^d(i)\pi_{t+1}^{-1}] - \gamma^d(i)w_t \right] \] (17)

\[ \mathbb{E}_t[\lambda_{t+1}\bar{r}_{t+1}^e(i)\pi_{t+1}^{-1}] = \mathbb{E}_t[\lambda_{t+1}r_{t+1}^L(i)\pi_{t+1}^{-1}] + \gamma^b(i)w_t . \] (18)

where \( \pi_{t+1} = P_{t+1}/P_t \) is the rate of inflation in period \( t+1 \). The optimal nominal interest rate on savings deposits is set such that the expected discounted real interest rate is equal to the expected discounted real opportunity cost of money minus the marginal cost of deposits times an optimal "markdown". The net nominal interest rate for loans is set such that the marginal expected discounted real return to loans is equal to the marginal expected discounted real opportunity cost of money plus the marginal cost of making loans; in short, \( \bar{r}_{t+1}^e(i) \) is the marginal cost of a bank loan in equilibrium to bank \( i \).

1.1.2 Aggregate Interest Rates and Interest Rate Spreads

The net nominal interest rate \( paid \) by entrepreneurs, \( r_{t+1}^e \), is equal to the total nominal repayments to banks per dollar borrowed such that

\[ r_{t+1}^eB_{t+1} = (1 - F_i(\bar{\omega}_{t+1}))r_{t+1}^bB_{t+1} + (1 - \mu)\bar{\phi}_{t+1}^y . \] (19)
The total net nominal payment to banks by entrepreneurs is equal to the nominal payments made by entrepreneurs who do not default, where $F_t(\omega_{t+1})$ is the proportion of entrepreneurs that do default, plus the nominal assets of the defaulted entrepreneurs, $\bar{\phi}^y_{t+1}$, minus the bankruptcy cost.

The aggregate marginal cost of loans, $\bar{r}^e_{t+1}$, can be obtained by aggregating equation (16) using equation (4):

$$\bar{r}^e_{t+1} = \left( \frac{\eta_b - 1}{\eta_b} \right) (1 - F_t(\omega_{t+1})) \bar{r}^b_{t+1} + B_{t+1}^{-1}(1 - \mu) \bar{\phi}^y_{t+1}. \quad (20)$$

By combining equations (19) and (20), we can write the net nominal interest rate paid by entrepreneurs as a function of the aggregate nominal cost of lending one dollar and the markup implied by monopolistic competition:

$$r^e_{t+1} = \frac{\eta_b}{\eta_b - 1} \bar{r}^e_{t+1} - \frac{1}{\eta_b} B_{t+1}^{-1}(1 - \mu) \bar{\phi}_{t+1}. \quad (21)$$

The second part of the equation is a correction term due to the fact that banks set a markup on the gross interest rate but not on the assets received through bankruptcy.

The endogenous interest rate spread between the rate faced by entrepreneurs, $r^e_{t+1}$, and the rate received by households, $r^d_{t+1}$, is increasing in firm market power and the marginal cost of producing loans. As seen by equations (17), (18), (21), increases in $\gamma^d(i)$ and decreases in $\eta_d$ decrease the rate received by households, $r^d_{t+1}$ relative to the interbank lending rates and increases in $\gamma^b(i)$ and decreases in $\eta_b$ increase the final entrepreneurial rate $r^e_{t+1}$ relative to the interbank lending rate. Additionally, if $\gamma^b(i) = \gamma^d(i) = 0$ for all banks and $\eta_d = \eta_b = \infty$, then one can show that $r^e_{t+1} = r^d_{t+1}$ and the banking sector collapses to the original BGG financial intermediary.
1.2 Entrepreneurs

Entrepreneurs own capital, hire labor, rent out their own labor, and produce wholesale output. They are risk-neutral and live for a finite number of periods. In each period, entrepreneurs have a constant survival probability \( \gamma \), which implies an expected lifetime of \( 1/(1-\gamma) \). At the end of period \( t \), entrepreneurs buy capital, \( K_{t+1} \), to be used at time \( t+1 \) at the real price per unit of capital, \( Q_t \). Capital purchases are financed through net worth, \( N_{t+1} \), (self-financing) and borrowing from banks, \( B_{t+1} \), (external financing). Capital bought in period \( t \) is combined with labor hired in period \( t+1 \) to produce output in period \( t+1 \). After the entrepreneur uses the capital in period \( t+1 \), it sells its un-depreciated capital back to the capital goods producers at the real price \( Q_{t+1} \).

1.2.1 Capital Demand and the Optimal Debt Contract

Entrepreneurs face an idiosyncratic shock to the real ex-post return on capital expenditures, \( \omega R_{t+1}^k \), where \( \omega \) is the idiosyncratic shock to the real ex-post gross aggregate return to capital expenditures, \( R_{t+1}^k \). \( \omega \) is i.i.d. across entrepreneurs and time with a cumulative distribution function given by

\[
\Pr[\omega \leq x] = F_t(x).
\]

Throughout the numerical simulations, \( F_t \) is a log-normal distribution with unit mean and standard deviation \( \sigma^F_t \). The standard deviation \( \sigma^F_t \) is allowed to change over time, representing movements in the riskiness of borrowers. A change in the riskiness of borrowers is a demand-side financial shock that will have a direct impact on the optimal debt contract between entrepreneurs and banks. We posit a first-order autoregressive process for the standard deviation of the idiosyncratic shocks:

\[
\sigma^F_t - \sigma^F = \rho_\sigma (\sigma^F_{t+1} - \sigma^F) + e^\sigma_t,
\]
where $e_t^\sigma$ is an i.i.d. white noise process with standard deviation $\sigma_{e_t}$.

The timing of the idiosyncratic shock introduces the financial accelerator credit friction. Entrepreneurs choose capital expenditures, and therefore external borrowing, before the realization of the idiosyncratic shock. Therefore, entrepreneurs who receive bad draws will not be able to completely pay back their debt. We use the same debt contract as in BGG, adjusted for the addition of our monopolistically competitive banking sector. Entrepreneurs receive a debt contract from banks specifying a nominal payment of $r_t^b$ for every dollar borrowed. Entrepreneurs who do not have the resources to pay back their loan declare bankruptcy and the collections agency receives the assets of bankrupt entrepreneurs minus the monitoring cost. For simplicity, we model the monitoring cost as a constant proportion $\mu$ of the realized gross real return to entrepreneurs, $R_{t+1}^k Q_t K_{t+1}$.

The variable $\bar{\omega}$ is defined as the cutoff value such that entrepreneurs who receive any value lower than the cutoff are unable to repay their loan in full:

$$\bar{\omega}_{t+1} R_{t+1}^k Q_t K_{t+1} = E_t \left[ r_t^b \pi_t^{-1} \right] B_{t+1}$$

(24)

Given $\bar{\omega}$, we can now define the real value of assets of defaulted entrepreneurs, $\phi_{t+1}^y$:

$$\phi_{t+1}^y = \Xi_t(\bar{\omega}_{t+1}) R_{t+1}^k Q_t K_{t+1},$$

(25)

where

$$\Xi_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega).$$

(26)

Given the real value of assets of defaulted entrepreneurs, the nominal value, $\bar{\phi}$, that appears in the bank problem is simply $\bar{\phi}_{t+1} = P_t \phi_{t+1}^y$.

BGG formulate an optimal debt contract that maximizes the return to the entrepreneurs subject to a zero-profit constraint on the financial intermediaries. This optimal debt con-
tract leads to the financial accelerator.\(^{10}\) Loosely defined, the financial accelerator is the elasticity of the credit spread, \(R_{t+1}^k / \mathbb{E}_t [r_{t+1}^e \pi_{t+1}^{-1}]\) with respect to the aggregate leverage ratio, \(Q_t K_{t+1} / N_{t+1}\). In equilibrium, the marginal return to capital expenditures must be equal to the marginal cost of external finance plus the external finance premium. Because the external finance premium is increasing in the aggregate leverage ratio of entrepreneurs, the optimal debt contract implies a positive elasticity of the credit spread with respect to the leverage ratio, and we allow this to vary over time in our model solution.

### 1.2.2 Production

Each entrepreneur combines its capital with hired labor to produce wholesale output using a constant returns to scale technology.\(^{11}\) The aggregate production function in period \(t\) is given by

\[
Y_t = A_t K_t^\alpha ((H_t^p)^\Omega (H_t^e)^{1-\Omega})^{1-\alpha},
\]

where \(Y_t\) is aggregate output, \(A_t\) is an aggregate productivity shock, \(K_t\) is the aggregate capital purchase in period \(t - 1\) by entrepreneurs, \(H_t^p\) is the labor input from households, and \(H_t^e\) is the labor input from entrepreneurs.\(^{12}\) The aggregate productivity shock follows...
a first-order autoregressive process:

$$\log A_t = \rho_a \log A_{t-1} + \epsilon_a^t,$$  \hspace{1cm} (28)

where $\rho_a \in (0, 1)$ is the persistence of the process and $\epsilon_a^t$ is normally distributed with mean zero and standard deviation $\sigma_{\epsilon_a}$.

Entrepreneurs sell their output to retailers at the nominal wholesale price of $P^w_t$ (and therefore receive the real price $P^w_t / P_t$). Entrepreneurs receive the rents that go to the capital owners; the marginal product of capital is

$$\alpha \left( \frac{P^w_t}{P_t} \right) \frac{Y_t}{K_t}. \hspace{1cm} (29)$$

Because entrepreneurs sell their un-depreciated capital back to capital producers at the end of every period, the real gross aggregate ex-post return to capital expenditures, $R^k_{t+1}$, must be equal to the sum of the capital rents and capital gains from reselling the capital:

$$R^k_{t+1} = \alpha \left( \frac{P^w_t}{P_t} \right) \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1} (1 - \delta) \frac{Q_t}{Q_t}. \hspace{1cm} (30)$$

Equating the marginal cost of labor with the marginal product of labor yields the first-order conditions for labor:

$$W_t = \Omega (1 - \alpha) \left( \frac{Y_t}{H^P_t} \right) \left( \frac{P^w_t}{P_t} \right), \hspace{1cm} (31)$$

$$W^e_t = (1 - \Omega) (1 - \alpha) A_t K^\alpha_t (H^P_t)^{(1-\alpha)\Omega} \left( \frac{P^w_t}{P_t} \right), \hspace{1cm} (32)$$

where $W_t$ is the real wage for household labor and $W^e_t$ is the real wage for entrepreneurial labor.
1.2.3 Net Worth

Entrepreneurs finance their capital expenditures through net worth and borrowing. The net worth of entrepreneurs is derived from entrepreneurial equity, $V_t$ (the wealth gained by operating their firms) and the wage from their labor, $W^e_t$. Total entrepreneurial net worth at the end of period $t$ (or the beginning of period $t+1$), $N_{t+1}$, is given by

$$N_{t+1} = \gamma V_t + W^e_t,$$  \hspace{1cm} (33)

where $\gamma V_t$ is the equity held by entrepreneurs at time $t$ who survive to period $t + 1$. Entrepreneurs who do not survive will simply consume their equity, $C^e_t = (1 - \gamma) V_t$.

Entrepreneurial equity is equal to gross real earnings less the real repayment of loans to banks less the bankruptcy cost:

$$V_t = R^k_{t} Q_{t-1} K_t - r^e_t \pi^{-1} B_t - \mu \phi^y_t.$$  \hspace{1cm} (34)

1.3 Government and Monetary Policy

Real government spending, $G_t$, follows a first-order autoregressive process:

$$G_t - G = \rho_e (G_{t-1} - G) + e^G_t$$  \hspace{1cm} (35)

where $e^G_t$ is an i.i.d white noise process with standard deviation $\sigma_{es}$.

Real government expenditures are financed through money creation and nominal lump-sum taxes such that

$$G_t = \frac{(M_t - M_{t-1}) + T_t}{P_t}.$$  \hspace{1cm} (36)

The government is able to target the nominal benchmark interest rate, $r^{cb}_{t+1}$, through mon-
etary policy.\textsuperscript{13} In our baseline model, the target central bank rate is set by a standard monetary policy rule that targets the level of inflation and the output gap,

\[
\frac{r_{t+1}^{cb}}{r^{cb}_t} = \left( \frac{\pi_t}{\pi} \right)^{\psi_\pi} \left( \frac{y_t}{y} \right)^{\psi_y} \exp(e^{em}_t) - 1.
\]  

(37)

The residual $e^{em}_t$ in the interest rate rule is an i.i.d. white noise process with standard deviation $\sigma_{e^{em}}$, and can be interpreted as a traditional monetary policy shock. Later, we will assume that this monetary policy shock follows a first-order autoregressive process in order to compare it to the rest of our shocks in the model. In Section 5, we augment the monetary policy by including interest rate spreads in the benchmark interest rate rule to assess whether policy that reacts to financial market interest rate spreads has better macroeconomic performance.

2 Calibration and Solution

Table 1 displays a full list of the model parameters and their values. For the traditional DSGE parameters, we choose fairly conventional values. We set the quarterly discount rate $\beta$ equal to 0.9875; this value implies an annual rate of interest of 5.18\% (the average rate on three-month Treasury bills from 1954Q3-2008Q1). The utility of leisure parameter, $\xi$, is set equal to 1.936 such that households spend exactly one third of their time working in the production sector. The capital share $\alpha$ is set to $1/3$, and the entrepreneur share of labor income is set equal to 0.01, implying the household share of income $(1 - \alpha)(1 - \Omega)$ is 0.66. The retail parameters $\eta$, the elasticity of substitution between final goods, and $\theta$, the Calvo price switching parameter, are set to 5 and 0.75 respectively; these parameter choices

\textsuperscript{13}The actual interest rate that is targeted by the bank changes across the different model specifications we look at in our later analysis. However, we are consistent in that the central bank rate is the risk-free rate in each of the economies.
imply a steady state markup of 25% and the average time between price adjustments of four quarters.

Following CD, we define the functional form of the capital goods production function, \( \Phi \), such that the production of capital goods is linear with quadratic adjustment costs:

\[
\Phi \left( \frac{I_t}{K_t} \right) K_t = I_t + \frac{\chi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t. 
\]

We set the marginal adjustment cost parameter \( \chi \) equal to the estimate from CD, 0.5582. Additionally, we set the depreciation rate \( \delta \) to the standard value of 0.025.

We jointly set the financial accelerator variables \( \mu \), \( \sigma^F \), and \( \gamma \) to match the default rate of 1.3 percent, the capital-to-net-worth ratio of 1.2, and the real rate of return on capital expenditures of 10 percent in the steady state.\(^{14}\) The bankruptcy cost \( \mu \) is set to 0.132, the steady state variance of the log-normal idiosyncratic shock distribution \( \sigma^F \) is set to 0.697, and the survival probability of entrepreneurs \( \gamma \) is set to 0.974.\(^{15}\)

In our model, the banking productivity parameters \( \gamma^d \) and \( \gamma^b \) are equal to total hours worked in each market divided by the total dollar value of loans or deposits. Using the Federal Reserve Bank of St. Louis’ FRED database, we use monthly data from January 1990 - February 2011 on total commercial and industrial loans, total demand deposits at commercial banks, the number of employees engaged in credit intermediation and related activities, and the number of employees engaged in depository credit intermediation to construct a monthly series for both banking productivity parameters \( \gamma^d \) and \( \gamma^b \).\(^{16}\)

\(^{14}\)The default rate is from CMR. McGrattan and Prescott (2004) show that the equity-to-debt ratio averaged 4.7 from 1960-1995, and began rising thereafter. A capital to net worth ratio of 1.2 is equivalent to an equity to debt ratio of 4.7. We set the real rate of return on capital expenditures to 10 percent due to a lack of an accurate and agreed-upon measure of the return.

\(^{15}\)Compared to the parameter values in BGG and CMR, our value of the bankruptcy costs is similar to BGG’s value of 0.12 but much smaller than CMR’s value of 0.33. Our value for \( \sigma^F \) is much closer to CMR’s value of 0.67 than BGG’s 0.28. And our value of \( \gamma \) is between the BGG value of 0.9728 and the CMR value of 0.9762.

\(^{16}\)We multiply number of employees in each activity by 160 to obtain an average monthly hours figure.
average value across the sample of $\gamma^d$ is $8.356 \times 10^{-4}$ and the average value of $\gamma^b$ is $4.954 \times 10^{-4}$.

Currently, there are no estimates of the level of competition among banks in either the deposit or loan markets. Given the values for the bank productivity parameters and historical interest rate data, we find the values of $\eta_b$ and $\eta_d$ to match historical entrepreneurial interest rates (we use the prime rate), $r^e$, the Fed Funds rate, $r^{cb}$, and the household savings deposit interest rate (we use the three-month Treasury bill rate), $r^d$, in the steady state. Using quarterly interest rates of 1.84%, 1.39%, and 1.27% for $r^e$, $r^{cb}$, and $r^d$, respectively, in the steady state, we find that the values $\eta_b = 260.36$ and $\eta_d = 7154.56$ are consistent with our model with no bank heterogeneity and steady state interest rates.\(^{17}\) The value for $\eta_b$ implies a steady state markup of 38.6 basis points and the value for $\eta_d$ implies a steady state markdown of only 1.4 basis points. We find that the larger wedge in the loan market is consistent with the fact that bank customers can take their deposits to another bank on short notice, but cannot move their existing loan to another bank.

The dynamics of the model will be driven by the parameterization of the shock processes. For the non-financial shocks, we take parameter values from papers with similar shock processes as those in our model. For the aggregate productivity shock, we follow Bloom, Floetotto, and Jaimovich (2010), who estimate macroeconomic uncertainty using the conditional heteroskedasticity of aggregate output, we set $\rho_a$ equal to 0.9627 and $\sigma_{e_a}$ equal to 0.018. For the government spending shock, we use the parameter values for the same AR(1) shock in CMR; we set $\rho_g$ equal to 0.92 and $\sigma_{e_g}$ equal to 0.02. For the standard deviation of the monetary policy shock, we choose $\sigma_{e_m}$ to match the value of 0.0058 from CD who estimate a similar monetary policy function in a model with the financial

\(^{17}\)In our model with heterogenous banks, we find $\eta_b = 260.23$ and $\eta_d = 4450.80$. While the estimate of the elasticity of substitution in the loan market does not change significantly, the estimate of the elasticity of substitution in the loan deposit market is much smaller. However, the change in the estimated markdown is less than one basis point.
accelerator.

For financial shocks, CMR also include a shock to $\sigma^F$, the variance of the distribution of idiosyncratic entrepreneurial shocks; following their estimates, we set $\rho_{\sigma^F}$ to 0.93 and $\sigma_{e^\sigma}$ to 0.035. For the bank loan productivity shock, we estimate $\rho_{\gamma^b}$ and $\sigma_{e^{\gamma^b}}$ using the loan productivity data used to calculate $\gamma^b$. From these estimates, we find that bank loan productivity is highly persistent with low variance; we estimate that the persistence parameter $\rho_{\gamma^b}$ is equal to 0.98 and the standard deviation of the shock $\sigma_{e^{\gamma^b}}$ is equal to $1 \times 10^{-5}$. Given that prior to the financial crisis LIBOR-OIS shocks were non-existent, it is harder to properly estimate the ex-ante distribution of those shocks. For this reason, we model the LIBOR-OIS shocks as a white noise process and set $\sigma_{e^l} = \sigma_{e^{\gamma^b}} = 1 \times 10^{-5}$.

Because the impact of shocks also depend critically on the exact monetary policy rule, we also calibrate the monetary policy parameters such that $\rho_r = 0.9$, $\psi_\pi = 1.38$, and $\psi_y = 0.62$ in all of our specifications. In Section 5, we will also calibrate the policy parameter associated with the interest rate spread such that $\psi_s = -0.86$.

We solve our full model both with and without bank heterogeneity. To solve each model, we use two steps. In the first step, we solve for the steady state values and parameters using GAMS. In the second step, we employ Dynare to use a first-order approximation of the model around the steady state found in step one. When we include bank heterogeneity, we use twelve banks. Within the twelve banks, we have three bank classes: small, medium, and large. We calibrate the ex-ante weight of each bank class to 0.05, 0.25, and 0.7, respectively, to approximate the bank size distribution in the U.S. Because we do not have good data on how heterogeneous banks are within each bank class, we simply use four

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18 We convert the monthly time series into a quarterly time series, HP filter the data to remove the increasing trend in bank loan productivity, and estimate a first-order autoregressive process.

19 We solve for the full financial accelerator with the complete debt contract and do not simply reduce the debt contract into a relationship between the credit spread and the leverage ratio as is done in other financial accelerator models. See Appendix C for a complete description as well as a discussion on the importance of using the full financial accelerator in the model.
banks within each class that are +/- 20% as productive as the average productivity parameters $\gamma^d$ and $\gamma^b$. Within each class of banks, we will have one bank that is more productive than average at both deposits and loans, two banks that have high productivity in one market and low productivity in the other market, and one bank that has bad productivity in both markets.

3 Banking and the Financial Accelerator

3.1 Banking and Non-Financial Shocks

We begin our analysis by looking at the role our banking sector has in propagating traditional monetary policy shocks, supply shocks, and demand shocks. To do this, we compare the response of the economy to these shocks in our baseline model to the response in the financial accelerator version of our model without banks (FA), as well as the response in an equivalent DSGE model with no financial accelerator (NO FA).\textsuperscript{20} Figures 1 - 3 plot the impulse response functions (IRFs) of 1% shocks to monetary policy, productivity, and government spending, respectively, on selected quantity, price, entrepreneurial, and interest rate variables.

The IRFs to a 1% expansionary monetary policy shock are shown in Figure 1. Across all models, a decrease in the benchmark central bank rate lowers the net nominal entrepreneurial interest rate, $r_{t+1}^e$, which leads to increased demand for new capital expenditures. This increased demand leads to an immediate increase in investment, output, consumption, price levels, and entrepreneurial net worth. For the models with the financial accelerator, the decrease in interest rates also lead to a decrease in the default rate of en-

\textsuperscript{20}In each of the two alternative models, we re-calibrate the utility of leisure parameter $\xi$ such that households spend one third of their time working in the production sector. In the FA model, we re-calibrate the financial accelerator variables to match our main calibration strategy: quarterly default rate of 1.3 percent, aggregate leverage ratio of 1.2, and a steady state return to capital expenditures of 10 percent.
entrepreneurs as well as a decrease in both the external finance premium, measured as the
bankruptcy costs relative to total borrowing $\mu \phi_t^y / B_{t-1}$, and the expected real credit spread,
$s_{t+1} = \mathbb{E}_t \left[ R_{t+1}^e / (r_{t+1}^e \pi_{t+1}^{-1}) \right]$. Interestingly, the decreases in the credit spread and finance
premium are greater in the FA model with no banks than the full banking model. In the full
banking model, even while the aggregate nominal entrepreneurial rate set by the banks falls
significantly in response to the shock, the spread between the aggregate nominal interest
rates $r_{t+1}^e$ and $r_{t+1}^d$ increases, implying that the household savings deposit rate falls by even
more than the net entrepreneurial interest rate. This is a result of the positive differential in
markups on loans over savings deposits.

While Figure 1 shows the differential effects of the monetary policy shock on the credit
spread and the finance premium between models, it looks as if the responses of a majority
of the variables are identical across models. While it is true that the responses are similar,
there is some important variation across models that is hard to see due to the scale of
the figure. The theory says that the financial accelerator should have an increasing effect
on the response of output and investment to a monetary policy shock because as interest
rates go down, the expected return to capital expenditures as measured by the credit spread
increases, capital expenditures increase, which increases the leverage ratio, which then
increases the credit spread (because in equilibrium the marginal cost of financing must be
equal to the marginal return to borrowing), and therefore increases capital demand further,
etc. BGG show that this upward spiral leads to an increase in the current period’s output
and investment relative to a model with no financial accelerator. In our model, this effect is
small but present: the FA model increases the first period response of output and investment
by 0.8 and 1.8 basis points, respectively. Increases in the response in the monopolistically
competitive banking model relative to the NO FA model are smaller: 0.5 and 1.1 basis
point increases for output and investment, respectively.\footnote{One reason the response to monetary policy shocks are so similar across models is because we set the}

\begin{footnote}{One reason the response to monetary policy shocks are so similar across models is because we set the}

on the initial response of output and investment to a monetary policy shock is a function of the weights in the monetary policy function. This is due to the fact that the credit spread that effects borrowing today is a function of today’s expectation of inflation tomorrow and inflation expectations are a function of monetary policy. Examining the initial responses of output and investment to monetary policy shocks leads us to conclude the policy weight on output has a greater effect on the size of the financial accelerator effect, and therefore policymakers can significantly reduce the size of the financial accelerator if they strongly target output in their interest rate rule. Perhaps more importantly for this paper, the size of the financial accelerator effect (as measured by the change in the first-period response to a monetary policy shock) is much smaller in our banking model than the FA model with no banks.

Figure 2 shows the IRFs to a 1% positive productivity shock. While each model shows the standard hump-shaped response of the economy to the positive supply shock, the size of the shock varies considerably across the three models. Relative to the NO FA model, both the FA model and our full MC bank model decrease the response of the economy to the shock. With the financial accelerator mechanism, capital expenditure demand is a function of the expected return to capital expenditures. While the supply shock increases the expected real return, $R_{t+1}^k$, the shock also decreases inflation expectations. Since deflation decreases the return to capital demand in the financial accelerator model (it increases the real net interest rate the entrepreneur is required to repay), the effect of a positive productivity shock is smaller in our two models with the financial accelerator than the DSGE model without the financial accelerator. Similar to the response to an expansionary productivity shock, our full model with MC banks shows a smaller response of the credit spread steady state leverage ratio to 1.2. This implies that only 16% of capital expenditures are externally financed. Using a leverage ratio of 2 in our calibration would imply 50% of capital expenditures are externally financed. This calibration effects the size of the increased response to monetary policy shocks because a larger fraction of capital demand will be sensitive to changes in the interest rate paid through external financing.
and finance premium than the FA model. As a result of the smaller increase in the credit spread, the effect of the positive productivity shock is greater in the banking model than the FA model.

The IRFs to a 1% positive government spending shock are shown in Figure 3. While output shows a positive hump-shaped response to shock in all models, the increase in output is derived entirely from increases in government spending as both private investment and consumption are crowded out by the increase in public spending. The decrease in investment is smaller in the two models with the financial accelerator than the NO FA model; as a result, the increase in output is greater in those two models, as well. Relative to the FA model, our banking model decreases the response to output by increasing the negative response to investment. The net entrepreneurial interest rate falls by slightly less in our banking model than the FA model, and therefore investment falls by slightly more in our banking model than the FA model.

In summary, relative to the NO FA model, the FA model increases the economy’s response to monetary policy shocks and government spending shocks and decreases the response to productivity shocks. With respect to all three shocks, adding our banking sector to the FA model mitigates, but does not flip, those responses. In other words, our full model also increases the response to monetary policy and government spending shocks and decreases the response to productivity shocks relative to the NO FA model, but it decreases the size of the changes in the IRFs for all three shocks relative to the FA model.²²

### 3.2 Banking and the Credit Spread-Leverage Ratio Elasticity

To further understand how our banking system mitigates the effect of the financial accelerator mechanism (the relationship between the credit spread and the aggregate leverage

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²²Quantitatively, the relative effects of the shocks in the FA model and the banking model depend on the calibration of the monetary policy rule and the debt contract parameters.
ratio), we simulate both the FA model and the MC bank model and calculate the implicit elasticity \( f_a_t \) each period:

\[
f_a_t = \frac{\log(R^e_{t+1}) - \log(r^e_{t+1}) + \log(\pi_{t+1})}{(\log(q_t) + \log(k_{t+1}) - \log(n_{t+1}))},
\]  

(39)

In the FA model with no banks, the mean of the implicit elasticity is 6.14%. In our full banking model, the mean of the implicit elasticity is 3.07\%.

To understand why the elasticity falls by nearly half when we add monopolistically competitive banks, we consider a few explanations. First, the FA model was re-calibrated to match the same steady state values for the default rate, capital-wealth ratio, and real rate of return on capital. Therefore, we first solve the FA model in which we assign the debt contract parameter values \( (\mu, \gamma, \sigma^F) \) from the full banking model and find an implicit elasticity of 4.4\%. Therefore, about 55.6\% of the difference in the implicit elasticity is due simply from recalibrating the FA model to match the same steady state values. However, 44.4\% of the difference in the implicit elasticity is due to banking sector. Second, to analyze the source of the smaller elasticity within the banking sector, we consider two extreme calibrations of the banking productivity and market power. In the first, we consider a version of our model with perfect competition \( (\eta^d = \eta^b = \infty) \) and calibrate the banking productivity parameters to match historical interest rate spreads. Again, we find that the average elasticity is 3.07\%. In the second calibration, we impose zero marginal costs \( (\gamma^d = \gamma^b = 0) \) and find the values of \( \eta^d \) and \( \eta^b \) to match historical interest rate spreads. Once more, we find an average elasticity of 3.07\%. Because we find that the average implicit elasticity is

\(^{23}\)BGG use a calibrated value of 5\% for the elasticity in their model.

\(^{24}\)While we previously showed that the effect of the financial accelerator is a function of monetary policy, the estimated elasticity is not a function of monetary policy. So, we conclude that monetary policy only affects the size of the shocks and does not affect the financial accelerator propagation mechanism directly.

\(^{25}\)In these examples, we maintain the parameter values of \( \mu, \gamma, \) and \( \sigma^F \) from the full banking model.

\(^{26}\)This calibration strategy will represent a lower bound on the values of \( \eta^d \) and \( \eta^b \) as it implies that interest rate spreads must entirely be explained by markups and markdowns. We find lower bound values of \( \eta^d = 884.01 \) and \( \eta^b = 224.25 \).
equal in these two calibration exercises that hold the interest rate spread constant, we con-
clude that the endogenous interest rate spread itself is causing a decline in the elasticity and
that we cannot attribute the change in elasticity exclusively to monopolistic competition or
positive marginal banking costs.

Finally, we find that any change in the parameterization of the banking sector that in-
creases the endogenous interest rate spread in the steady state (increases in $\gamma^d$ and $\gamma^b$ or
decreases in $\eta^b$ and $\eta^d$) will decrease the implicit elasticity of the credit spread with re-
spect to the leverage ratio in the model. By solving the full financial accelerator model
with an endogenous elasticity of the credit spread with respect to the leverage ratio, we
have been able to show how this important elasticity crucially varies with the endogenous
interest rate spread introduced through the banking sector.

4 Financial Shocks

With a basic understanding of our model with a representative, monopolistically compet-
itive bank, we now aim to understand the relationship between financial shocks and the
intermediation role of the banking sector.

4.1 Demand-Side Financial Shocks

Demand-side financial shocks originate from the riskiness facing entrepreneurs each pe-
period. This riskiness is characterized by the parameter $\sigma^F$, the standard deviation of the
distribution of idiosyncratic shocks hitting entrepreneurs each period. Shocks to riskiness
are captured by equation (23), which postulates $\sigma^F$ as a first-order autoregressive process.

Figure 4 plots the IRFs of our major variables to a 1% increase in riskiness. The blue
line represents the FA model, while the dashed red line represents the model with a rep-
resentative monopolistically competitive bank. A nearly universal result is that shocks are
mitigated in the model with a monopolistically competitive bank. As an example, the relative response of output is approximately 50% less in the banking model relative to the simple FA model. Qualitatively similar results hold for other macroeconomic variables. An analogous story happens on the financial side of the economy. The initial reaction of the entrepreneurial interest rate is almost three times as high in the simple FA model, with the credit spread response almost two times as high. In line with evidence from the previous section, we also find that the response of the financial accelerator is higher in the FA model than in our banking model.

To better illustrate the effect of a shock to borrower riskiness, Figure 5 maps out the effect of this shock on the rest of the economy. With an increase in $\sigma^F$, the debt contract implies a higher default rate and lower capital-to-wealth ratio $k$, and banks offset the increased default rate with a higher $r^b$. However, there is no direct impact of this shock on $r^d$ or $\bar{r}^e$. Due to the marginal cost correction term coming from bank production, there will be a small decrease in $r^e$, but the general equilibrium effects do not offset the decline in the optimal capital-to-wealth ratio. This decline in $k$ leads to a decrease in both output and investment, the latter which triggers the financial accelerator mechanism: the price of capital falls, lowering wealth, and beginning a cycle of continually lower investment. Finally, monetary policy responds to the decreased output by lowering the central bank rate, which in turns lowers $r^d$ and $r^e$. The decrease in $r^d$ increases consumption through the Euler equation, while the decrease in $r^e$ increases the credit spread, offsetting most (but not all) of the direct negative effects of the shock on investment. In the end, we find that the general equilibrium effects of this shock are small in our model.
4.2 Supply-Side Financial Shocks

Next, we turn to the supply side of the financial sector. There are two different supply-side shocks we examine. The first is a shock to bank loan productivity, or an increase in the cost of bank intermediation. The second is a shock to the spread between the interbank interest rate and the central bank rate.

4.2.1 Bank Loan Productivity Shocks

With regards to bank production, we allow the parameter $\gamma_b^i(t)$, which governs how real labor resources are used to produce loans, to vary over time according to a first-order autoregressive process (given by equation (9)). A positive shock to $\gamma_b^i(t)$ implies a decrease in loan production, or an increase in the cost of bank intermediation, given the specification for loan production in equation (8).

Figure 6 displays the IRFs to a 1% increase in $\gamma_b^i$ in the model with a representative, monopolistically competitive bank. Given the negative impact this shock has on loan production, we see decreases in both output and investment. This, in turn, has impacts on both net worth and default rates of entrepreneurs. The net worth of entrepreneurs falls by over 1% after this shock, while the default rate decays over time away from its steady state. Indeed, this decay effect is related to the response of the external finance premium, which is falling after this shock. A shrinking of the premium is, in fact, in line with the decrease in default rates. Interestingly, there is a hump-shaped response of the net entrepreneurial rate, which combined with the increase in inflation causes a decay in the credit spread after the shock. A similar set of results holds once we look at the full, monopolistically competitive banking model with heterogeneous banks.

To better highlight the important role that these intermediation cost shocks play, Figure

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27 The shock standard deviation is assumed constant across all banks.
8 shows how this shock propagates through the economy. An increase in the marginal cost of loan origination causes immediate increases in $r^e$ and $p^e$. These then cause the expected credit spread $\mathbb{E}_t \left[ R^{k}_{t+1}/(r^e_{t+1} p_{t+1}) \right]$ to decrease, as agents expect that the real return on capital will decrease by less than the increase in real borrowing rates. This change in the expected credit spread decreases the capital-to-wealth ratio $k$ and investment, which then triggers the financial accelerator mechanism: the price of capital falls, wealth falls, beginning the cycle of lower investment. Monetary policy responds to the decreased output by lowering the central bank rate, which partially offsets the increase in $r^e$ and causes a drop in $r^d$. The decrease in $r^d$ increases consumption through the Euler equation. However, the fact that this shock operates through the intensive margin of investment means that while the decrease in $r^e$ increases the credit spread, offsetting most (but not all) of the direct negative effects of the shock on investment, the flow of this shock goes all the way back to its impact on the expected credit spread, and a second larger cycle begins again.

### 4.2.2 Interbank Lending Shocks

A second supply-side shock we examine is an increase in the spread between interbank rates and policy rates. We compare this spread to the commonly mentioned LIBOR-OIS spread. Namely, this is an exogenous shock that induces a spread between the bank-specific interbank lending rate $r^L_{t+1}(i)$ and the benchmark nominal interest rate set by the central bank $r^c_{t+1}(i)$. The shock itself is identified by the i.i.d white noise process $e^L_t$, with standard deviation $\sigma_e$. Again, we don’t take a stand on where this shock originates from, but we know from empirical evidence during the crisis that monetary policy was no longer effective in pinning down other interbank lending rates, thus inducing a non-negligible spread between these interbank rates and the policy instrument itself.

Figure 7 displays the IRFs to a 1% increase in the interbank lending spread in the model with a representative, monopolistically competitive bank. A positive shock to the interbank
lending spread reflects an increase in the opportunity cost of money in the interbank lending market, which implies banks who would like to produce more loans via borrowing in the interbank market may not be able to do so because it is too expensive (i.e., the returns from the strategy of borrowing in the interbank market and lending in the entrepreneurial loan market are not positive). The shock causes decreases across the board in output, investment, and consumption. We see a fall in net worth of entrepreneurs, and the typical relationship between the external finance premium and the default rate. Interestingly enough, the tick up in inflation along with the not one-for-one increase in the entrepreneurial interest rate cause an increase in the credit spread. Breaking it down among different banking sectors, the large bank shocks dominate; most of the impact felt by the variables we look at are coming from shocks to the large bank sector.

Figure 9 illustrates how this interbank lending shock works through the economy. An increase in the interbank lending spread causes $r^d$ and $r^e$ to increase, but the spread between them is unchanged. To begin, the increase in $r^d$ decreases consumption through the Euler equation, which implies a decrease in a variety of macro variables including output, the price of capital, the wage, hours worked, and importantly $R_k$. The decrease in the rental rate of capital decreases both entrepreneurial equity $V$ and net worth $N$. Turning to the increase in $r^e$, the monetary policy rule implies that expectations of the credit spread will fully adjust after one period, which implies no change in the intensive investment margin through the capital-to-wealth ratio $k$. Combined with the effects of the change in $r^d$, a constant $k$ and decrease in $N$ lead to decreases in aggregate capital expenditures, investment, and borrowing. The fall in investment triggers the financial accelerator mechanism: the price of capital falls, wealth falls, beginning the cycle of lower investment. Policy responds to the fall in output, with the central bank rate dropping to partially offset the increase in the interbank lending spread and as a result the general equilibrium increases in $r^d$ and $r^e$ are less than implied by the size of the initial shock.
In summary, shocks to both the demand and supply sides of the financial sector of the economy have non-trivial effects. Supply-side shocks have larger impacts than demand-side shocks, in particular the shock to the cost of bank intermediation through the production function. Next, we will examine how important these financial shocks are, and how they compare to the standard macroeconomic shocks to monetary policy, aggregate productivity, and government spending that we previously examined.\textsuperscript{28}

### 4.3 The Importance of Bank Intermediation Cost Shocks

The results from the previous subsection shed light on the importance of different types of financial shocks. Due to its effect on the intensive investment margin, the shock to bank productivity, or an increase in the cost of financial intermediation, had the largest effect. However, this effect could be coming from our calibration of the shock rather than from its true economic impact. To alleviate this concern, we normalize the size of all of the financial shocks to have the same persistence and standard deviation in an effort to examine the relative importance of each shock fairly.

To begin, we specify each of the financial shocks to follow a first-order autoregressive process with persistence parameter of 0.9 and an i.i.d. shock standard deviation of 0.00001. Figure 10 compares the effect of each of the three financial shocks in the model with a monopolistically competitive banking sector with no heterogeneity. In line with the fact that the bank productivity shock works through the intensive investment margin, a margin that the other two financial shocks do not have an effect on, the magnitude of the impulse responses of output, investment, and consumption are much larger after a shock

\textsuperscript{28}De Graeve (2008) focuses on the business cycle relationship between credit spreads and output. We find a countercyclical correlation of -0.03 between the credit spread and output, in line with previous work. In addition, we do a variance decomposition of the credit spread in our full, heterogenous banking model, and find that approximately 94% of the forecast error variance of the credit spread is due to the financial shocks (not inclusive of monetary policy shocks). Therefore, what De Graeve (2008) really might be capturing in its credit spread analysis is financial shocks, which we have shown are fundamentally important to the analysis.
to the intermediation cost than to riskiness or interbank lending. Looking at the financial variable responses, we see that they are larger and much longer-lived after shocks to the intermediation cost. This result is robust to different persistence parameters, as well as when we move to the model with bank heterogeneity.

To go a step further, we then specify every shock in the model to follow a first-order autoregressive process with persistence parameter of 0.9 and an i.i.d shock standard deviation of 0.00001. In doing so, it allows us to compare not only the financial shocks to each other, but also to our standard monetary policy, aggregative productivity, and government spending shocks. Table 2 displays the variance decomposition of the twelve bank, monopolistically competitive banking sector model. Each row of the table identifies how much of the forecast error variance can be attributed to each of the shocks in the model. By looking at the model with bank heterogeneity, it allows us to breakdown how much of forecast error variance of our variables of interest are due to the banking-sector-specific shocks. At first glance, one thing stands out: for all but the policy rate, the majority of forecast error variance of the variables can be attributed to bank productivity shocks, mainly to the bank productivity shock originating in the large banking sector. Even when normalizing all of the shocks to follow the same, exogenous process, it is clear that this cost of intermediation shock is crucial for our understanding of how shocks propagate through our economy. Also interesting is the fact that the interbank lending shocks explain more of the forecast error variance of the central bank policy rate than the monetary policy shock itself.

In short, we find that shocks to the cost of intermediation through bank productivity are not only the most important shock within the set of financial shocks, but also within the set of all shocks in our model. To the best of our knowledge, this is a unique and important result, and it highlights the need going forward for better understanding of both the qualitative and quantitative implications of the costs and benefits of bank intermediation.
5 A Monetary Policy Experiment

In an effort to identify the effect of different monetary policy rules in an environment with a heterogeneous, monopolistically competitive banking sector and financial shocks, this section performs a monetary policy experiment using policy rules that differ in their reaction to financial market interest rate spreads. Specifically, we propose three different policy rules. The first is our benchmark policy rule that responds only to the lagged policy rate, inflation, and the output gap. This is what we refer to as our sub-optimal (or baseline) policy rule, as we hypothesize that there may be other things that policy can react to that will result in macroeconomic benefits.

Our second proposed rule is one that responds to the lagged policy rate, inflation, the output gap, and the interest rate spread between $r^e$ and $r^d$. It is this spread that we have identified as the catalyst to intermediation cost shocks being the most important, and thus it seemed a natural candidate as something that policy might react to. Lastly, our third proposed rule responds to the lagged policy rate, inflation, the output gap, and the expected credit spread $\mathbb{E}_t \left[ \frac{R^{k}_{t+1}}{r^e_{t+1}} \pi_{t+1} \right]$. We also know that this expected credit spread is important, and so we want to ascertain how different policies might look if they respond to each of these spreads separately. We calibrate the new policy parameter in an effort to minimize macroeconomic variances. Holding the benchmark policy parameters fixed, we choose a uniform interest rate spread parameter $\psi = -0.86$ for our two new policy rules. Intuitively, this parameter should be negative; an increase in either of these interest rate spreads implies some sort of financial market stress, which should in turn be alleviated via expansionary policy.

Figures 11 though 15 plot the IRFs of our macroeconomic and financial variables to shocks to monetary policy, aggregate productivity, riskiness, bank productivity, and inter-bank lending, respectively. In response to the monetary policy, aggregative productivity,
and interbank lending shocks, the three different policy rules have nearly identical implications for macroeconomic and financial market responses, with only slight variation for the response of the net entrepreneurial interest rate to the aggregate productivity shock under the third policy rule reacting to the expected credit spread.

Once we begin examining the shocks to riskiness and bank productivity, the importance of the policy rule begins to come to light. Figure 13 plots the IRFs to the riskiness shock, and the policy rule reacting to the credit spread seems to perform the best in terms of dampening the effect of the shock on the macroeconomy (via output and investment). The opposite result holds when we examine the bank productivity shock. Here, policy that reacts to the spread between interest rates $r^e$ and $r^d$ has better macroeconomic performance. These results are not meant to advise policymakers on what their policy rule should look like, but rather are meant as an illustration of how, in the face of financial shocks, we can adjust our standard policy rules in a performance-enhancing way. Since bank productivity shocks were shown in the last section to be the most important shock in the context of volatility in our model, even after normalization, perhaps new policy rules that are able to react to such an interest rate spread are worth examining as reliable alternatives to our standard policy rules.

6 Conclusion

This paper augments the standard BGG financial accelerator model with the inclusion of a fully-specified, heterogeneous, monopolistically competitive banking sector. With this banking sector, we are able to address the impact of shocks originating from the financial side of the economy (both demand and supply shocks) on the real macroeconomy. First, we show that the inclusion of our banking sector mitigates (but does not negate) the impact of the credit friction on macroeconomic fluctuations by affecting both the magnitude and
persistence of non-financial shocks. This is achieved by effectively reducing the elasticity of the expected credit spread with respect to the leverage ratio through the introduction of a positive interest rate spread. Second, we find that financial shocks, both on the demand and supply sides, can cause severe macroeconomic fluctuations, with supply-side shocks to bank intermediation costs via bank production having the largest impact. Even after normalizing all of the shocks in our economy as the same exogenous process, the impact of intermediation costs on the intensive investment margin and the financial accelerator imply that it is the most important shock in explaining variation in the macroeconomy. Using a monetary policy experiment with variations in policy rules, we highlight that policy that reacts to financial market interest rate spreads can improve macroeconomic performance above and beyond a simple policy rule that only reacts to macroeconomic variables.

This paper is not meant to provide an explanation of the financial crisis. This model is able to qualitatively explain the behavior of the macroeconomy around its steady state, and incorporates a rich-enough financial market via the banking sector to understanding the implications of large financial market shocks. Going forward, we have multiple avenues of continued study. First, we aim to estimate this type of model to understand how to quantify the size, competition, and importance of the banking sector in this type of model. The most important aspect of this, undoubtedly, is to quantify intermediation costs. A second focus of ongoing research includes examining the relationship between bond markets and the overall economy in order to ascertain how movements in the term structure are related to the shocks we describe in this paper. We also wish to address the idea that credit history can have an impact on shock propagation. In the end, we believe the future of this research area remains fruitful given the policy questions aimed at the banking sector.
References


Crisis,” Unpublished Manuscript.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>Panel A: Households, Retailers, Government</td>
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<tr>
<td>$\beta$</td>
<td>0.9875</td>
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<td>Panel B: Production</td>
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<tr>
<td>$\alpha$</td>
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<td>Panel C: Entrepreneurs</td>
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<td>$\sigma^F$</td>
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<td>Panel D: Banking Sector</td>
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<td>$\eta^{d^*}$</td>
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<td>Elasticity of substitution in deposit market without (with) bank heterogeneity</td>
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<td>Panel E: Shock Processes</td>
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<td>$\sigma_{e^L}$</td>
<td>$1 \times 10^{-5}$</td>
<td>S.D. of interbank shocks</td>
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Table 1: Full Model Parameters
Table 2: Variance Decomposition of the model with the twelve bank, monopolistically-competitive banking sector. All shocks are normalized as AR(1) processes with persistence parameter of 0.9 and i.i.d. shock standard deviation of 0.00001. In each row, the elements in the columns represent what percent of the forecast error variance is attributable to the given shock. $e^m$ is the monetary policy shock, $e^a$ is the aggregate productivity shock, $e^g$ is the government spending shock, $e^o$ is the riskiness shock, $e^{gb}(1)$ is the bank productivity shock in the small banking sector, $e^{gb}(5)$ is the bank productivity shock in the medium banking sector, $e^{gb}(9)$ is the bank productivity shock in the large banking sector, $e^{L}(1)$ is the interbank lending shock in the small banking sector, $e^{L}(5)$ is the interbank shock in the medium banking sector, $e^{L}(9)$ is the interbank shock in the large banking sector.
Figure 1: Impulse responses functions (in percentage) to a 1% expansionary monetary policy shock. The solid blue line represents the DSGE model with no financial accelerator, the dashed red line represents the DSGE model with the full financial accelerator but no banks, and the dash-dot green line represents the full model with a monopolistically competitive banking sector.
Figure 2: Impulse responses functions (in percentage) to a 1% positive productivity shock. The solid blue line represents the DSGE model with no financial accelerator, the dashed red line represents the DSGE model with the full financial accelerator but no banks, and the dash-dot green line represents the full model with a monopolistically competitive banking sector.
Figure 3: Impulse responses functions (in percentage) to a 1% positive government spending shock. The solid blue line represents the DSGE model with no financial accelerator, the dashed red line represents the DSGE model with the full financial accelerator but no banks, and the dash-dot green line represents the full model with a monopolistically competitive banking sector.
Figure 4: Impulse responses functions (in percentage) to a 1% positive riskiness (demand-side) financial shock. The solid blue line represents the DSGE model with the full financial accelerator and the red line represents the model with a monopolistically competitive banking sector with no heterogeneity.
Shock to $\sigma^F$: Borrower Riskiness

No changes to $r^L$, $\gamma^b$, and $\gamma^d$ imply no direct impact of shock on bank rates $r^d$ and $r^e$.

$\gamma^e$ decreases (slightly) due to MC correction term (Equation (21)).

Banks offset increased default rate with higher $r^d$. (Equation (20)).

Decrease in $r_e$ does not offset decline in optimal capital-wealth ratio $k = QK/N$.

Decrease in $k$ leads to decrease in investment and output.

Decrease in investment triggers financial accelerator mechanism: decreased price of capital lowers wealth, which lowers investment, etc.

Policy responds to decreased output by decreasing the target interest rate, lowering $r^d$ and $r^e$.

Decrease in $r^e$ increases consumption today through Euler equation.

Decrease in $r^d$ increases credit spread, offsetting most of, but not all of the direct negative effects of shock on investment.

Given no changes to $r^k$ and $r^e$, debt contract implies higher default rate, lower capital to wealth ratio $k = QK/N$.

Figure 5: The dynamics of a shock to borrower riskiness.
Figure 6: Impulse responses functions (in percentage) to a 1% positive supply-side shock to the bank loan production function. The solid blue line represents the model with a monopolistically competitive banking sector with no heterogeneity.
Figure 7: Impulse responses functions (in percentage) to a 1% interbank lending shock. The solid blue line represents the model with a monopolistically competitive banking sector with no heterogeneity.
Shock to bank loan productivity $\gamma^b$

Increase in marginal costs of loan origination increase rates $\bar{r}$ and $r^e$

The expected credit spread $E_t \left[ \frac{R_{t+1}^e}{(r_{t+1}^e+\pi_{t+1})} \right]$ decreases. Agents expect real return on capital to decrease by less than increase in real borrowing rate.

Change in expected credit spread implies decrease in capital to wealth ratio, $k = QK/N$, and investment

Decrease in investment triggers financial accelerator mechanism: decreased price of capital lowers wealth, which lowers investment, etc

Policy responds to decrease in output; central bank target rate decreases to partially offset increase in $r^e$, also causes drop in $r^d$

Decrease in $r^d$ increases consumption today through Euler equation

Decrease in $r^d$ partially offsets decrease in expected credit spread, partially offsetting declines in investment and output

Figure 8: The dynamics of a shock to bank productivity.
Figure 9: The dynamics of a shock to the interbank lending spread.
Figure 10: Impulse responses functions (in percentage) in the model with a monopolistically competitive banking sector with no heterogeneity. All shocks are normalized as AR(1) processes with persistence parameter of 0.9 and i.i.d. shock standard deviation of 0.00001. The solid blue line represents the response to a 1% positive shock to riskiness, the dashed red line represents the response to a 1% positive shock to bank productivity, and the dash-dot green line represents the response to a 1% positive shock to interbank lending.
Figure 11: Impulse responses functions (in percentage) to a 1% expansionary monetary policy shock in the model with the twelve bank, monopolistically-competitive banking sector. The solid blue line represents the response in the model with a traditional monetary policy rule, the dashed red line represents the response with the monetary policy rule that reacts to the interest rate spread $r^e / r^d$, and the dash-dot green line represents the response with the monetary policy rule that reacts to the expected credit spread $E_t \left[ r^e_{t+1} / r^d_{t+1} \pi_{t+1} \right]$. 
Figure 12: Impulse responses functions (in percentage) to a 1% positive productivity shock in the model with the twelve bank, monopolistically-competitive banking sector. The solid blue line represents the response in the model with a traditional monetary policy rule, the dashed red line represents the response with the monetary policy rule that reacts to the interest rate spread $r^e/r^d$, and the dash-dot green line represents the response with the monetary policy rule that reacts to the expected credit spread $\mathbb{E}_t \left[ R_{t+1}^k / \left( R_{t+1}^e + \pi_{t+1} \right) \right]$. 
Figure 13: Impulse responses functions (in percentage) to a 1% positive riskiness shock in the model with the twelve bank, monopolistically-competitive banking sector. The solid blue line represents the response in the model with a traditional monetary policy rule, the dashed red line represents the response with the monetary policy rule that reacts to the interest rate spread $r^e / r^d$, and the dash-dot green line represents the response with the monetary policy rule that reacts to the expected credit spread $E_t \left[ R_{t+1}^k / r_{t+1}^d \right]$. 

$55$
Figure 14: Impulse responses functions (in percentage) to a 1% positive bank productivity shock to the large banking sector in the model with the twelve bank, monopolistically-competitive banking sector. The solid blue line represents the response in the model with a traditional monetary policy rule, the dashed red line represents the response with the monetary policy rule that reacts to the interest rate spread $r^s/r^d$, and the dash-dot green line represents the response with the monetary policy rule that reacts to the expected credit spread $E_t [R^e_{t+1}/r_{t+1}^1\pi_{t+1}]$. 
Figure 15: Impulse responses functions (in percentage) to a 1% positive interbank lending shock to the large banking sector in the model with the twelve bank, monopolistically-competitive banking sector. The solid blue line represents the response in the model with a traditional monetary policy rule, the dashed red line represents the response with the monetary policy rule that reacts to the interest rate spread $r^e/r^l$, and the dash-dot green line represents the response with the monetary policy rule that reacts to the expected credit spread $E_t[r^e_{t+1}/r^l_{t+1} \pi_{t+1}]$. 
A The Model: Capital Good Producers, Households, Retailers, and Government

This appendix provides a complete description of the capital good producers, households, retailers. It also describes the necessary market clearing conditions.

A.1 Capital Goods Producers

At the end of period $t$, capital goods producers buy investment goods and existing, undepreciated capital $(1 - \delta)K_t$ from entrepreneurs and combine these to create capital for the next period, $K_{t+1}$, such that

$$K_{t+1} = I_t + (1 - \delta)K_t.$$  \hspace{1cm} (40)

New capital goods are produced from the retail final good according to the production function $I_t = \Phi(I_t/K_t)K_t$, such that $\Phi(I_t/K_t)K_t$ units of the final good are required to make $I_t$ new units of capital. $\Phi(\cdot)$ is increasing, concave, and $\Phi(x) \geq x$. Next period capital $K_{t+1}$ is then sold to entrepreneurs. From the capital goods producer’s first-order conditions, the equilibrium price of a unit of capital, $Q_t$ must be equal to the marginal cost of producing a new unit of capital,

$$Q_t = \Phi'(I_t/K_t).$$  \hspace{1cm} (41)

A.2 Households

There is a continuum of identical households normalized to one. Each household is infinitely lived, works, consumes, holds money, and saves by depositing its savings into a bank that pays a riskless rate of return. In each period, the households choose consump-
max \( \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k [\ln(C_{t+k}) + \xi \ln(M_{t+k}/P_{t+k}) + \zeta \ln(1 - H_{t+k})] \), \quad (42)

subject to the budget constraint

\[
P_t C_t + D_{t+1} \leq w_t H_t - T_t + \Pi^R_t + \Pi^I_t + \Pi^B_t + r^d_t D_t + (M_{t-1} - M_t), \quad (43)
\]

where \( P_t \) is the price level, \( w_t \) is the nominal wage, \( T_t \) is nominal lump-sum taxes, \( \Pi^R_t, \Pi^I_t \) and \( \Pi^B_t \) are the dividends received by ownership of retailers, capital goods producers, and banks, respectively, and \( r^d_t \) is the nominal return on period \( t-1 \) savings deposits \( D_t \).

The first-order conditions for consumption, labor, real money balances, and savings deposits are given by

\[
\beta^k C_{t+k}^{-1} P_{t+k}^{-1} = \lambda_{t+k} \quad (44)
\]

\[
\xi \frac{1}{1 - H_t} = \lambda_t \left( \frac{w_t}{P_t} \right) \quad (45)
\]

\[
\zeta \frac{1}{M_t} = \lambda_t - \mathbb{E}_t [\lambda_{t+1}] \quad (46)
\]

\[
\lambda_t = \mathbb{E}_t [\lambda_{t+1} r^d_{t+1}] \quad (47)
\]

The first-order equations can be rewritten as

\[
C_t^{-1} = \beta \mathbb{E}_t [C_{t+1}^{-1} r^d_{t+1} \pi^{-1}_{t+1}] \quad (48)
\]

\[
\xi \frac{1}{1 - H_t} = C_t^{-1} w_t \quad (49)
\]

\[
\frac{M_t}{P_t} = \zeta C_t \left( 1 - (r^d_{t+1})^{-1} \right)^{-1}. \quad (50)
\]
Equation (48) is the intereuler equation equating real consumption across time periods, equation (49) is the intraeuler equation determining the tradeoff between real consumption and leisure, and equation (50) is the optimal demand for nominal money balances.

A.3 Retailers

There exists a unit mass of monopolistically competitive retailers. Retailers purchase wholesale goods from entrepreneurs at the nominal wholesale price, $P^w_t$, and resell them at their own retail price. Let $Y_t(i)$ denote the quantity of output resold by retailer $i$ and let $P_t(i)$ denote the nominal price the retailer receives. Total final goods are a composite of individual retail goods

$$Y_t = \left[ \int_0^1 Y_t(i)^{(\eta-1)/\eta} \, di \right]^{\eta-1}, \tag{51}$$

and the corresponding aggregate price index is given by

$$P_t = \left[ \int_0^1 Y_t(i)^{1-\eta} \, di \right]^{1/\eta}, \tag{52}$$

where $\eta > 1$ is the elasticity of substitution in the retail market.

The standard monopolistic competition demand curve for individual retailers is

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\eta} Y_t. \tag{53}$$

The retailer chooses its sale price, $P_t(i)$, to maximize its profits taking as given the aggregate demand, price level, and wholesale good price.

To introduce price stickiness, we introduce Calvo pricing such that retailers are only free to change their price each period with probability $1-\theta$. Let $P^*_t(i)$ denote the price chosen by retailers who are able to change their price. Retailers choose $P^*_t(i)$ to maximize
expected profits, given by

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} \theta^k \lambda_{t+k} \left( \frac{P^*_t - P^w_{t+k}}{P_{t+k}} \right) Y^*_{t+k}(i) \right],$$

where $Y^*_{t+k}(i)$ is the demand in period $t + k$ given price $P^*_t$. The first-order conditions from maximizing expected profits can be rewritten such that

$$P^*_t = \frac{\eta}{\eta - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \lambda_{t+k} X_{t+k}^{-1} Y_{t+k} P^\eta_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \lambda_{t+k} Y_{t+k} P^\eta_{t+k}}.$$  \hspace{1cm} (55)

where $X_t$ is the optimal price markup such that $P_t = X_t P^w_t$.

To numerically implement the Calvo pricing equations without log-linearization, we summarize the optimal pricing equation with two recursive equations linked by the optimal pricing equation (55). To begin, note that the numerator $n_t$ and denominator $d_t$ in equation (55) can be written recursively as

$$n_t = P^\eta_t Y_t X_t^{-1} + \theta \mathbb{E}_t [\lambda_{t+1} n_{t+1}]$$ \hspace{1cm} (56)

$$d_t = Y_t P_t^{-1} + \theta \mathbb{E}_t [\lambda_{t+1} d_{t+1}].$$ \hspace{1cm} (57)

Therefore, the optimal pricing rule can be simply rewritten as

$$P^*_t = \frac{\eta}{\eta - 1} \frac{n_t}{d_t}.$$ \hspace{1cm} (58)

Let $\hat{P}_t = P^*_t / P_t$ and $F_{1,t} = P_t^{-\eta} n_t$. From equation (56), $F_{1,t}$ is written recursively as

$$F_{1,t} = Y_t X_t^{-1} + \theta \mathbb{E}_t [\lambda_{t+1} \lambda_{t+1}^\eta F_{1,t+1}] .$$ \hspace{1cm} (59)
Substituting $F_{1,t}$ into the rewritten optimal pricing rule (58) yields

$$P_t^* P_t^{-\eta} = \frac{\eta}{\eta - 1} P_t^{-\eta} n_t. \tag{60}$$

Let $F_{2,t} = P_t^* P_t^{-\eta} d_t = \hat{P}_t P_t^{1-\eta} d_t$. From equation (57), $F_{2,t}$ is written recursively as

$$F_{2,t} = Y_t \hat{P}_t + \theta \mathbb{E}_t \left[ \lambda_{t+1} \left( \frac{\hat{P}_t}{\hat{P}_{t+1}} \right) \pi_{t+1}^{\eta-1} F_{2,t+1} \right]. \tag{61}$$

Using the variables $F_{1,t}$ and $F_{2,t}$, the optimal pricing rule is simply

$$F_{2,t} = \frac{\eta}{\eta - 1} F_{1,t}. \tag{62}$$

Notice that $P_t^*$ is the same for all retailers in each period. Therefore, in each period, $1 - \theta$ retailers reset their price to $P_t^*$ and the aggregate price will evolve according to

$$P_t = \left[ \theta P_{t-1}^{1-\eta} + (1 - \theta)(P_t^*)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{63}$$

### A.4 Market Clearing

Finally, the model has two market clearing constraints

$$Y_t = C_t + C_t^c + G_t + \Phi \left( \frac{I_t}{K_t} \right) K_t + \mu \phi_t^y \tag{64}$$

$$H_t = H_t^p + H_t^d + H_t^b. \tag{65}$$

The first equation is the market clearing condition for the final good. Total output must equal total consumption (by households and existing entrepreneurs), government spending, total resources used to create new capital goods, and the bankruptcy cost. Because households are indifferent between production labor and banking labor, the second equa-
tion states that labor market clears simply when the total demand for labor equals the supply of labor.
B The Optimal Debt Contract

Recalling Sections 2.1 and 2.2, the derivation of the optimal debt contract with the Townsend (1979) costly state verification method used in BGG is essentially unchanged with the introduction of a monopolistically competitive banking sector.

To understand this derivation, recall that entrepreneur \( j \) faces an idiosyncratic shock \( \omega_j \) to their real ex-post gross aggregate return which is i.i.d across entrepreneurs and over time, such that the return to capital expenditures at the end of period \( t \) for entrepreneur \( j \) is \( \omega_j R^k_{t+1} \). Entrepreneurs with a draw of \( \omega_j \) greater than or equal to an endogenously determined cutoff \( \bar{\omega} \) will be able to pay back their loans. They receive \( \omega_j R^k_{t+1} Q_t K^j_{t+1} - R^b_{t+1} B^j_{t+1} \) and the banking sector receives \( R^b_{t+1} B^j_{t+1} \). Those entrepreneurs with draws lower than \( \bar{\omega} \) will default and receive nothing, leaving the banking sector with \( (1 - \mu) \omega_j R^k_{t+1} Q_t K^j_{t+1} \), where \( \mu \) is the monitoring cost (interpreted as the cost of bankruptcy). Therefore, the cut-off variable \( \bar{\omega} \) is determined by the following indifference condition:

\[
\bar{\omega}_{t+1} R^k_{t+1} Q_t K_{t+1} = E_t [r^b_{t+1} \pi^{-1}_{t+1}] B_{t+1}.
\] (66)

Given this, the net nominal entrepreneurial interest rate paid by entrepreneurs is the summation of principal and interest on repaid loans and assets received by defaulting entrepreneurs divided by total loans (total nominal repayments to banks per dollar borrowed), such that

\[
(1 - F_t(\bar{\omega}_{t+1})) r^b_{t+1} B_{t+1} + (1 - \mu) \bar{\phi}^y_{t+1} = r^e_{t+1} B_{t+1}.
\] (67)

Note that the real value of assets of defaulted entrepreneurs is defined as

\[
\phi^y_{t+1} = \Xi_t(\bar{\omega}_{t+1}) R^k_{t+1} Q_t K_{t+1},
\] (68)
such that the nominal value is $\bar{\phi}_t^y = P_t \phi_t^y$, and $\Xi_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_t} \omega dF_t(\omega)$.

Equation (67) states that the total net nominal payments to banks by entrepreneurs is equal to the nominal payments made by non-defaulting entrepreneurs (those who draw a high $\bar{\omega}$, and who are represented in the distribution by $1 - F_t(\bar{\omega}_{t+1})$), plus nominal assets of the defaulting entrepreneurs ($\bar{\phi}_t^y$), and minus the bankruptcy cost ($\mu \bar{\phi}_t^y$). In section 1.1.2, we show that the nominal aggregate marginal costs of loans $\bar{r}_{t+1}$ is given by

$$\bar{r}_{t+1} = \left( \frac{\eta_b - 1}{\eta_b} \right) (1 - F_t(\bar{\omega}_t + 1)) R_t^b + B_{t+1}^{-1}(1 - \mu) \bar{\phi}_t^y. \quad (69)$$

Combining (67) with (69) provides a very useful expression:

$$r_{t+1}^e = \frac{\eta_b}{\eta_b - 1} \bar{r}_{t+1} - \frac{1}{\eta_b} B_{t+1}^{-1}(1 - \mu) \bar{\phi}_t^y. \quad (70)$$

Equation (70) states that the marginal benefit of originating a loan to entrepreneurs ($r_{t+1}^e$) is equal to a markup over marginal cost ($\eta_b / (\eta_b - 1)) \bar{r}_{t+1}$ minus a correction term that arises from the fact that banks set markups based on gross interest rates and not on assets received through bankruptcy. In BGG, $r_{t+1}^e$ is simply the policy rate set by the central bank according to some policy rule. With our monopolistically competitive banking sector, $r_{t+1}^e$ is a function of marginal cost, bank market power, and monetary policy.

In an effort to determine the optimal debt contract in this setup, we will formulate the problem in real terms. Even though borrowing and lending are formulated in nominal terms, we define the optimal debt contract in real terms because entrepreneurs only care about their real payoff. Rewriting (67) allows us to define the real net entrepreneurial interest rate $R_{t+1}^e = E_t[r_{t+1}^e \pi_{t+1}^{-1}]$ such that:

$$(1 - F_t(\bar{\omega}_{t+1})) R_{t+1}^b B_{t+1} + (1 - \mu) \bar{\phi}_t^y = R_{t+1}^e B_{t+1}. \quad (71)$$
Combining (66), (68), and (71) yields the following equation:

$$
\mathbb{E}_t[(\Gamma_t(\bar{\omega}_{t+1}) - \mu \Xi_t(\bar{\omega}_{t+1})) R^k_{t+1} Q_{t+1} K_{t+1}] = \mathbb{E}_t[R^e_{t+1} B_{t+1}],
$$

(72)

where

$$
\Gamma_t(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1}[1 - F_t(\bar{\omega}_{t+1})] + \Xi_t(\bar{\omega}_{t+1}).
$$

(73)

$1 - \Gamma_t$ is the share of entrepreneurial earnings kept by the entrepreneur, and $\Gamma_t - \mu \Xi_t$ is the share of entrepreneurial earnings that are obtained by the banking sector. Equation (72) can be though of as the incentive compatibility constraint for the banking sector. The left hand side represents the aggregate marginal real return to entrepreneurial loans and the right hand side represents the aggregate marginal cost of entrepreneurial loans plus markups. With a perfectly competitive banking sector, equation (72) would be the zero-profit condition for the representative bank.

The optimal debt contract will be the loan that is optimal from the entrepreneur’s perspective; that is, the optimal loan will be the one that maximizes the entrepreneur’s expected share of the payoff subject to (72). Rather than writing the optimal contracting problem in terms of the loan amount and interest rate, it is simpler to write the problem in terms of capital purchased and the cutoff value:

$$
\max_{K_{t+1}, \bar{\omega}_{t+1}} \mathbb{E}_t\left[(1 - \Gamma_t(\bar{\omega}_{t+1})) R^k_{t+1} Q_{t+1} K_{t+1}\right],
$$

(74)

subject to

$$
\mathbb{E}_t[(\Gamma_t(\bar{\omega}_{t+1}) - \mu \Xi_t(\bar{\omega}_{t+1})) R^k_{t+1} Q_{t+1} K_{t+1}] = R^e_{t+1} (Q_{t+1} K_{t+1} - N_{t+1}).
$$

(75)

Rewriting (74) in terms of the capital-to-wealth ratio $k_{t+1} = Q_{t+1} K_{t+1} / N_{t+1}$ yields the fol-
ollowing first-order conditions:

\[
\bar{\omega}_{t+1} : \Gamma'_{t+1}(\bar{\omega}_{t+1}) = \lambda_{t+1}[\Gamma'(\bar{\omega}_{t+1}) - \mu \Xi'(\bar{\omega}_{t+1})] \quad (76)
\]

\[
k_{t+1} : \mathbb{E}_t \left\{ (1 - \Gamma_t(\bar{\omega}_{t+1})) + \lambda_{t+1}[\Gamma_t(\bar{\omega}_{t+1})
- \mu \Xi_t(\bar{\omega}_{t+1})] \left( \frac{R^k_{t+1}}{R^e_{t+1}} \right) - \lambda_{t+1} \right\} = 0. \quad (77)
\]

Combining (76) and (77) yields

\[
0 = \mathbb{E}_t \left\{ (1 - \Gamma_t(\bar{\omega}_{t+1})) \left( \frac{R^k_{t+1}}{R^e_{t+1}} \right) \right.
+ \frac{\Gamma'_{t+1}(\bar{\omega}_{t+1})}{\Gamma'_{t+1}(\bar{\omega}_{t+1}) - \mu \Xi'_{t+1}(\bar{\omega}_{t+1})} \left[ (\Gamma_t(\bar{\omega}_{t+1}) - \mu \Xi_t(\bar{\omega}_{t+1})) \left( \frac{R^k_{t+1}}{R^e_{t+1}} \right) - 1 \right] \left. \right\}. \quad (78)
\]

Rewriting the zero-profit condition constraint equation (72) in terms of \( k_{t+1} \) yields

\[
\Gamma_t(\bar{\omega}_{t+1}) - \mu \Xi_t(\bar{\omega}_{t+1}) = \left( \frac{R^e_{t+1}}{R^k_{t+1}} \right) \left( 1 - \frac{1}{k_{t+1}} \right). \quad (79)
\]

Equations (78) and (79) pin down the optimal debt contract variables \( \bar{\omega}_{t+1} \) and \( k_{t+1} \) (note that \( B_{t+1} = N_{t+1}(k_{t+1} - 1) \)). These two debt contract equations introduce the financial accelerator into the model.

Recall from the main text that we defined the distribution for \( \omega \), the idiosyncratic shock to capital expenditures, as a log-normal distribution with unit mean and standard deviation \( \sigma^F_t \). Following BGG, we assume that \( \ln(\omega_{t+1}) \sim N(-0.5(\sigma^F_t)^2, (\sigma^F_t)^2) \); given this assumption, BGG show that the distributions in the debt contract equations can be written as

\[
z_{t+1}(\bar{\omega}_{t+1}) = \frac{\ln(\bar{\omega}_{t+1}) + 0.5(\sigma^F_t)^2}{\sigma^F_t} \quad (80)
\]
\[ \Gamma_t(\bar{\omega}_{t+1}) = \Phi^N(z_{t+1}(\bar{\omega}_{t+1}) - \sigma^F_t) + \bar{\omega}_t + 1(1 - \Phi^N(z_{t+1}(\bar{\omega}_{t+1}))) \] (81)

\[ \Gamma'_t(\bar{\omega}_{t+1}) = 1 - \Phi^N(z_{t+1}(\bar{\omega}_{t+1})) \] (82)

\[ \Xi_t(\bar{\omega}_{t+1}) = \Phi^N(z_{t+1}(\bar{\omega}_{t+1}) - \sigma^F_t) \] (83)

\[ \Xi'_t(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1}f(\bar{\omega}_{t+1}) \] (84)

where \( \Phi^N(\cdot) \) is the standard Normal CDF and \( f \) is the pdf of the log-normal distribution.
C  Full vs. Reduced Form Financial Accelerator

In the derivation of the optimal debt contract, the following two equations pinned down the optimal cutoff value $\bar{\omega}_{t+1}$ and optimal capital-to-wealth ratio $k_{t+1} = Q_tK_{t+1}/N_{t+1}$:

$$0 = \mathbb{E}_t \left\{ (1 - \Gamma_t(\bar{\omega}_{t+1})) \left( \frac{R^k_{t+1}}{R^e_{t+1}} \right) + \frac{\Gamma'_{t+1}(\bar{\omega}_{t+1})}{\Gamma'_{t+1}(\bar{\omega}_{t+1}) - \mu \bar{z}_t(\bar{\omega}_{t+1})} \left[ (\Gamma_t(\bar{\omega}_{t+1}) - \mu \bar{z}_t(\bar{\omega}_{t+1})) \left( \frac{R^k_{t+1}}{R^e_{t+1}} \right) - 1 \right] \right\}$$  \hspace{1cm} (85)

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu \bar{z}_t(\bar{\omega}_{t+1}) = \left( \frac{R^e_{t+1}}{R^k_{t+1}} \right) \left( 1 - \frac{1}{k_{t+1}} \right).$$  \hspace{1cm} (86)

The joint solution can be summarized by two much simpler equations. Taking equation (85), we can rewrite this as

$$\frac{R^k_{t+1}}{R^e_{t+1}} = \rho(\omega_{t+1}).$$  \hspace{1cm} (87)

We can also rewrite equation (86) as

$$k_{t+1} = \Psi \left( \frac{R^k_{t+1}}{R^e_{t+1}} \right).$$  \hspace{1cm} (88)

Combining these two equations (by inverting equation (88)) yields the reduced form financial accelerator equation used in BGG, CD, and others who estimate and/or calibrate New-Keynesian Models with the financial accelerator:

$$\mathbb{E}_t \left[ \frac{R^k_{t+1}}{R^e_{t+1}} \right] = S(k_{t+1}).$$  \hspace{1cm} (89)

In particular, the non log-linearized equation used in those papers is given in the following form:

$$\mathbb{E}_t[R^k_{t+1}] = \bar{\psi} \mathbb{E}_t \left[ \left( \frac{N_{t+1}}{Q_tK_{t+1}} \right)^{-\bar{\psi}} R^e_{t+1} \right].$$  \hspace{1cm} (90)
where $\psi$ represents the financial accelerator in reduced form and $\psi$ is a scalar. $\psi$ is the elasticity of the credit spread with respect to the leverage ratio. Given that $\psi$ drops out of the log-linearized equation, authors in the financial accelerator literature simply calibrate or estimate the elasticity $\psi$.

The reduced form method is convenient because it eliminates $\omega_{t+1}$ from the model and all related distributions of $\omega$. The reduced form method, however, has two significant drawbacks. First, because $\omega_{t+1}$ is eliminated from the model, we cannot calculate the effects of shocks on the default rate (measured as $F_t(\omega_{t+1})$). Second, and more importantly, the reduced form method reduces the optimal debt contract to an exogenous elasticity. This has two important effects. The first is that it assumes the elasticity of the credit spread with respect to leverage ratio is fixed. This elasticity, $\psi$, however, is time-varying with a non-zero variance in our model. In our model without financial shocks, the standard deviation of the implicit elasticity is 0.17%, and in the model with financial shocks the standard deviation of the implicit elasticity is 1.08%. The standard deviation of the implicit elasticity increases when adding financial shocks primarily because the implicit elasticity exhibits a significant negative response to the shock to bank productivity. This finding confirms that the addition of marginal costs to bank production are the primary reason that the implicit elasticity changes when we introduce positive marginal costs of bank deposit and loan production.

Most importantly, fixing the elasticity of the credit spread with respect to the leverage ratio exogenously assumes that the financial intermediary has no effect on the elasticity; the finding that the implicit elasticity in our model varies with the bank productivity shock shows that this assumption is false. By including the full debt contract equations, (equations (103) - (109) in the model block of Appendix D), we set the elasticity endogenously and we allow for the elasticity to vary across our banking models. As a result, we are able to show that the introduction of a banking sector with positive marginal costs of bank deposit
and loan production reduces the elasticity of the credit spread with respect to the leverage ratio, and as a direct result the impact of the financial accelerator is reduced.
D The Full Model Equation Block

This appendix provides the full model equations with bank heterogeneity with twelve banks and all financial shocks. Below, we list the equations used for the first-order approximation solution method of the model. Note that variables without time subscripts denote steady state values. Notes are made for sets of equations that apply to each of the individual twelve banks.

Capital Producers

\[
K_{t+1} = I_t + (1 - \delta)K_t \quad (91)
\]

\[
Q_t = 1 + \chi \left( \frac{I_t}{K_t} - \delta \right) \quad (92)
\]

\[
AC_t = \left( \frac{\chi}{2} \right) \left( \frac{I_t}{K_t} - \delta \right)^2 \quad (93)
\]

Entrepreneurs

\[
B_{t+1} = Q_tK_{t+1} - N_{t+1} \quad (94)
\]

\[
Y_t = \exp(A_t)K_t^\alpha(H^p_t)^{(1 - \alpha)\Omega} \quad (95)
\]

\[
R^k_{t+1} = \frac{\alpha X_t^{-1} \left( \frac{Y_{t+1}}{K_t} \right) + Q_{t+1}(1 - \delta)}{Q_t} \quad (96)
\]

\[
W_t = \Omega(1 - \alpha) \left( \frac{Y_t}{H^p_t} \right) X_t^{-1} \quad (97)
\]

\[
W_t^e = (1 - \Omega)(1 - \alpha) \exp(A_t)K_t^\alpha(H^p_t)^{(1 - \alpha)\Omega}X_t^{-1} \quad (98)
\]

\[
V_t = R^k_tQ_{t-1}K_t - r_t^e\pi_t^{-1}B_t - \mu \phi_t^y \quad (99)
\]

\[
\phi_t^y = \Xi_{t-1}(\bar{\omega}_t)R^k_tQ_{t-1}K_t \quad (100)
\]

\[
N_{t+1} = \gamma V_t + W_t^e \quad (101)
\]
\[ C_t^* = (1 - \gamma)V_t \] (102)

**Debt Contract**

\[ \mathbb{E}_t[r_{t+1}^e \tau_{t+1}^{-1} B_{t+1}] = \mathbb{E}_t[(\Gamma_t(\omega_{t+1}) - \mu \Xi_t(\omega_{t+1})) R_{t+1}^k Q_t K_{t+1}] \] (103)

\[ 0 = \mathbb{E}_t \left\{ (1 - \Gamma_t(\omega_{t+1})) \left( \frac{R_{t+1}^k}{r_{t+1}^e \tau_{t+1}^{-1}} \right) \right. \\
\left. + \frac{\Gamma_{t+1}'(\bar{\omega}_{t+1})}{\Gamma_{t+1}'(\bar{\omega}_{t+1}) - \mu \Xi_{t+1}'(\bar{\omega}_{t+1})} \left[ (\Gamma_t(\omega_{t+1}) - \mu \Xi_t(\omega_{t+1})) \left( \frac{R_{t+1}^k}{r_{t+1}^e \tau_{t+1}^{-1}} \right) - 1 \right] \right\} \] (104)

\[ z_{t+1} = \frac{\ln(\bar{\omega}_{t+1}) + 0.5(\sigma_t^F)^2}{\sigma_t^F} \] (105)

\[ \Xi_t(\omega_{t+1}) = \Phi^N(z_{t+1} - \sigma_t^F) \] (106)

\[ \Gamma_t(\bar{\omega}_{t+1}) = \Phi^N(z_{t+1} - \sigma_t^F) + \bar{\omega}_{t+1}(1 - \Phi^N(z_{t+1})) \] (107)

\[ \Xi_{t+1}'(\omega_{t+1}) = \left( \frac{1}{\sigma_t^F \sqrt{2\pi}} \right) \exp \left( -\frac{(\ln(\bar{\omega}_{t+1}) + 0.5(\sigma_t^F)^2)^2}{2(\sigma_t^F)^2} \right) \] (108)

\[ \Gamma_{t+1}'(\bar{\omega}_{t+1}) = 1 - \Phi^N(z_{t+1}) \] (109)

**Households**

\[ C_t^{-1} = \beta \mathbb{E}_t[C_{t+1}^{-1} r_{t+1}^d \tau_{t+1}^{-1}] \] (110)

\[ \lambda_t = \beta \left( \frac{C_t}{C_{t-1}} \right) \] (111)

\[ \xi \frac{1}{1 - H_t} = C_t^{-1} W_t \] (112)
Retailers

\[ F_{1,t} = Y_t X_t^{-1} + \theta \mathbb{E}_t [\lambda_{t+1} \pi^\eta_{t+1} F_{1,t+1}] \]  
\[ F_{2,t} = Y_t \hat{\rho}_t + \theta \mathbb{E}_t \left[ \lambda_{t+1} \left( \frac{\hat{\rho}_t}{\hat{\rho}_{t+1}} \right) \Pi_{t+1} \eta_{t+1} F_{2,t+1} \right] \]  
\[ F_{2,t} = \frac{\eta}{\eta - 1} F_{1,t} \]  
\[ \hat{\rho}_t = \left( \frac{1 - \theta \pi_t^{\eta-1}}{1 - \theta} \right)^{\frac{1}{\eta-1}} \]

Banking Sector:

The first six equations hold for all \( i = 1, \ldots, 12 \):

\[ \mathbb{E}_t [\lambda_{t+1} r_{t+1}^d (i) \pi_{t+1}^{-1}] = \left( \frac{\eta_d}{\eta_d + 1} \right) \left( \mathbb{E}_t [\lambda_{t+1} r_{t+1}^L (i) \pi_{t+1}^{-1}] - \gamma^d (i) w_t \right) \]  
\[ \mathbb{E}_t [\lambda_{t+1} r_{t+1}^e (i) \pi_{t+1}^{-1}] = \mathbb{E}_t [\lambda_{t+1} r_{t+1}^L (i) \pi_{t+1}^{-1}] + \gamma^e (i) w_t \]  
\[ \mathbb{E}_t [r_{t+1}^b (i) \pi_{t+1}^{-1}] = \mathbb{E}_t \left[ \frac{r_{t+1}^e (i) \pi_{t+1}^{-1} - B_{t+1}^{-1} (1 - \mu) \Xi_t (\tilde{\omega}_{t+1}) R_{t+1}^K Q_t K_{t+1}}{\Gamma_t (\tilde{\omega}_t) \left( \frac{\eta_b - 1}{\eta_b} \right)} \right] \]  
\[ D_{t+1} (i) = \xi (i) D_{t+1} \left( \frac{r_{t+1}^d (i)}{r_{t+1}^d (i)} \right)^{-\eta_d} \]  
\[ B_{t+1} (i) = \xi (i) B_{t+1} \left( \frac{r_{t+1}^b (i)}{r_{t+1}^b (i)} \right)^{\eta_b} \]  
\[ L_y (i) = B_y (i) - D_y (i) \]  
\[ H^y_t = \sum_{i=1}^{12} \gamma^y (i) B_{t+1} (i) \]  
\[ H^d_t = \sum_{i=1}^{12} \gamma^d (i) D_{t+1} (i) \]  
\[ r_{t+1}^d = \left( \sum_{i=1}^{12} \xi (i) \left( r_{t+1}^d (i) \right)^{\eta_d - 1} \right)^{\frac{1}{\eta_d - 1}} \]
\( \bar{r}_{t+1} = \left( \sum_{i=1}^{12} \zeta(i) \left( \bar{r}_{t+1}^i \right)^{1-\eta_b} \right)^{\frac{1}{1-\eta_b}} \) (126)

\( r_t^b = \left( \sum_{i=1}^{12} \zeta(i) \left( r_t^{b+1}^i \right)^{1-\eta_b} \right)^{\frac{1}{1-\eta_b}} \) (127)

\[ \mathbb{E}_t [ r_t^{cb+1} \pi_t^{t-1} ] = \mathbb{E}_t \left[ \left( \frac{\eta_b}{\eta_b - 1} \right) \bar{r}_{t+1}^c \pi_t^{t-1} - \left( \frac{1}{\eta_b - 1} \right) B_{t+1}^{t-1} \left( 1 - \mu \right) \phi_t^y \right] \] (128)

\( L_t = \sum_{i=1}^{12} L_{t+1}^i \) (129)

**Market Clearing**

\[ Y_t = C_t + I_t + G_t + C_t^e + \mu \phi_t^y \] (130)

\[ D_t = B_t \] (131)

\[ H_t = H_t^p + H_t^b + H_t^d \] (132)

**Monetary Policy**

\[ \log \left( \frac{r_t^b}{r_{t}^{cb}} \right) = \rho_t \log \left( \frac{r_{t-1}^b}{r_t^{cb}} \right) + \psi_\pi \log \left( \frac{\pi_t}{\pi} \right) + \psi_y \log \left( \frac{y_t}{y} \right) - mpshock_t \] (133)

**Shocks**

The first two equations hold for all \( i = 1, \ldots, 12 \):

\[ \log \left( \frac{r_t^i}{r_t^{cb}} \right) - \log \left( \frac{r_{t-1}^i}{r_{t-1}^{cb}} \right) = \rho_t \log \left( \frac{r_{t-1}^i}{r_{t-1}^{cb}} \right) - \log \left( \frac{r_{t-1}^i}{r_{t-1}^{cb}} \right) + e_{t+1}^i \] (134)

\[ \gamma_t^b(i) - \gamma_t(i) = \rho_t \left( \gamma_{t-1}^b(i) - \gamma_t^b(i) \right) + e_t^b \] (135)

\[ A_t = \rho_a A_{t-1} + e_t^a \] (136)
\[ G_t - G = \rho g(G_{t-1} - G) + e_t^G \]  
(137)

\[ \sigma_t^F - \sigma^F = \rho \sigma(\sigma_{t-1}^F - \sigma^F) + e_t^\sigma \]  
(138)

\[ mpshock_t = \rho mmpshock_{t-1} + e_m \]  
(139)

**Miscellaneous Financial Variables**

\[
Default_t = 1 - \Gamma_{t-1}^{\rho}(\bar{\theta}_{t+1})
\]  
(140)

\[
s_t+1 = \mathbb{E}_t \left[ \frac{R_{t+1}^c}{r_{t+1}^c \pi_{t+1}^{-1}} \right]
\]  
(141)

\[
EFP_t = \mu \phi^\gamma B_{t-1}^{-1}
\]  
(142)

\[
\bar{s}_{t+1} = \frac{r_{t+1}^{r^d}}{r_{t+1}^d}
\]  
(143)

In much of our qualitative and quantitative analysis in the paper, we focus on the following twelve variables:

1. Output \(Y_t\)

2. Investment \(I_t\)

3. Consumption \(C_t\)

4. Inflation \(\pi_t\)

5. Capital price \(Q_t\)

6. Wage \(W_t\)

7. Net worth \(N_t\)

8. Default rate \(Default_t\)
9. External finance premium $EFP_t$

10. Policy rate $r^{ch}_t$

11. Credit spread $s_{t+1}$

12. Interest Rate Spread $\tilde{s}_{t+1}$